Probabilities and Data

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COLLEGE OF INFORMATION STUDIES

Slides adapted from Dave Blei and Lauren Hannah

Roadmap

- What are probabilities
  - Discrete
  - Continuous
- How to manipulate probabilities
- Properties of probabilities

Preface: Why make us do this?

- Probabilities are the language we use to describe data
- A reasonable (but geeky) definition of data science is how to get probabilities we care about from data
- Later classes will be about how to do this for different probability models and different types of data
- But first, we need key definitions of probability

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- But first, we need key definitions of probability
- So pay attention!
- Also, ya'll need to get your environments set up

Outline

Card problem (from David MacKay)

- There are three cards
  - Red/Red
  - Red/Black
  - Black/Black

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- I go through the following process.
  - Close my eyes and pick a card
  - Pick a side at random
  - Show you that side
- Suppose I show you red. What's the probability the other side is red too? (Write down your answer!)

Random variable

- Probability is about *random variables.*
- A random variable is any "probabilistic" outcome.
- For example,
  - The flip of a coin
  - The height of someone chosen randomly from a population
- We'll see that it's sometimes useful to think of quantities that are not strictly probabilistic as random variables.
  - The temperature on 11/12/2013
  - The temperature on 03/04/1905
  - The number of times "streetlight" appears in a document

Random variable

- Random variables take on values in a *sample space*.
- They can be *discrete* or *continuous*:
  - Coin flip: $\{H, T\}$
  - Height: positive real values $(0, \infty)$
  - Temperature: real values $(-\infty, \infty)$
  - Number of words in a document: Positive integers $\{1, 2, \ldots\}$
- We call the outcomes *events*.
- Denote the random variable with a capital letter; denote a realization of the random variable with a lower case letter.
- E.g., X is a coin flip, x is the value (H or T) of that coin flip.

Discrete distribution

- A discrete distribution assigns a probability to every event in the sample space
- For example, if X is an (unfair) coin, then

$$\begin{aligned} P(X=H) &= 0.7 \\ P(X=T) &= 0.3 \end{aligned}$$

- The probabilities over the entire space must sum to one

$$\sum_x P(X=x) = 1$$

- And probabilities have to be greater than 0
- Probabilities of disjunctions are sums over part of the space. E.g., the probability that a die is bigger than 3:

$$P(D>3) = P(D=4) + P(D=5) + P(D=6)$$

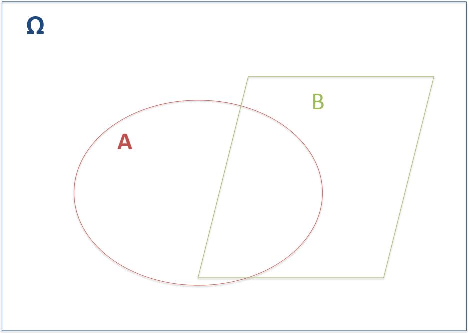
Outline

Events

An *event* is a set of outcomes to which a probability is assigned, for example, getting a card with Red on both sides.

Intersections and unions:

- Intersection: $P(A \cap B)$
- Union: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$



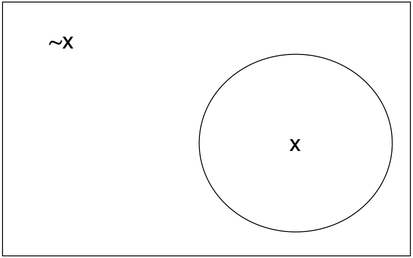
Joint distribution

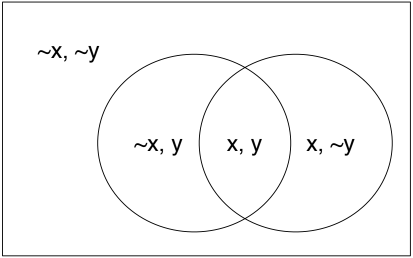
- Typically, we consider collections of random variables.
- The joint distribution is a distribution over the configuration of all the random variables in the ensemble.
- For example, imagine flipping 4 coins. The joint distribution is over the space of all possible outcomes of the four coins.

$$
\begin{aligned}
P(HHHH) &= 0.0625 \\
P(HHHT) &= 0.0625 \\
P(HHTH) &= 0.0625 \\
&\dots
\end{aligned}
$$

- You can think of it as a single random variable with 16 values.

Visualizing a joint distribution





Marginalization

If we are given a joint distribution, what if we are only interested in the distribution of one of the variables?

We can compute the distribution of $P(X)$ from $P(X, Y, Z)$ through *marginalization*:

$$\begin{aligned}\sum_y \sum_z P(X, Y = y, Z = z) &= \sum_y \sum_z P(X) P(Y = y, Z = z | X) \\ &= P(X) \sum_y \sum_z P(Y = y, Z = z | X) \\ &= P(X)\end{aligned}$$

Marginalization (from Leyton-Brown)

Joint distribution

temperature (T) and weather (W)

	T=Hot	T=Mild	T=Cold
W=Sunny	.10	.20	.10
W=Cloudy	.05	.35	.20

Marginalization allows us to compute distributions over smaller sets of variables:

- $P(X,Y) = \sum_z P(X,Y,Z=z)$
- Corresponds to summing out a table dimension
- New table still sums to 1

- Marginalize out weather
- Marginalize out temperature

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T=Hot	T=Mild	T=Cold
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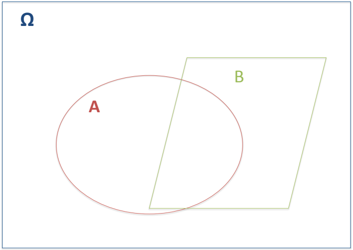
- Marginalize out temperature

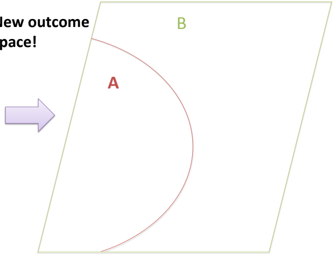
W=Sunny	.40
W=Cloudy	.60

Conditional Probabilities

The *conditional probability* of event A given event B is the probability of A when B is known to occur,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$





Conditional Probabilities

Example

What is the probability that the sum of two dice is six given that the first is greater than three?

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- $A \equiv$ First die
- $B \equiv$ Second die

	B=1	B=2	B=3	B=4	B=5	B=6
A=1	2	3	4	5	6	7
A=2	3	4	5	6	7	8
A=3	4	5	6	7	8	9
A=4	5	6	7	8	9	10
A=5	6	7	8	9	10	11
A=6	7	8	9	10	11	12

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A=6	7	8	9	10	11	12

$$P(A > 3 \cap B + A = 6) = \frac{2}{36}$$

$$P(B > 3) = \frac{3}{6}$$

$$P(A > 3 | B + A = 6) = \frac{\frac{2}{36}}{\frac{3}{6}} = \frac{2}{36}\frac{6}{3}$$

$$= \frac{1}{9}$$

Outline

The chain rule

- The definition of conditional probability lets us derive the *chain rule*, which let's us define the joint distribution as a product of conditionals:

$$\begin{aligned} P(X, Y) &= P(X, Y)\frac{P(Y)}{P(Y)} \\ &= P(X|Y)P(Y) \end{aligned}$$

- For example, let Y be a disease and X be a symptom. We may know $P(X|Y)$ and $P(Y)$ from data. Use the chain rule to obtain the probability of having the disease and the symptom.
- In general, for any set of N variables

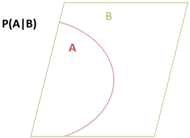
$$P(X_1, \ldots, X_N) = \prod_{n=1}^{N} P(X_n | X_1, \ldots, X_{n-1})$$

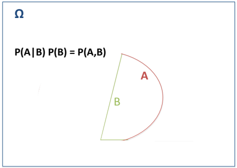
Bayes' Rule

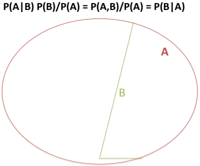
What is the relationship between $P(A|B)$ and $P(B|A)$?

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

1. Start with $P(A|B)$
2. Change outcome space from B to Ω
3. Change outcome space again from Ω to A







Independence

Random variables X and Y are independent if and only if $P(X = x, Y = y) = P(X = x)P(Y = y)$.

Conditional probabilities equal unconditional probabilities with independence:

- $P(X = x \mid Y) = P(X = x)$
- *Knowing Y tells us nothing about X*

Mathematical examples:

- If I draw two socks from my (multicolored) laundry, is the color of the first sock independent from the color of the second sock?
- If I flip a coin twice, is the first outcome independent from the second outcome?

Independence

Intuitive Examples:

- Independent:
  - you use a Mac / the Green line is on schedule
  - snowfall in the Himalayas / your favorite color is blue
- Not independent:
  - you vote for Mitt Romney / you are a Republican
  - there is a traffic jam on the Beltway / the Redskins are playing

Independence

Are these independent?

- the values of two dice
- the value of the first die and the sum of the values
- whether it is raining and the number of taxi cabs
- whether it is raining and the amount of time it takes me to hail a cab
- the first two words in a sentence

Independence

Example: two coins, C_1, C_2 with

$$P(H \mid C_1) = 0.5, \qquad P(H \mid C_2) = 0.3$$

Suppose that I randomly choose a number $Y \in \{1, 2\}$ and take coin C_Z. I flip it twice, with results (X_1, X_2)

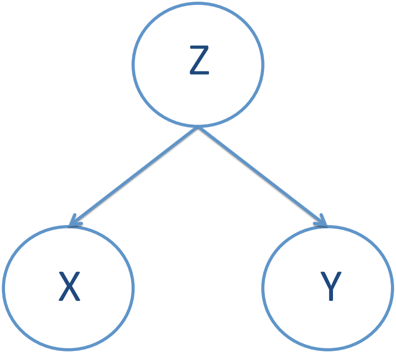
- are X_1 and X_2 independent?
- what about if I know Y?

Conditional Independence

Two random variables (or events) X and Y are *conditionally independent* given Z if and only if

$$P(X = x, Y = y|Z) = P(X = x|Z)P(Y = y|Z)$$

Graphical model notation:



Outline

Returning to the card problem

- Now we can solve the card problem.
- Let X_1 be the random side of the random card I chose
- Let X_2 be the other side of that card
- Compute $P(X_2 = \mathrm{red} | X_1 = \mathrm{red})$

$$P(X_2 = \mathrm{red} | X_1 = \mathrm{red}) = \frac{P(X_1 = R, X_2 = R)}{P(X_1 = R)} \tag{1}$$

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$$P(X_2 = \text{red} | X_1 = \text{red}) = \frac{P(X_1 = R, X_2 = R)}{P(X_1 = R)} \tag{1}$$

- Numerator is 1/3: Only one card has two red sides.
- Denominator is 1/2: There are three possible sides of the six that are red.

Conditional Probabilities

I have 5 socks in my dryer: 3 gray, 2 blue.

- pull one out
- pull second one out
- possible outcomes: GG, GB, BG, BB
- Socks:
  $P(2nd\ Sock\ G \mid 1st\ Sock\ G) =$
- Dice: I roll 2 dice, look at total:
  $P(Total = 7 \mid Total \leq 7) =$

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- Dice: I roll 2 dice, look at total:
  $P(Total\ = 7 \mid Total\ \leq 7) = \frac{6/36}{21/36} = \frac{6}{21}$

Joint Probability Distributions

A *joint probability distribution* is a probability distribution over a set of random variables.

Example: let X be the color of the first sock, Y the color of the second. I have 5 socks in my dryer: 3 gray, 2 blue.

		Y G	Y B
X	G		
X	B		

Can extend conditional probabilities to conditional distributions: $P(Y \mid X = G)$ or $P(X \mid Y = B)$

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		Y	
		G	B
X	G	$3/5 \cdot 2/4 = 3/10$	$3/5 \cdot 2/4 = 3/10$
	B	$2/5 \cdot 3/4 = 3/10$	$2/5 \cdot 1/4 = 1/10$

Can extend conditional probabilities to conditional distributions: $P(Y \mid X = G)$ or $P(X \mid Y = B)$

Outline

Continuous random variables

- We've only used discrete random variables so far (e.g., dice)
- Random variables can be continuous.
- We need a *density* $p(x)$, which *integrates* to one.
  E.g., if $x \in \mathbb{R}$ then
  $$\int_{-\infty}^{\infty} p(x)dx = 1$$
- Probabilities are integrals over smaller intervals. E.g.,
  $$P(X \in (-2.4, 6.5)) = \int_{-2.4}^{6.5} p(x)dx$$
- Notice when we use P, p, X, and x.

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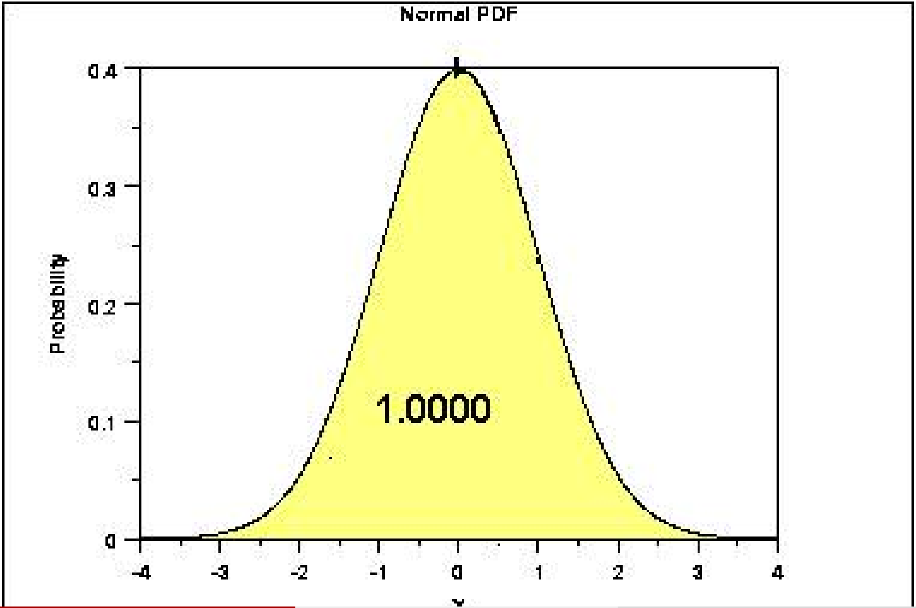
- Notice when we use P, p, X, and x.
- Integrals? I didn't sign up for this!

Integrals?



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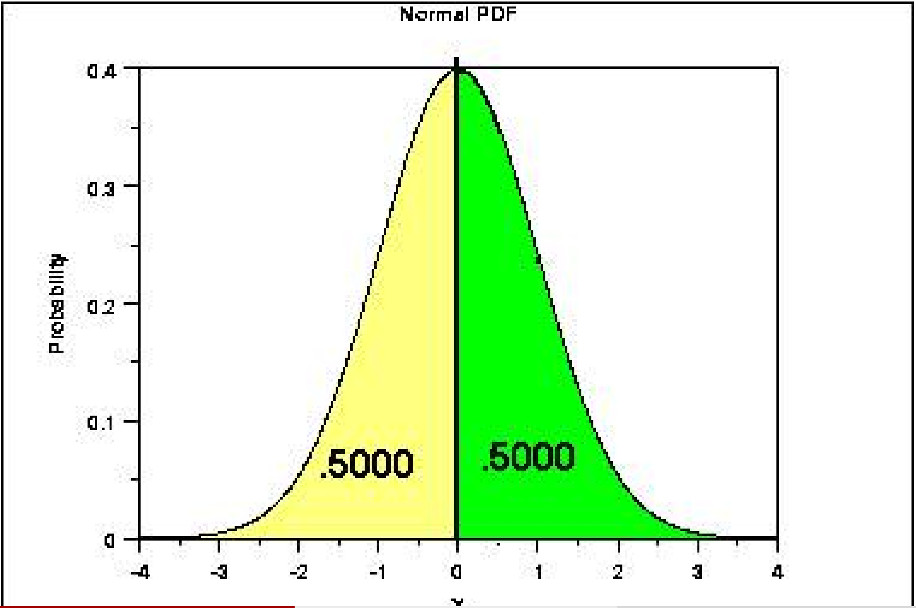

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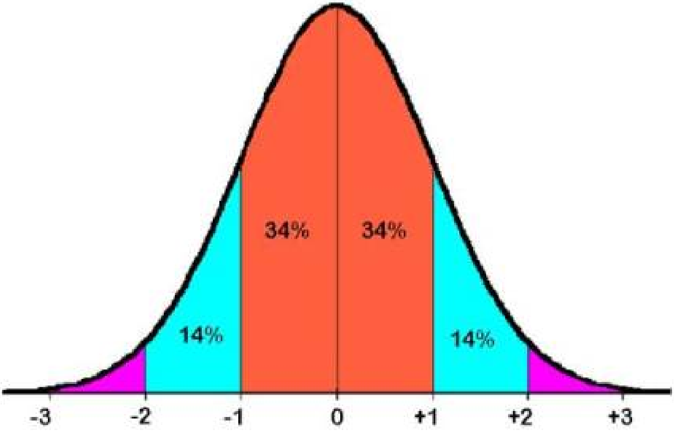


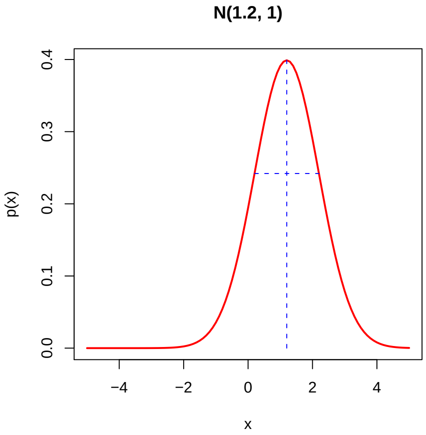
The Gaussian distribution

- The Gaussian (or Normal) is a continuous distribution.

$$p(x|\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}$$

- The density of a point x is proportional to the negative exponentiated half distance to μ scaled by σ^2.
- μ is called the *mean*; σ^2 is called the *variance*.

Gaussian density



- The mean μ controls the location of the bump.
- The variance σ^2 controls the spread of the bump.

Outline

Expectation

An *expectation* of a random variable is a weighted average:

$$
\begin{aligned}
\mathrm{E}[f(X)] &= \sum_{x=1}^{\infty} f(x)\, p(x) && \text{(discrete)} \\
&= \int_{-\infty}^{\infty} f(x)\, p(x)\, dx && \text{(continuous)}
\end{aligned}
$$

Alternate formulation for positive random variables:

$$
\begin{aligned}
\mathrm{E}[X] &= \sum_{x=1}^{\infty} P(X > x) && \text{(discrete)} \\
&= \int_{0}^{\infty} P(X > x)\, dx && \text{(continuous)}
\end{aligned}
$$

Expectation

Expectations of constants or known values:

- $\mathrm{E}[a] = a$
- $\mathrm{E}[Y \mid Y = y] = y$

Expectation

Example: Gaussian distribution $X \sim N(\mu, \sigma^2)$

$$
\begin{aligned}
\mathrm{E}[X] &= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \\
&=
\end{aligned}
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Expectation

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$$\begin{aligned} \mathrm{E}[X] &= \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \\ &= \mu \end{aligned}$$

Expectation of die / dice

What is the expectation of the roll of die?

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One die

$$1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{6} + 3 \cdot \frac{1}{6} + 4 \cdot \frac{1}{6} + 5 \cdot \frac{1}{6} + 6 \cdot \frac{1}{6} =$$

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Expectation of die / dice

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What is the expectation of the sum of two dice?

Two die

$$2 \cdot \frac{1}{36} + 3 \cdot \frac{2}{36} + 4 \cdot \frac{3}{36} + 5 \cdot \frac{4}{36} + 6 \cdot \frac{5}{36} + 7 \cdot \frac{6}{36} + 8 \cdot \frac{5}{36} + 9 \cdot \frac{4}{36} + 10 \cdot \frac{3}{36} + 11 \cdot \frac{2}{36} + 12 \cdot \frac{1}{36} =$$

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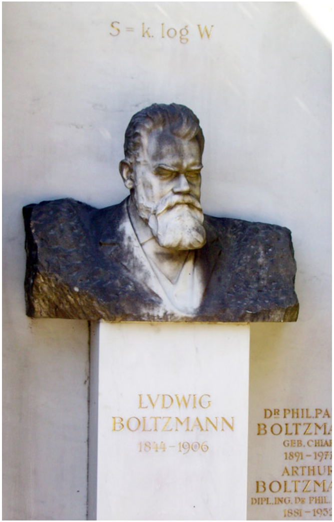
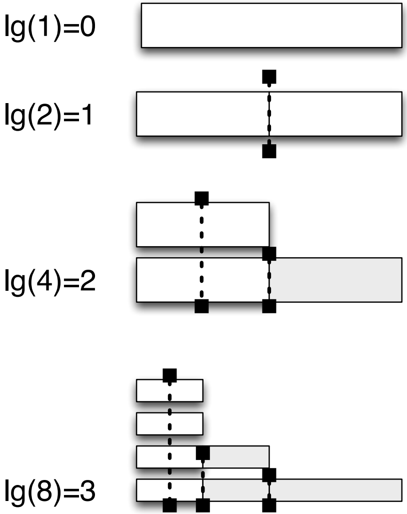
Entropy

- Measure of disorder in a system
- In the real world, entroy in a system tends to increase
- Can also be applied to probabilities:
  - Is one (or a few) outcomes certain (low entropy)
  - Are things equiprobable (high entropy)
- In data science
  - We look for features that allow us to *reduce* entropy (decision trees)
  - All else being equal, we seek models that have *maximum* entropy (Occam's razor)



Aside: Logarithms

- $\lg(x) = b \Leftrightarrow 2^b = x$
- Makes big numbers small
- Way to think about them: cutting a carrot



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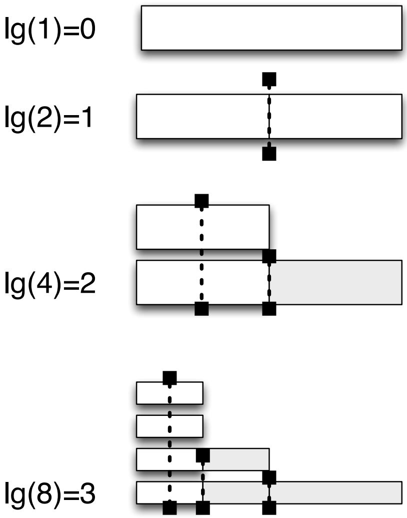

Aside: Logarithms

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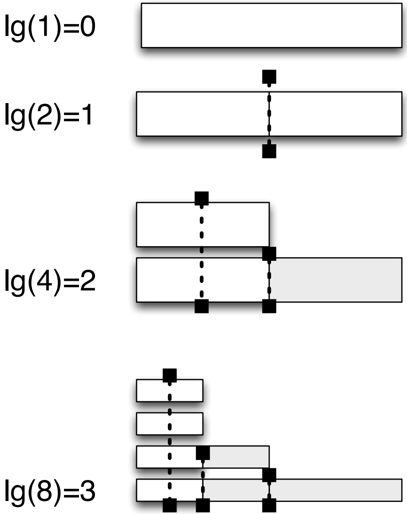
- Makes big numbers small
- Way to think about them: cutting a carrot
- Negative numbers?
- Non-integers?



Entropy

Entropy is a measure of uncertainty that is associated with the distribution of a random variable:

$$\begin{aligned} H(X) &= -\mathrm{E}\left[\lg(p(X))\right] \\ &= -\sum_x p(x)\lg(p(x)) && \text{(discrete)} \\ &= -\int_{-\infty}^{\infty} p(x)\lg(p(x))\,dx && \text{(continuous)} \end{aligned}$$

Does not account for the values of the random variable, only the spread of the distribution.

- $H(X) \geq 0$
- uniform distribution = highest entropy, point mass = lowest
- suppose $P(X=1)=p,\ P(X=0)=1-p$ and $P(Y=100)=p,\ P(Y=0)=1-p$: X and Y have the same entropy

Entropy of a die / dice

What is the entropy of a roll of a die?

Entropy of a die / dice

What is the entropy of a roll of a die?

One die

$$-\left(\frac{1}{6}\lg\left(\frac{1}{6}\right)+\frac{1}{6}\lg\left(\frac{1}{6}\right)+\frac{1}{6}\lg\left(\frac{1}{6}\right)+\frac{1}{6}\lg\left(\frac{1}{6}\right)+\frac{1}{6}\lg\left(\frac{1}{6}\right)+\frac{1}{6}\lg\left(\frac{1}{6}\right)\right)=2.58$$

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What is the entropy of the sum of two die? Tricky question: will it be higher or lower than the first one?

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Two die

$$\begin{aligned}-\Bigg(\frac{1}{36}\lg\left(\frac{1}{36}\right)&+\frac{2}{36}\lg\left(\frac{2}{36}\right)+\frac{3}{36}\lg\left(\frac{3}{36}\right)+\frac{4}{36}\lg\left(\frac{4}{36}\right)+\frac{5}{36}\lg\left(\frac{5}{36}\right)\\&+\frac{6}{36}\lg\left(\frac{6}{36}\right)+\frac{5}{36}\lg\left(\frac{5}{36}\right)+\frac{4}{36}\lg\left(\frac{4}{36}\right)+\frac{3}{36}\lg\left(\frac{3}{36}\right)\\&+\frac{2}{36}\lg\left(\frac{2}{36}\right)+\frac{1}{36}\lg\left(\frac{1}{36}\right)\Bigg)=3.27\end{aligned}$$

Whew!

- That's it for now
- You don't have to be an expert on this stuff (there are other classes for that)
- This is to get your feet wet and to know the concepts when you see the math

First assignment

- Find some data
- Find interesting relationships in your data (next week!)
- Use Rattle to display those relationships (be creative and thorough!)