Analytical Physics Equations from 2nd Edition Purcell
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Chapter One: The Electric Field

(1.1) **Coulomb's Law –** *Two stationary electric charges repel or attract one another with a force given by:*

$$\overrightarrow{F_2} = \frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

(1.9) **Potential Energy of a System of Charges –** *Factor of ½ is to account for counting each pair twice. Superposition of the potential energies of each pair of particles (Total of N charges).*

$$U = \frac{1}{2}\sum_{j=1}^{N}\sum_{k\neq j}\frac{q_k q_j}{r_{jk}}$$

(1.21) **Gauss's Law –** *The electric flux through a surface enclosing some charge is equal to four pi times the enclosed charge, regardless of the geometry.*

$$\int_{Surface}\vec{E}\cdot d\vec{a} = 4\pi q_{enc}$$

(1.26) **Electric field from a line of Charge –** Wire with linear charge distribution lambda extending infinitely in either direction a distance r away points in the radial direction outward with magnitude:

$$\vec{F}_2 = \frac{\lambda}{r^2}\hat{r}$$

(1.38) **The Energy of an Electric Field -** *The potential energy U of a system of charges which is the total work required to assemble the system, can be calculated from the electric field itself by simply assigning an amount of energy to every volume element and integrating over all space where there is electric field.*

$$U = \frac{1}{2}\int_{\substack{Entire\\Field}} E^2 dv$$

Chapter Two: The Electric Potential

(2.6) **Electric Potential** – *Definition, independent of path:*

$$\phi_{21} = -\int_{p_i}^{p_f} E \cdot d\vec{s} \qquad E = -\nabla \phi$$

(2.20) **Electric Potential of Uniformly Charged Disk** l*ocated on the xz plane with uniform surface charge density sigma.*

$$\phi(0,\pm y,0) = 2\pi\sigma\left(\sqrt{y^2 + a^2} \mp y\right)$$

(2.39) **Gauss's Differential Form** – *relation between charge density and electric field.*

$$\nabla \cdot E = 4\pi\rho$$

(2.56) **Constraints of electrostatic field -** *Vector operator is known as Laplacian – this follows from impossibility of constructing a confinement of a charge using just electrostatic fields. Also, the curl of the electric field must be zero everywhere* *<u>there is no charge</u>.*

$$\nabla^2\phi = 0 \qquad \nabla \times E = 0 \qquad \int_{\substack{closed\\loop}} \vec{E} \cdot d\vec{s} = 0$$

Chapter Three: Electric Fields Around Conductors

(3.11) **Capacitance of conductor**

$$Q = C\Delta\phi$$

(3.16) **Capacitance of various geometries:**

(1) Area of plates is A, separation of plates is s
(2) Radius of inner cylinder is a, outer is b, length L
(3) Radius of inner sphere is a, outer is b

$$C_{\substack{parallel\\plates}} = \frac{A}{4\pi s} \qquad C_{\substack{concentric\\cylinders}} = \frac{L}{\log\left(\frac{b}{a}\right)} \qquad C_{\substack{concentric\\spheres}} = \frac{ab}{b-a}$$

Electric field between parallel plate capacitors: $\mathrm{E} = \frac{\Delta \mathrm{V}}{\mathrm{s}} = \frac{[\text{stat-volts}]}{[\text{cm}]} = 4\pi\sigma$

(3.25) **Energy in a capacitor –** *Can be expressed in terms of capacitance and voltage or electric field and volume.*

$$U = \frac{1}{2}C\phi^2 = \frac{E^2}{8\pi}V$$

The electric field is always zero in a conductor and perpendicular to the surface when it leaves the conductor. Capacitances add inversely in series and directly in parallel.

(4.5) **Current Density –** *Assume some number of carriers of charge in a current are electrons moving at an average velocity. The current density is given as:*

$$\vec{J}_e = -eN_e\vec{u}_e$$

(4.6) **Properties of the Current –** *An electric current going through an area is defined as the surface integral. The divergence of the current density is also related to the charge density because of charge conservation. Also note that in (2) the divergence is zero when there is no time dependence.*

$$I = \int_s \vec{J}\cdot d\vec{a} \qquad \nabla\cdot\vec{J} = \frac{\partial\rho}{\partial t}$$

(4.12) **Resistance of Material –** *The resistance of a material can be expressed in terms of Ohm's law (1), a constant electric field (2), conductivity (3), and resistivity (4). In each case the cross sectional area is A, the length is L, the current density is J, and the voltage is V.*

$$R = \frac{V}{I} = \frac{LE}{AJ} = \frac{L}{A\sigma} = \frac{\rho L}{A}$$

(4.21) **Relative Populations –** *The ratio of populations of energy levels at thermal equilibrium is related to the energy difference, the temperature and Boltzman's constant (k = 1.381e-16 erg/Kelvin).*

$$\frac{p_1}{p_2} = e^{\frac{-\Delta E}{kT}}$$

(4.22) **Adding Resistors** – *Resistors add directly in series and inversely in parallel*

$$R_{series} = \sum_{i=1}^{N} R_i \qquad \frac{1}{R_\parallel} = \sum_{i=1}^{N}\frac{1}{R_i} \qquad R_{\parallel\, N=2} = \frac{R_1R_2}{R_1+R_2}$$

(4.23) **Circuit laws –** *These laws can be used to simplify complex circuits.*

1. The current through each element must equal the voltage across that element divided by the resistance of the element.

2. At a node of the network, a point where three or more connect wires meet, the algebraic sum of the currents into the node must be zero from charge conservation.

3. The sum of the potential differences taken in order around a loop of the network is zero.

(4.24) **Power dissipated by a current –** *When current flows through a resistor, some power is lost to heat.*

$$P = I^2R = \frac{V^2}{R} = VI$$

(4.33) **RC Circuits –** *When a switch is thrown allowing a charged capacitor to dissipate its energy, the system is characterized by a time constant tau in the following exponential equation.*

$$\tau = RC \qquad Q(t) = CV_0 e^{\frac{-t}{\tau}} \qquad I(t) = \frac{V_0}{R}e^{\frac{-t}{\tau}}$$

Chapter Five: The Fields of Moving Charges

(5.1) **The force on a moving charge –** *A test charge moving in electric and magnetic fields experiences a force (where c, the speed of light, is 2.998e10 cm/s in cgs).*

$$\vec{F} = q\vec{E} + \frac{q}{c}\vec{v} \times \vec{B}$$

(5.12) **The field from a moving charge –** *Let theta prime denote the angle between this radius vector and the velocity of the charge Q, which is moving in the positive x prime direction in the primed frame.*

$$E' = \frac{Q}{r'^2} \frac{1-\beta^2}{(1-\beta^2 \sin^2 \theta)^{3/2}}$$

Chapter Six: The Magnetic Field

(6.3) **Magnetic field from wire –** *The field direction is given by the right hand rule and is direction in the theta direction.*

$$\vec{B} = \frac{2I}{rc}\hat{\theta}$$

(6.7) **The force on wires with parallel currents –** *The attractive / repulsive force on a length l of the second wire from another. The sign is reversed if the currents are anti-parallel.*

$$F_{21} = \frac{2I_1 I_2}{c^2 r}$$

(6.10) **Ampere's Law**

$$\int \vec{B} d\vec{s} = \frac{4\pi I}{C}$$

(6.20) **Magnetic Vector Potential**

$\nabla \times \left(\nabla \times \vec{A}\right) = \frac{4\pi \vec{J}}{c}$	$B = \nabla \times A$	$\vec{A}(x_1, y_1, z_1) = \frac{1}{c}\int \frac{\vec{J}(x_1, y_1, z_1) dv_2}{r_{12}}$

(6.38) **Force on any charge carrying wire –** *We can integrate over the entire length of the wire in order find the magnetic field.*

$$dB = \frac{I d\vec{l} \times \hat{r}}{cr^2}$$

(6.42) **Fields of Rings and Coils –** *The following are derived from Biot-Savart: current carrying ring of radius b on axis (1), inside solenoid (2), where n is turns per unit length.*

$$B_z = \frac{2\pi b^2 I}{c(b^2 + z^2)^{3/2}} \qquad B_z = \frac{4\pi In}{c}$$

(6.46) **Magnetic Change at a Current Sheet –** *A magnetic field is changed when you move from one side of a current sheet to the other based on how much current is going through the sheet.*

$$\Delta B = \frac{4\pi J}{c}$$

(6.60) **Transformation of Fields –** The following give the transformation to a frame moving at velocity beta in a direction oriented with the associated fields in the following manner. To transform from a moving frame, switch the primes and the plus/minus signs.

$$\vec{E}'_{perp} = \gamma(\vec{E}_{perp} + \beta \times \vec{B}_{perp}) \qquad \vec{B}'_{perp} = \gamma(\vec{B}_{perp} - \beta \times \vec{E}_{perp})$$

$$\vec{E}'_{\parallel} = \vec{E}_{\parallel} \qquad \vec{B}'_{\parallel} = \vec{B}_{\parallel}$$

(6.64) **The Hall Effect –** *Suppose there are m mobile charge carriers per cubic centimeters and denote the charge of each by q. Then the current density is nqv. If we now substitute in for the average velocity, we can relate this transverse field to the directly measurable quantities J and B:*

$$E_t = \frac{-\vec{J} \times \vec{B}}{nqc}$$

Chapter Seven: Electromagnetic Induction

(7.7) **Flux and Induced EMF –** *A coil with N turns that has a change in flux experiences an induced voltage. The third formulation is an equivalent formulation.*

$$\Phi = \int \vec{B} \cdot d\vec{a} \qquad \varepsilon = -\frac{1}{c}\frac{d\Phi}{dt} \qquad \nabla \times E = -\frac{1}{c}\frac{\partial B}{\partial t}$$

(7.38) **Mutual Inductance and Reciprocity –** *Two circuits in a fixed configuration generate some magnetic field and also have some area through which a changing magnetic field can flux. The mutual inductance and the accompanying equivalence – for any configuration - of one inductance of one circuit to the other are given below.*

$$\varepsilon_{21} = -M_{21}\frac{dI}{dt} \qquad M_{21} = M_{12}$$

(7.54) **Self Induction -** *When the current is changing, there is a change in the flux through a circuit itself, and there is an electromotive force that is induced whatever the source. This is called the self inductance and is given by the following expressions: general form (1), toroidal coil of rectangular cross section, inner radius a, height h, and outer radius b (2), and a solenoid of radius a (3).*

$$\varepsilon_{11} = -L\frac{dI}{dt} \qquad L_{torus} = \frac{2N^2 h}{c^2}\ln\left(\frac{b}{a}\right) \qquad L_{solenoid} = \frac{4\pi^2 N^2 a^2}{c^2}$$

(7.66) **Transient Behavior of RL Circuits –** *the current in circuit with a resistor and inductor disconnected from a battery that has created a current within the circuit is given by an exponential expression with time constant R/L.*

$$\tau = R / L \qquad I(t) = I_0 e^{-\tau t}$$

(7.70) **Energy in Magnetic Field –** The inductor is the analog to the electric energy stored in a capacitor.

$$U = \frac{1}{2}LI^2 = \frac{1}{8\pi}\int B^2 dv$$

Chapter Eight: Alternating Current Circuits

(8.10) Resonant Circuit – *a LRC oscillates in a damped manner according to the following equations.*

$$V(t) = e^{-\alpha t}\left(A\cos \omega t + B\sin \omega t\right) \qquad \omega^2 = \frac{1}{LC} - \frac{R^2}{4L^2}$$

$$I(t) = AC\omega\left(A\cos \omega t + B\sin \omega t\right)e^{-\alpha t}$$

(8.15) Resonant Frequency in LC circuit – *in a circuit without a resistor the circuit oscillates with a resonant frequency.*

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

(8.19) Driven LR circuit – *A sinusoidal voltage epsilon is given to a circuit containing a resistor and an inductor experiences the following current:*

$$\cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} \qquad I_0 = \frac{\varepsilon_0}{\sqrt{R^2 + \omega^2 L^2}} \qquad I(t) = I_0 \cos(\omega t - \phi)$$

(8.41) Driven LRC circuit – Similarly, we can express the behavior of an LRC circuit in the same way.

$$I(t) = \frac{\varepsilon_0}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)}}\cos(\omega t - \phi) \qquad \tan\phi = \frac{1}{R\omega C} - \frac{1}{\omega C}$$

(8.57) Networks in Alternating Circuits – *We can reduce the components in an AC circuit into their complex impedances and then add the complex values as resistors obeying the circuit laws for DC circuits. The impedance of common elements:*

$$Z_R = R \qquad Z_C = \frac{1}{C\omega i} \qquad Z_L = L\omega i$$

(8.60) Power and Energy – *Since the of cos-squared over many cycles is ½ and we have the voltage proportional to V/R, the following expressions explain how energy is dissipated by an AC circuit.*

$$\overline{P} = RI^2 = \frac{V_0^2}{2R} = \frac{V_{rms}^2}{R}$$

Chapter Nine: Maxwell's Equations

(9.15) Maxwell's Equations – *The relationship between the electric, magnetic, charge density rho, and current density J is given by the following equations.*

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial B}{\partial t} \qquad \nabla \times \vec{B} = -\frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

$$\nabla \cdot E = 4\pi\rho \qquad \nabla \cdot B = 0$$

(9.27) Poynting Vector – *The power density S is aligned with **E** x **B**, in the direction of propagation.*

$$S = \frac{\overline{E^2}c}{4\pi} = \frac{E_0^2 c}{8\pi}$$

(9.33) **EM Wave Transformation** – *If a frame is moving in the x-direction with respect to a frame of an observed EM wave propagating in x, then the electric and magnetic fields in y and z will be:*

$$E_y^{/} B_z^{/} = E_0 \sqrt{\frac{1-\beta}{1+\beta}} \qquad B_z^{/} = E_0 \sqrt{\frac{1-\beta}{1+\beta}} \qquad \lim_{\beta \to c}\left(E_y^{/} = B_z^{/} = 0\right)$$

Chapter Ten: Electric Fields in Matter

(10.10) **The Dipole Moment** – *Even when the net charge of a distribution is zero, it can still create an electric field because of the distribution of internal charges. This dipole distribution is independent of where the coordinates are chose (always points from –q to +q).*

$$\vec{p} = \int \vec{r}\rho dv = \sum_i q_i \vec{r}_i$$

(10.15) **Electric and Potential Fields –** *A dipole distribution creates both a potential and expressed electric field. Since the electric field is somewhat complex, it is broken into radial and angular components, where theta is measured from the alignment of the dipole vector.*

$$\phi(\vec{r}) = \frac{\hat{r}\cdot\vec{p}}{r^2} \qquad E_r = \frac{2p}{r^3}\cos\theta \qquad E_\theta = \frac{p}{r^3}\sin\theta$$

(10.18) **Torque on Dipole –** *A dipole will want to align itself with an electric field. Thus, the torque exerted on a dipole and its energy in a field is given by:*

$$\vec{N} = \vec{p}\times\vec{E} \qquad U = -\vec{p}\cdot\vec{E}$$

(10.23) **Force on Dipole** – *A dipole will not experience a force in a constant electric field. It will, however, experience a force in a non-constant field. Note: we have a vector operator working on E, not p times the gradient of E.*

$$\vec{F} = \left(\vec{p}\cdot\nabla\right)\vec{E}$$

(10.57) **Electric Susceptibility –** *The polarization of matter can be characterized by a polarization density, which is the dipole moment per unit volume. This P is also the product of the susceptibility times the electric field.*

$$\vec{P} = N\alpha\vec{E} = \vec{p}V \qquad \chi_e = \frac{\vec{P}}{\vec{E}} = N\alpha$$

(10.58) **Dielectric Constant –** *A dielectric in an electric field causes the electric field in the region to decrease by a factor the constant, while the effective capacitance increases by the constant.*

$$\varepsilon = 1 + 4\pi\chi_e \qquad C = C_0\varepsilon \qquad E = \frac{E_0}{\varepsilon}$$

(10.61) **Bound-Charge Current –** *A changing polarization of a substance actually creates a movement of charge within a polarized substance.*

$$\vec{J} = \frac{d\vec{P}}{dt}$$

(10.70) **EM Wave in Dielectric –** *If there is an EM wave within a dielectric, it moves slower than if it were in a vacuum. The following constraints apply:*

$$v = \frac{\omega}{k} = \frac{c}{\varepsilon} \qquad B_0 = \sqrt{\varepsilon}E_0$$

(11.9) **Magnetic Dipole Moment –** *A magnetic dipole moment is a vector whose direction is normal to a loop of current – i.e. aligned with the directed area vector a.*

$$\vec{m} = \frac{\vec{I}}{c}\vec{a}$$

(11.10) **Magnetic Dipole Potential –** *We again define a potential function for the field produced by a magnetic dipole.*

$$A = \frac{\vec{m}\times\hat{r}}{r^2}$$

(10.15) **Field, Force, Torque –** *The following are the same algebraically as for a dipole configuration.*

$$\vec{N} = \vec{m}\times\vec{E} \qquad U = -\vec{m}\cdot\vec{E} \qquad \vec{F} = \left(\vec{m}\cdot\nabla\right)\vec{E}$$

$$E_r = \frac{2\vec{m}}{r^3}\cos\theta \qquad E_\theta = \frac{\vec{m}}{r^3}\sin\theta$$

(11.32) **Atom Moments –** *The change in the magnetic moment for an atom of mass M from an applied magnetic field is:*

$$\Delta\vec{m} = \frac{-q^2r^2}{4Mc^2}\vec{B}_1$$

(11.39) **Magnetic Susceptibility –** *An applied electric field creates a magnetic polarization, which in turn creates a current density around the edge of the magnetized substance.*

$$\vec{M} = V\vec{m} = \chi_m\vec{B} \qquad \vec{J} = \vec{M}c = c\nabla\times\vec{M}$$

(11.54) **The H-Field –** *A field is created abstractly that is produced from currents under our control which satisfies Ampere's Law:*

$$\int_C \vec{H}\cdot d\vec{l} = \frac{4\pi}{c}I$$

(11.57) **True B-Field –** *We want to know the true magnetic field within a substance, however, so we use the relationship described in 11.39 (replacing B with H) and then we can derive an expression in terms of the susceptibility as well as the permeability, mu.*

$$\vec{B} = \vec{H} + 4\pi\vec{M} = (1+4\pi\chi_m)\vec{H} = \mu\vec{H}$$