Resolve the apparent paradox that the most effective approach for controlling space-charge effects is to apply strong focusing, which makes the beam-size small and increases the space-charge force. T.P. Wangler, 2008

We use the equation for a round continuous beam of rms size a:

$$a'' + k_0 a - \frac{\varepsilon^2}{a^3} - \frac{K}{4a} = 0 \cdot$$

The second term $k_0 a$ is the focusing force, where k_0 is the zero-current phase-advance per unit length, and the fourth term is the space-charge force, where K is the generalized perveance.

The important quantity is not the space charge force, but the ratio ρ of the space-charge force to the focusing force, which is

$$\rho = \frac{(K/4a)}{(k_0^2 a)} = \frac{K}{4k_0^2 a^2} \cdot$$

To see what happens to ρ when k_0 is increased, we need to find out what the matched beam size a will do. The matched beam size is given by $a^2=\varepsilon/k$, where k is the phase advance per unit length including space charge for the equivalent uniform beam, which is defined as:

$$k^2 = k_0^2 - \frac{K}{4a^2},$$

so the equation for ρ becomes

$$\rho = \frac{K}{4k_0^2}\frac{k}{\varepsilon} = \frac{K}{4\varepsilon k_0}\frac{k}{k_0},$$

The matched solution gives the tune depression

$$\frac{k}{k_0} = \frac{1}{u + \sqrt{1+u^2}}$$

where $u = K/8\varepsilon k_0$ is the space-charge parameter. Then we can express the parameter ρ as a function of the space charge parameter u.

$$\rho = \frac{K}{4k_0^2}\frac{k}{\varepsilon} = \frac{K}{4\varepsilon k_0}\frac{k}{k_0} = \frac{2u}{u + \sqrt{1+u^2}} = \frac{2}{1 + \sqrt{1/u^2 + 1}} \cdot$$

Meanwhile, the squared matched beam size is

$$a^2 = \frac{\varepsilon}{k} = \frac{\varepsilon/k_0}{k/k_0} = \frac{\varepsilon}{k_0}\left(u + \sqrt{1+u^2}\right)$$

As the focusing strength k_0 increases (with K and ε constant), the space-charge parameter u decreases. The matched beam size a decreases and the space-charge force K/4a increases. However, ρ, the ratio of the space-charge force to the focusing force, decreases. The latter is what is relevant in determining the importance of the space-charge force. **Thus, to reduce the effect of space charge, one needs strong focusing which makes the beam size small and makes the ratio of space charge force to focusing force small.**