Trace-Space Particle Distributions and Emittance

PhSp-Dist-Emitt-Linear-Simplified.x
mcd

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Set array origin and fix seed for random # generator:

$ORIGIN := 1 \qquad Seed(1) = 2.683 \times 10^9$

NUMBER OF PARTICLES: $N := 100 \qquad i := 1..N$

INITIAL PARTICLE POSITIONS: $x_{0_i} := 0 \qquad j := 1..N$

$rand := 0 \qquad \alpha := 1$

DEFINE UNIFORM SYMMETRICAL DISTRIBUTION OF INITIAL SLOPES FOR THE PARTICLES:

$k := 1..\frac{N}{2} \qquad \delta := \frac{2 \cdot \alpha}{N}$

$x_{p1_k} := -\alpha + (k-1) \cdot \delta$

$x_{prand_j} := (-\alpha \cdot rnd(1) + \alpha \cdot rnd(1)) \qquad x_{p2_k} := -\alpha + \left(k + \frac{N}{2}\right) \cdot \delta$

RANDOM OR UNIFORM DISTRIBUTION:

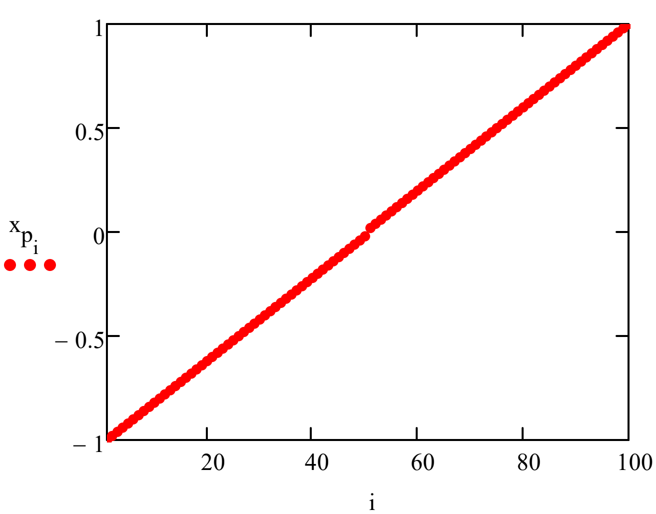
$$x_p := \begin{cases} x_{prand} & \text{if } rand = 1 \\ stack(x_{p1}, x_{p2}) & \text{otherwise} \end{cases}$$

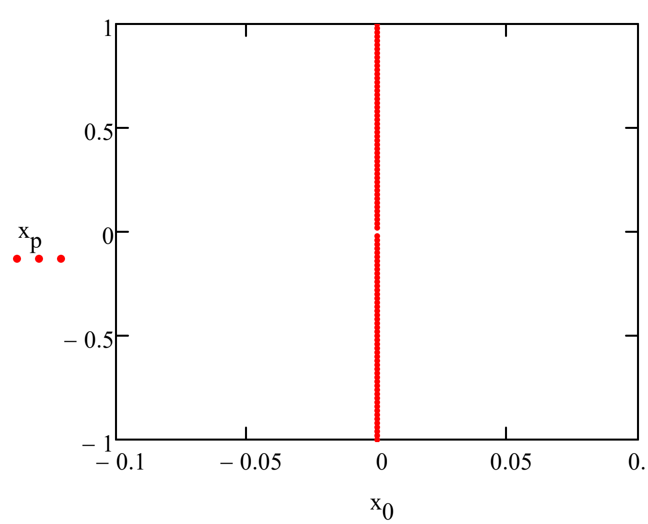
Element check:

$x_{p_1} = -1 \qquad x_{p_{\frac{N}{2}+1}} = 0.02$

$x_{p_{\frac{N}{2}}} = -0.02 \qquad x_{p_N} = 1$

FOCUSING CONSTANT IN m^-1: $k_0 := 10$





Given

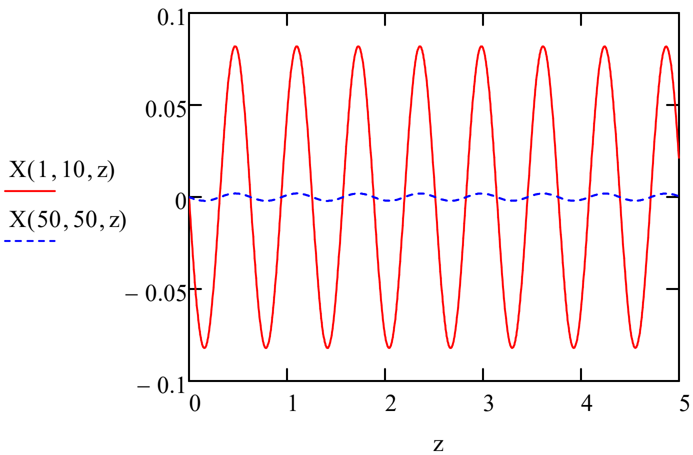
$$\frac{d^2}{dz^2}x(z) = -k_0{}^2\,x(z)$$

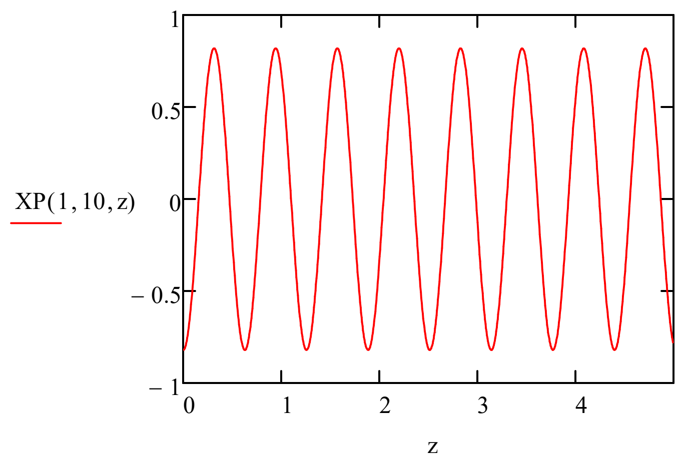
$x(0) = x_{0_i}$ $\qquad$ $x'(0) = x_{p_j}$

$x(i,j) := \text{Odesolve}(z, 5)$

$$X(i,j,z) := \left|\begin{array}{l} X1 \leftarrow x(i,j) \\ \overrightarrow{X1(z)} \end{array}\right.$$

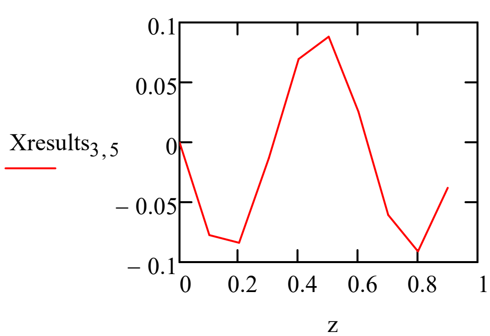
$$XP(i,j,z) := \frac{d}{dz}X(i,j,z)$$





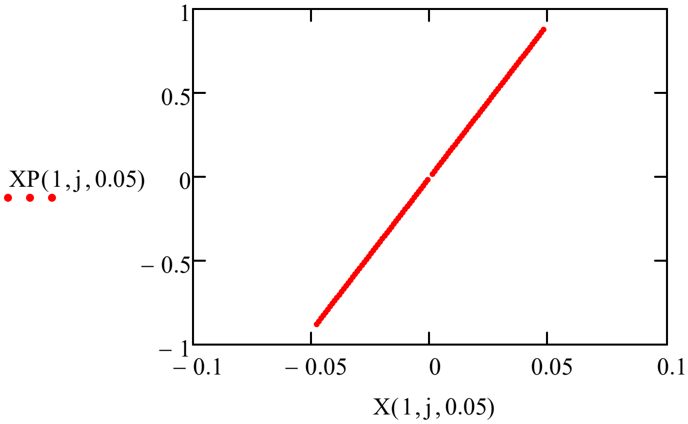
$$X2(N,z) := \left|\begin{array}{l} \text{for } i \in 1..N \\ \quad \text{for } j \in 1..N \\ \qquad \left|\begin{array}{l} X1 \leftarrow X(i,j) \\ X2_{i,j} \leftarrow \overrightarrow{X1(z)} \end{array}\right. \\ X2 \end{array}\right.$$

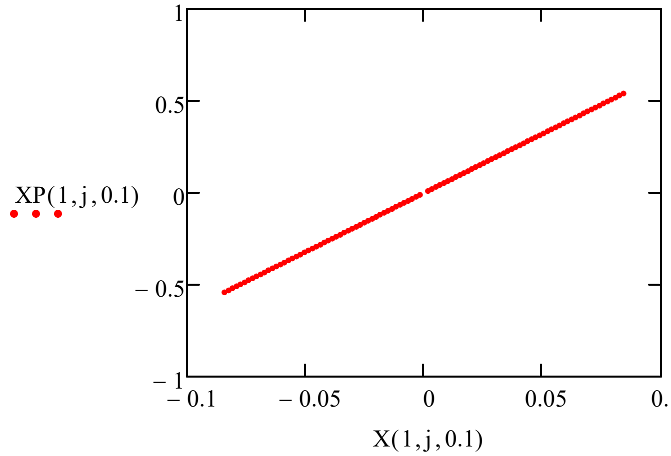
$k := 1..10$ $\qquad$ $z_k := \frac{k-1}{10}$ $\qquad$ $Xresults := X2(N,z)$

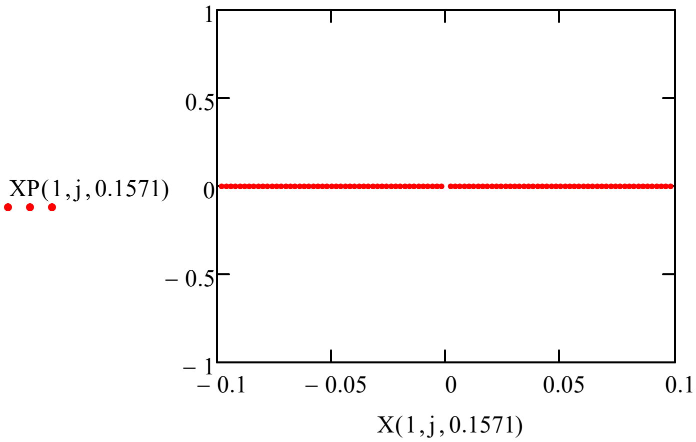


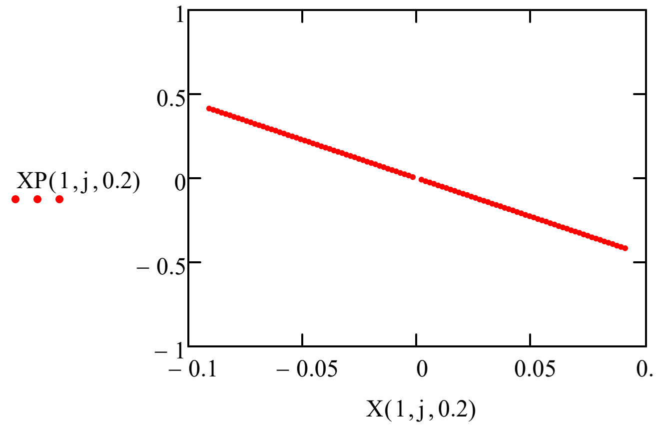
$\lambda := \frac{2\cdot\pi}{k_0} = 0.6283$ $\qquad$ $\frac{\lambda}{4} = 0.1571$

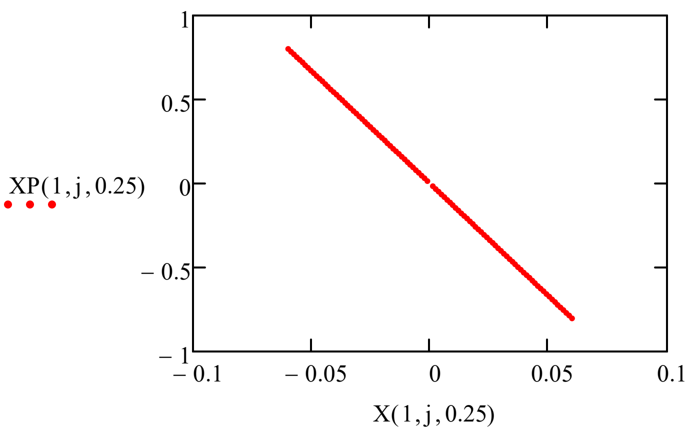
$\frac{\lambda}{2} = 0.3142$ $\qquad$ $\frac{3\cdot\lambda}{4} = 0.4712$ $\qquad$ $5\cdot\lambda = 3.142$

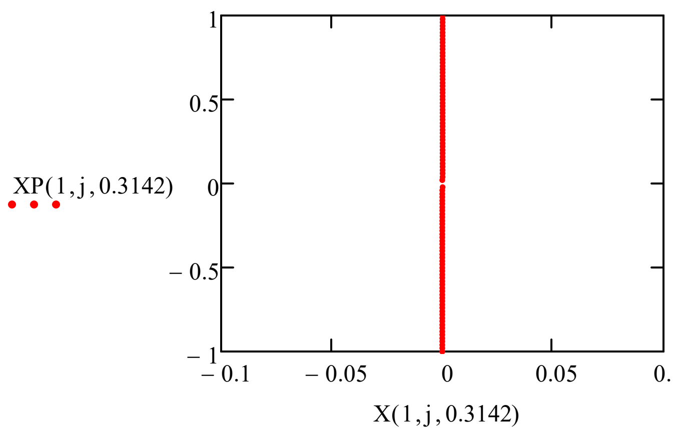

XP(1, j, 0.05)
X(1, j, 0.05)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1

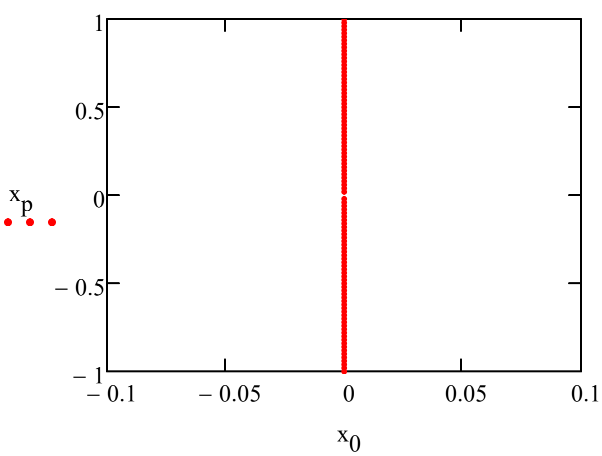

XP(1, j, 0.1)
X(1, j, 0.1)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1

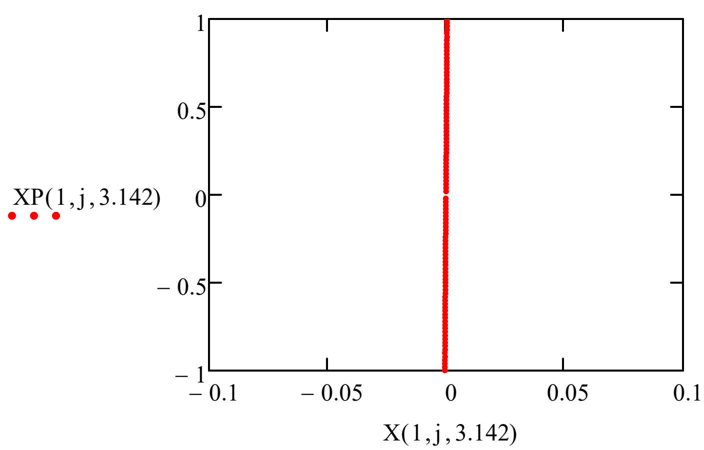

XP(1, j, 0.1571)
X(1, j, 0.1571)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1

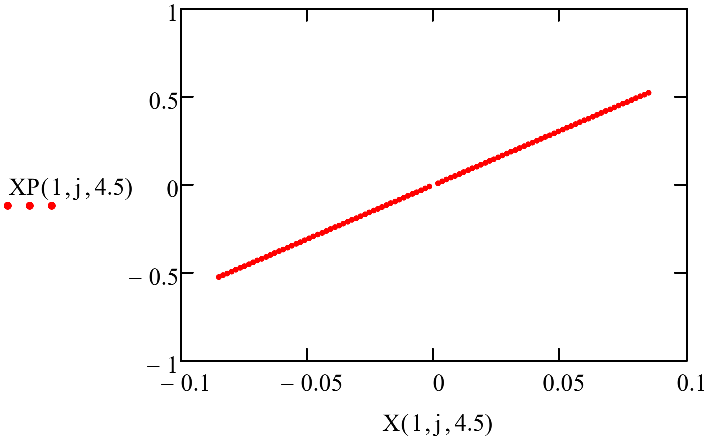

XP(1, j, 0.2)
X(1, j, 0.2)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1


XP(1, j, 0.25)
X(1, j, 0.25)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1


XP(1, j, 0.3142)
X(1, j, 0.3142)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1


xp
x0
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1


XP(1, j, 3.142)
X(1, j, 3.142)
1
0.5
0
− 0.5
− 1
− 0.1
− 0.05
0
0.05
0.1



$$X_{ave} := \frac{1}{N} \cdot \sum_{j} X\left(1, j, \frac{\lambda}{2}\right) = 0$$

$$XP_{ave} := \frac{1}{N} \cdot \sum_{j} XP\left(1, j, \frac{\lambda}{2}\right) = 2.646 \times 10^{-15}$$

$$X_{rms} := \sqrt{\frac{1}{N} \cdot \sum_{j} \left(X\left(1, j, \frac{\lambda}{2}\right) - X_{ave}\right)^2} = 3.439 \times 10^{-6}$$

$$XP_{rms} := \sqrt{\frac{1}{N} \cdot \sum_{j} \left(XP\left(1, j, \frac{\lambda}{2}\right) - XP_{ave}\right)^2} = 0.586$$

$$XXPcorr := \frac{1}{N} \cdot \sum_{i} \left[\sum_{j} \left[\left(X\left(1, i, \frac{\lambda}{2}\right) - X_{ave}\right) \cdot \left(XP\left(1, j, \frac{\lambda}{2}\right) - XP_{ave}\right)\right]\right] = \blacksquare$$

$$XXPcorr := 0$$

$$RMS\_Emitt := \sqrt{X_{rms}{}^2 \cdot XP_{rms}{}^2 - XXPcorr^2} = 2.015 \times 10^{-6}$$

$$Eff\_Emitt := 4 \cdot RMS\_Emitt = 8.0615 \times 10^{-6}$$

$$X_{ave2} := \frac{1}{N} \cdot \sum_{j} X(1, j, 5 \cdot \lambda) = 4.712 \times 10^{-8}$$

$$XP_{ave2} := \frac{1}{N} \cdot \sum_{j} XP(1, j, 5 \cdot \lambda) = -5.873 \times 10^{-7}$$

$$X_{rms2} := \sqrt{\frac{1}{N} \cdot \sum_{j} \left(X(1, j, 5 \cdot \lambda) - X_{ave2}\right)^2} = 3.642 \times 10^{-5}$$

$$XP_{rms2} := \sqrt{\frac{1}{N} \cdot \sum_{j} \left(XP(1, j, 5 \cdot \lambda) - XP_{ave2}\right)^2} = 0.585$$

$$XXPcorr2 := \frac{1}{N} \cdot \sum_{i} \left[\sum_{j} \left[\left(X(1, i, 5 \cdot \lambda) - X_{ave2}\right) \cdot \left(XP(1, j, 5 \cdot \lambda) - XP_{ave2}\right)\right]\right] =$$

$$XXPcorr2 := 0$$

$$RMS\_Emitt2 := \sqrt{X_{rms2}{}^2 \cdot XP_{rms2}{}^2 - XXPcorr2^2} = 2.132 \times 10^{-5}$$

$$Eff\_Emitt2 := 4 \cdot RMS\_Emitt2 = 8.529 \times 10^{-5}$$