
The Dimensions of Stock Returns

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Over the years, researchers have tried to identify factors that explain the wide range in individual stock returns. The goal is to find factors that explain not only the average stock returns for each time period but also the changes in average stock returns across time periods. If a factor does both, it can be said to explain the common variation of returns. We will consider a factor to be a dimension of returns if it explains the common variation in returns and if its explanatory power seems to be related to risk and independent of other factors.

One factor that has been researched is the stock beta, a measure of relative return volatility. If there were a positive relation between betas and average returns, beta could be considered a dimension of returns. However, research has not shown a reliable relation between betas and average returns.

Recent research has identified several factors that do explain some of the common variation in stock returns. Professors Eugene F. Fama and Kenneth R. French [1991 and 1992] of the University of Chicago find that two factors taken together, company size and the book-to-market ratio (BtM), explain a large amount of the common variation in stock returns and that the differences in average returns are related to risk. Based on their results, we conclude that size and BtM are two dimensions of stock returns.

Note: Appendix 2 contains a glossary defining some of the technical terms used in this paper.

Their results have important implications for equity portfolio construction. Equity “market portfolios,” such as the Standard and Poors 500 Index, the Russell 3000 and the Wilshire 5000 are heavily weighted towards big stocks and low BtM stocks. The Fama and French results suggest these market portfolios are not appropriate for investors that are sensitive to return volatility. It is our opinion that most institutional portfolios should have higher commitments to small stocks and high BtM stocks than represented by market portfolios.

1 Identifying the Dimensions of Stock Returns

Fama and French [1991] report results for individual factors that are similar to the findings of previous researchers. They find that four factors - size, BtM ratio, earnings yield (E/P) and leverage - when considered individually, seem to explain common variation in stock returns (see *Appendix 1*). Betas do not. Their results begin in 1963, the earliest time for which book value data is available on Compustat, their data source.

They also examine these factors together. When size and BtM are used together as explanatory variables, each continues to explain some of the common variation of returns. However, none of the other factors continues to have explanatory power when considered along with size and BtM. Fama and French conclude that BtM and size, taken together, explain the meaningful information content of all four factors. We conclude that these two factors are the relevant dimensions of stock returns.

2 Relating Factor Returns to Economic Fundamentals

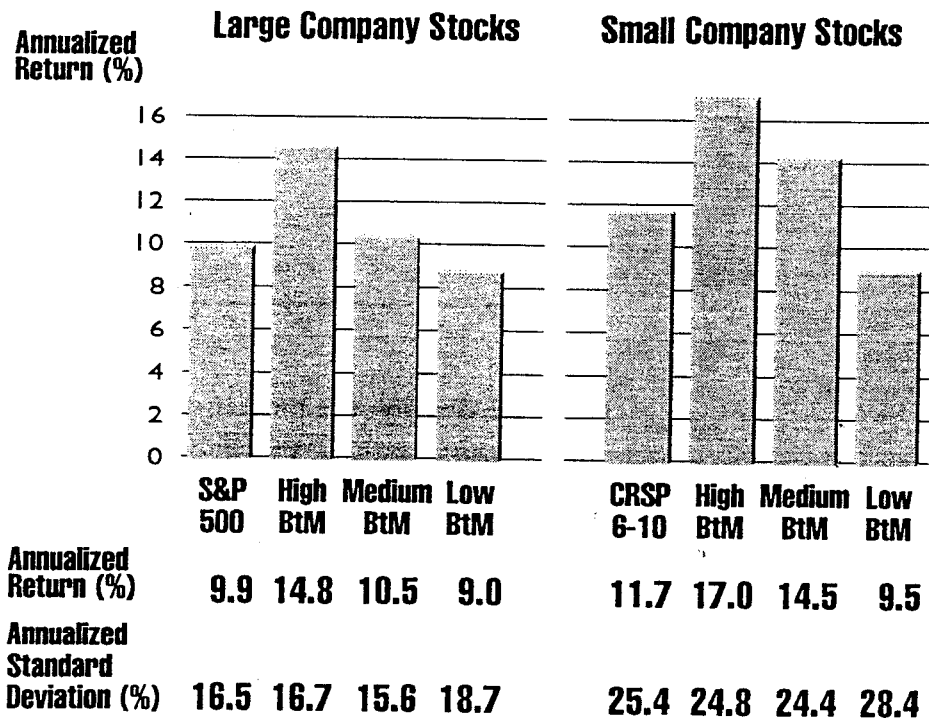
The higher returns for small and high BtM stocks must either be the rewards for taking risk or the result of inefficient pricing. Fama and French [1992] investigate the relation between risk and the factor returns and conclude the results are consistent with market efficiency.

The Dimensions of Stock Returns

Exhibit 1 displays returns and standard deviations for six stock series developed by Fama and French, three BtM categories for each of two size categories. The small stock results are what we would expect. Adjusting for book-to-market ratios, the standard deviations of small stock returns are greater than the standard deviations of large stock returns.

The book-to-market results do not show a relation between returns and return volatility. Adjusting for size, the standard deviations of all three book-to-market categories are about the same. The differences in returns must be due to risk factors that are not reflected in the standard deviations.

Exhibit 1
Size and Book-to-Market
Summary Statistics
 January 1964 through December 1990



Calculated using quarterly returns.

2 Relating Factor Returns to Economic Fundamentals

Fama and French turn to economic fundamentals for an explanation. Exhibit 2 displays the earnings behavior of big and small stocks, and high and low BtM stocks.

The patterns suggest that high BtM stocks have above-average amounts of economic stress. On average, high BtM stocks have lower earnings than low BtM stocks. Their earnings tend to decline for the five years before the year following portfolio formation. The earnings of low BtM stocks are high by comparison, and rise prior to portfolio formation.

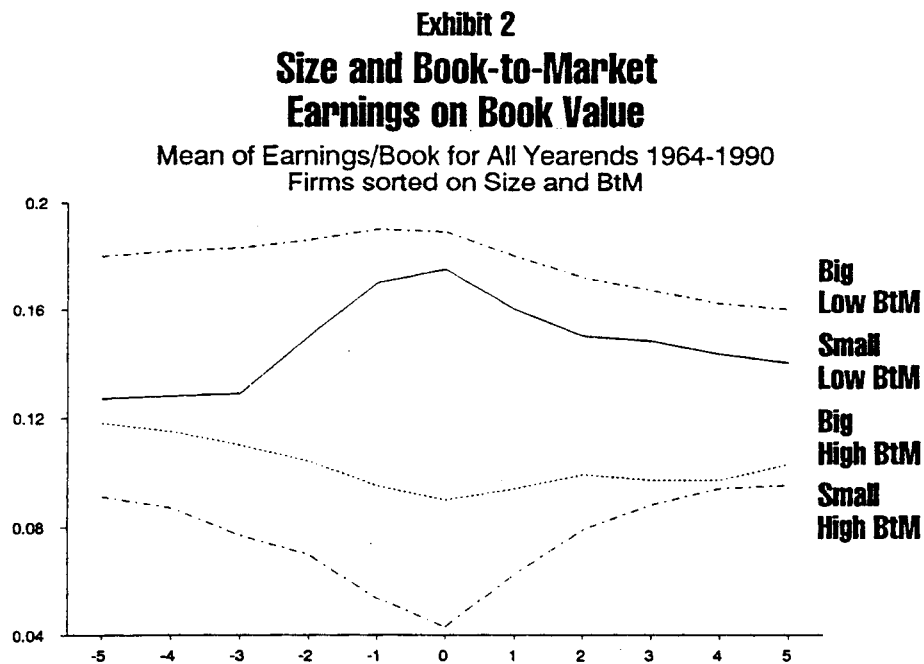
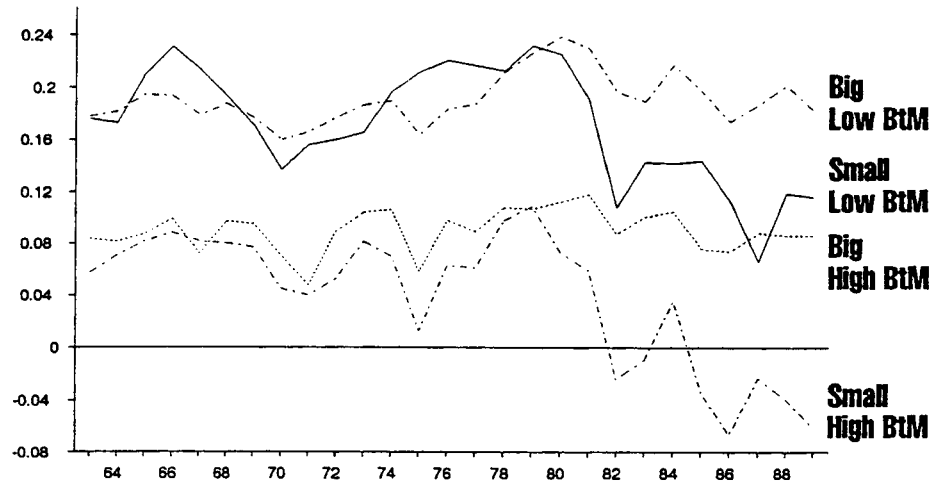


Exhibit 3 displays the pattern of earnings for the four categories for the individual years 1964-1990. Small stock earnings are more volatile than big stock earnings and high BtM stocks have lower earnings than low BtM stocks. Most noticeably, the earnings for each of the four categories behave very differently for long periods of time. This indicates an underlying risk factor relating to earnings.

The earnings behavior of BtM and the above-average returns for high BtM stocks are consistent with market efficiency. Size and BtM seem to be associated with risk factors relating to earnings prospects.

Exhibit 3
Size and Book-to-Market
Earnings on Book Value

For December Yearends
 Firms sorted on Size and BtM



BtM and size are associated with risk factors. They are not themselves risk factors. Small stocks are not riskier than big stocks simply because they are small. High BtM stocks are not always riskier than low BtM stocks. Instead, size and BtM seem to be associated with some underlying risk factor or factors that has yet to be identified. The challenge in the future will be to identify these underlying risk factors.

3 Implications for Portfolio Construction

Our way of using these findings is to think of stock returns as having three dimensions. The first dimension is the overall behavior of the stock market. This dimension is used to decide how much of a portfolio should be invested in stocks in general. The market's performance appears to be a function of two additional dimensions, company size and book-to-market ratio.

Exhibit 4 displays how Fama and French develop the three dimensions by forming 25 portfolios. *S1-S5* are five size portfolios, with *S1* being the smallest quintile of NYSE stocks. *B/M1-B/M5* are five book-to-market portfolios, with *B/M1* being the lowest book-to-market quintile of NYSE stocks.

3 Implications for Portfolio Construction

Exhibit 4

Regressions on the Excess Returns of 25 Portfolios

Formed on Size and Book-to-Market Equity on the Excess Return on the Market Portfolio and on the Returns on the Portfolios for Size and Book-to-Market Equity: 7/63-12/90 Obs=330

$$R(t) - RF(t) = a + b [RM(t) - RF(t)] + sSMB(t) + hHML(t) + e(t)$$

return minus 30 day t-Bill return = average excess return + beta[market return minus t-bill return] + sensitivity to size[small minus big] + sensitivity to BtM[high BtM minus low BtM] + residual

		Book-to-Market					Book-to-Market				
		(lowest)				(highest)	(lowest)				(highest)
		1	2	3	4	5	1	2	3	4	5
		average excess return: a					t(a)				
(smallest)	1	-0.37	-0.07	-0.00	0.06	0.05	-3.33	-0.88	-0.11	0.89	0.70
	2	-0.05	-0.06	0.05	0.09	-0.00	-0.55	0.74	0.60	1.24	-0.03
Size	3	-0.07	0.14	-0.07	0.15	0.04	-0.82	1.63	-0.84	1.96	0.44
	4	0.11	-0.09	-0.05	0.07	0.03	1.26	-1.07	-0.56	0.71	0.27
(largest)	5	0.19	-0.08	-0.09	-0.09	-0.03	2.73	-1.13	-1.03	-1.14	-0.29
		sensitivity to market (beta): b					t(b)				
(smallest)	1	1.02	0.99	0.92	0.89	0.94	36.68	48.94	52.75	55.63	52.60
	2	1.10	1.05	1.00	1.00	1.08	50.21	56.06	52.67	54.04	57.24
Size	3	1.10	1.01	1.00	0.98	1.07	52.60	48.36	47.14	50.21	47.80
	4	1.06	1.08	1.03	1.02	1.22	48.92	50.71	44.86	40.86	43.04
(largest)	5	0.95	1.00	0.99	0.98	1.05	56.08	55.26	43.99	50.86	39.64
		sensitivity to size: s					t(s)				
(smallest)	1	1.40	1.23	1.16	1.12	1.20	36.03	43.54	47.55	49.93	48.23
	2	1.00	0.97	0.84	0.75	0.92	32.73	36.90	31.49	29.02	34.88
Size	3	0.73	0.67	0.61	0.49	0.66	24.84	22.79	20.69	18.00	21.25
	4	0.37	0.31	0.28	0.24	0.45	12.10	10.58	8.60	6.98	11.33
(largest)	5	-0.15	-0.12	-0.20	-0.16	-0.07	-6.12	-4.62	-6.42	-5.94	-1.78
		sensitivity to BtM: h					t(h)				
(smallest)	1	-0.27	0.07	0.23	0.38	0.63	-5.78	1.93	7.81	14.03	20.73
	2	-0.51	0.02	0.22	0.40	0.70	-13.70	0.52	6.71	12.87	21.94
Size	3	-0.38	-0.04	0.28	0.47	0.70	-10.66	-1.11	7.81	14.38	18.46
	4	-0.44	0.03	0.30	0.53	0.74	-11.99	0.81	7.67	12.42	15.39
(largest)	5	-0.46	0.00	0.22	0.54	0.74	-15.87	0.18	5.65	16.51	17.05
		correlation: R ²					s(e)				
(smallest)	1	0.93	0.96	0.96	0.96	0.96	1.97	1.43	1.24	1.14	1.26
	2	0.95	0.96	0.95	0.95	0.96	1.54	1.32	1.34	1.31	1.33
Size	3	0.95	0.93	0.92	0.93	0.92	1.48	1.48	1.50	1.37	1.58
	4	0.93	0.92	0.90	0.87	0.89	1.53	1.50	1.63	1.76	2.00
(largest)	5	0.94	0.92	0.87	0.89	0.84	1.20	1.28	1.59	1.36	1.87

The Dimensions of Stock Returns

The returns of the 25 portfolios are regressed against three factors, 1) the stock market (excess) return, 2) a “size effect” factor and 3) a “book-to-market” factor. The slope coefficients (b , s and h) measure a portfolio’s exposure to each factor.

- 1) market factor b . Most portfolios have a b close to 1.0. As a result, b s will not explain much of the differences in portfolio returns.
- 2) size effect s . The slope coefficients on the size effect, the s , are positive for small stocks and negative for large stocks. Adjusting for BtM (going down the rows), there is a large size effect. Adjusting for size (going across the columns), there is no additional BtM effect.
- 3) book-to-market effect h . The slope coefficients on the book-to-market effect, the h , are negative for low book-to-market stocks and positive for high book-to-market stocks. Adjusting for size (going across the columns), there is a large book-to-market effect. Adjusting for BtM (going down the rows), there is no additional size effect.

The regressions typically explain over 90% of the variation in stock returns. The R^2 range from 0.85 to 0.96.

The regression intercepts, the a s, measure the average stock returns unexplained by the regressions. If $a=0$, the regressions explain all of the average returns. Only two of the portfolios have intercepts that are too big to be considered non-zero. 23 of the 25 have intercepts that can be considered to be zero.

Fama and French conclude that the three-factor model is a good description of stock returns. It explains a large part of the differences in average returns (a s are close to zero) and a large part of the variation in returns (R^2 s are close to 1.0).

The three-factor model provides an interesting way to evaluate active management results. By regressing manager returns against the three-factor model returns, the impact of management can be measured by the intercept term a . The reliability of management can be measured by the intercept’s t-statistic.

4 Comparison to Value and Growth Stock Returns

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It is common practice to classify stocks as being either “value” or “growth” stocks (Frank Russell uses “price driven” and “earnings growth” instead of “value” and “growth”). Whatever the label, the returns for the two stock groupings behave differently. The labels also correspond to the Fama and French BtM categories. High BtM stocks appear to be similar to value stocks, while low BtM stocks appear to be similar to growth stocks. Exhibit 5 compares the Fama & French returns to big stock index returns created by Frank Russell & Co. and by Wilshire Associates. The returns and standard deviations of the value indices are very similar to the high

Exhibit 5
**Comparison of Fama & French,
 Frank Russell and Wilshire Associates Strategies**
 Quarterly Data: 1979-1990

	High Book-to-Market			Low Book-to-Market		
	Fama&French High BtM	Russell Price-Driven Index	Wilshire Large Co. Value	Fama&French Low BtM	Russell Earnings Growth	Wilshire Large Co. Growth
Annualized Return (%)	17.7	16.5	16.8	10.0	15.5	15.2
Standard Deviation (%)	14.5	15.4	14.2	26.3	19.6	18.8
Correlations (R²)						
Fama & French High BtM	1.000					
Russell Price-Driven	.925	1.000				
Wilshire Large Value	.949	.919	1.000			
Fama & French Low BtM	.713	.822	.716	1.000		
Russell Earnings Growth	.648	.756	.646	.953	1.000	
Wilshire Large Growth	.735	.842	.746	.965	.934	1.000

Wilshire data courtesy Wilshire Associates.

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BtM category. The returns and standard deviations of the growth indices are similar to the low BtM category. As we have seen, high BtM companies might more appropriately be labeled “stressed” or “stretched” companies instead of value companies. And since low BtM stocks have higher earnings but lower returns than high BtM stocks, maybe they should be labeled “healthy” instead of growth.

Given the Fama and French results, we reach different conclusions about portfolio and benchmark construction than previous researchers who have shown only that growth and value stocks behave differently. Knowing only that growth and value stocks behave differently, we might use a market portfolio as our benchmark, such as the Wilshire 5000 or the Russell 3000.

Market portfolios are weighted heavily towards big stocks and low BtM stocks. This appears to be a poor way to form a portfolio for anyone who associates risk with return volatility. We believe investors can achieve greater returns for a given standard deviation of returns by increasing commitments to small stocks and high book-to-market stocks.

5 Portfolio Construction

In developing a portfolio benchmark or in constructing a portfolio, there is no optimal way to weight the stock dimensions. Investment returns depend on other risk factors in addition to return volatility, making optimizing programs useless. One way to develop a portfolio benchmark is to consider different weightings of the following four indices:

- 1) large cap market index, such as the S&P 500, the Schwab 1000 or the Russell 1000;
- 2) big stock high BTM index of the Fama and French big-high BtM stocks;
- 3) small cap market index, as Dimensional’s 9-10 or 6-10 Portfolios or the Russell 2000;
- 4) small stock high BtM index of the Fama and French small-high BtM stocks.

5 Portfolio Construction

The four indices conform loosely to the notion of a market portfolio plus book-to-market effect and size effect portfolios. Indices 3 and 4 can be used to emphasize the size effect. Indices 2 and 4 can be used to emphasize the book-to-market effect.

Exhibit 6 displays the results for three portfolio combinations. The first is the S&P 500 Index alone. The second is a blended market portfolio of 90% S&P 500 and 10% small stocks. The third keeps a market weighting in small stocks, but invests half of each size range in high BtM stocks. The fourth doubles the commitment to small stocks.

Exhibit 6
Comparison of Three Portfolios
Quarterly Data: 1964-1990

	S&P 500 Index	"The Market" 90% S&P 10% D6-10	45% S&P 45% BH 5% D6-10 5% SH	40% S&P 40% BH 10% D6-10 10% SH
Annualized Returns (%)	9.9	10.2	12.7	13.0
Annualized Standard Deviation (%)	16.5	17.1	16.8	17.5
S&P	S&P 500 Index			
BH	Big Stocks, High Book-to-Market			
D 6-10	Small Stocks, Dimensional's 6-10			
SH	Small Stocks, High Book-to-Market			

As can be seen from Exhibit 6, the returns increase with greater commitments to high BtM and small stocks. Small stocks increase the standard deviation of returns, high book to market stocks do not. The tradeoff between returns and standard deviation favors high commitments to high BtM and small stocks.

Which portfolio combination to select depends on individual preferences about market, book-to-market and size risks. It is our opinion that most institutional clients will want greater investments in small stocks and high BtM stocks than they achieve with their market portfolios.



A1 Summary of Fama and French Results

July 1963-
December 1990

	Factor	Result
1	Beta	Not meaningful.
2	Size	Size is significant. Small stocks have higher returns and higher standard deviations than large stocks.
3	Beta & Size	Size is significant. Not only is beta not meaningful, but it works in the wrong direction. Adjusted for size, low beta stocks have higher returns than high beta stocks.
4	Book Value	Book value is significant. Stocks with high book-to-market ratios (BtM) have higher returns than stocks with low BtM ratios.
5	Leverage	Leverage is significant. Adjusted for book value of assets, the smaller the company and the greater the book value of equity assets, the greater the stock return.
6	Earnings	Earnings are significant. Stocks with high earnings yields (E/P) have higher returns than stocks with low earnings yields.
7	Size & Book-to-Market	Both factors are significant, and in the same way as when each factor is considered separately.
8	Size, Book-to-Market & Earnings Yield	Only size and BtM are significant when the three factors are taken together. Earnings yield is not meaningful.

A2 Glossary of Fundamental Data

- Beta** Beta measures the sensitivity of rates of return on a portfolio or a given stock compared with the rates of return of the market as a whole. A beta of 1.5 roughly forecasts a 1.5% change in an asset's return for an simultaneous 1% change in the return on the market as a whole.
- Book-to-Market Ratio (BE/ME)** A ratio comparing the book value per share of a firms' common stock with the market price of the firms' common stock. This means that the book to market ratio relates the value the firms' accountants place on the firm (using accepted accounting principles) with the value the stock market places on the firm. A high ratio indicates that investors believe the firms' assets have been overvalued on its financial statements. Fama and French believe book-market is at least partially driven by earnings. High book-to-market stocks have lower earnings on book value than low book-to-market stocks, and therefore sell at a discount. In this sense, high book-market firms are relatively distressed. Their poor economic performance may be the risk factor compensated for by their higher expected returns.
- Correlation (R or R²)** A measure of how closely two variables move through time. An R or R² of 1.00 indicates identical movement; 0.00 or a negative R indicates less related movement. A correlation (R²) of returns between two assets of .52 would indicate that, over the time period, 52% of the variance of return of the one asset is associated with the return of the other asset. Usually, a security will have a high returns correlation with another security if the same forces affect their returns.
- Earnings-to-Book Ratio (E/BE)** A ratio comparing the current (or sometimes projected) earnings per share of a firm with the book value per share of the firm's common stock. In *Dimensions of Stock Returns* Earnings-to-Book (E/BE) is used to measure earnings instead of Earnings-to-Price (E/P) because E/BE measures changes in a firm's earnings without reflecting corresponding changes in the firm's market price, which are reflected in the denominator of the Earnings-to-Price ratio.
- Earnings-to-Price Ratio (E/P)** A ratio comparing the current (or sometimes the projected) earnings per share of a firm with the current price of its common stock. A low earnings to price ratio generally indicates that investors believe the firm's earnings are likely to grow, or that there is little uncertainty about its earnings.

Appendix A3

Glossary - continued

Earnings Yield See Earnings to Price Ratio.

Expected Return The expected rate of return on an asset is the sum of each possible rate of return times its respective probability of occurring.

Leverage The amount of fixed costs (assets, debt etc.) a firm incurs relative to the firm's market value or book value. Leverage is often expressed as a ratio, such as debt to equity, assets to price or assets to book. In a leveraged situation where borrowed funds are used to finance an investment, there is increased risk from a low yield on the investment or an increase in interest rates.

Regression A measurement of the relationship between two variables, called a dependant variable and an independent variable. In the book-to-market study, Fama and French study a regression between average returns and book-to-market equity. In this case the average return is the dependant variable and the book-to-market equity is the independent variable. The slope of the regression line is expressed as a figure called the "t-statistic". A high t-statistic indicates a strong relationship between the two variables.

A3 References

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ADDENDUM I

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