This completes the proof of the complete reducibility of all (finite and infinite dimensional) representations of the rotation group or unimodular unitary group. It is clear also that the same consideration applies for all closed groups, i.e., whenever the invariant integral $\int dR$ converges.

The result for the inhomogeneous Lorentz group is: For every positive numerical value of $P$, the representations of the little group can be, in an irreducible representation, only the $D^{(0)}$, $D^{(\frac{1}{2})}$, $D^{(1)}$, $\cdots$, both for $P_+$ and for $P_-$. All these representations have been found already by Majorana and by Dirac and for positive $P$ there are none in addition to these.

### B. Representations of the two dimensional Euclidean group

This group, as pointed out in Section 6, has a great similarity with the inhomogeneous Lorentz group. It is possible, again[24], to introduce "momenta", i.e. variables $\xi$, $\eta$ and $v$ instead of the $\zeta$ in such a way that

$$t(x, y)\varphi(p_0, \xi, \eta, v) = e^{i(x\xi+y\eta)}\varphi(p_0, \xi, \eta, v). \tag{90}$$

Similarly, one can define again operators $R(\beta)$

$$R(\beta)\varphi(p_0, \xi, \eta, v) = \varphi(p_0, \xi', \eta', v), \tag{91}$$

where

$$\begin{aligned} \xi' &= \xi \cos\beta - \eta \sin\beta, \\ \eta' &= \xi \sin\beta + \eta \cos\beta. \end{aligned} \tag{91a}$$

Then $\delta(\beta)R(\beta)^{-1} = S(\beta)$ will commute, on account of (71c), with $t(x, y)$ and again contain $\xi$, $\eta$ as parameter only. The equation corresponding to (57a) is

$$\delta(\beta)\varphi(p_0, \xi, \eta, v) = \sum_w S(\beta)_{vw}\varphi(p_0, \xi', \eta', w). \tag{92}$$

One can infer from (90) and (92) again that the variability domain of $\xi$, $\eta$ can be restricted in such a way that all pairs $\xi$, $\eta$ arise from one pair $\xi_0$, $\eta_0$ by a rotation, according (91a). We have, therefore two essentially different cases:

a.) $\xi^2 + \eta^2 = \Xi \neq 0$

b.) $\xi^2 + \eta^2 = \Xi = 0,$ i.e. $\xi = \eta = 0.$

The positive definite metric in the $\xi$, $\eta$ space excludes the other possibilities of section 6 which were made possible by the Lorentzian metric for the momenta, necessitated by (55).

Case b) can be settled very easily. The "little group" is, in this case, the group of rotations in a plane and we are interested in one and two valued irreducible representations. These are all one dimensional ($e^{is\beta}$)

$$S(\beta) = e^{is\beta} \tag{93}$$

where $s$ is integer or half integer. These representations were also all found by Majorana and by Dirac. For $s = 0$ we have simply the equation $\Box\varphi = 0$,