Trace-Space Particle Distributions and Emittance

PhSp-Dist-Emitt-II3-Simplified.xmcd   S. Bernal 10-20-2014, 2-18-2022

Set array origin and fix seed for random # generator:

$$\text{ORIGIN} := 1 \qquad \text{Seed}(1) = 2.683 \times 10^{9}$$

NUMBER OF PARTICLES: $N := 100 \qquad i := 1 .. N$

INITIAL PARTICLE POSITIONS: $x0_i := 0 \qquad j := 1 .. N$

rand := 1 $\alpha := 1$

DEFINE UNIFORM SYMMETRICAL DISTRIBUTION OF INITIAL SLOPES FOR THE PARTICLES:

$$k := 1 .. \frac{N}{2} \qquad \delta := \frac{2 \cdot \alpha}{N}$$

$$xp1_k := -\alpha + (k - 1) \cdot \delta$$

$$xprand_j := (-\alpha \cdot rnd(1) + \alpha \cdot rnd(1)) \qquad xp2_k := -\alpha + \left(k + \frac{N}{2}\right) \cdot \delta$$

RANDOM OR UNIFORM DISTRIBUTION:

$$x_p := \begin{vmatrix} x_{prand} & \text{if } rand = 1 \\ stack(x_{p1}, x_{p2}) & \text{otherwise} \end{vmatrix}$$

**Element check:** $x_{p_1} = 0.192$ $x_{p_{\frac{N}{2}+1}} = 0.141$

$x_{p_{\frac{N}{2}}} = 0.259$ $x_{p_N} = 9.273 \times 10^{-4}$

FOCUSING CONSTANT IN m^-1: $k_0 := 10$
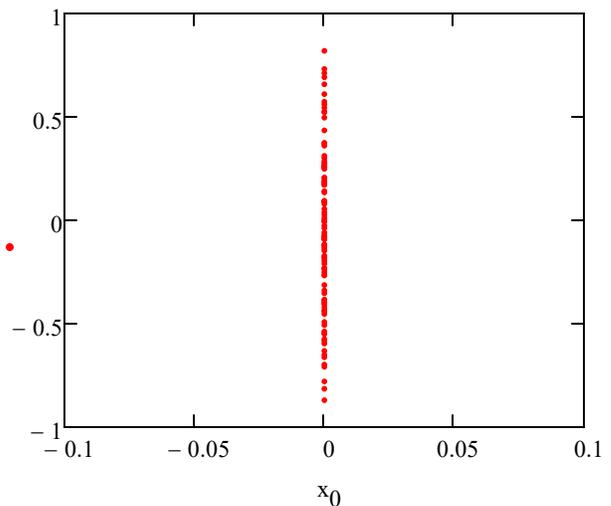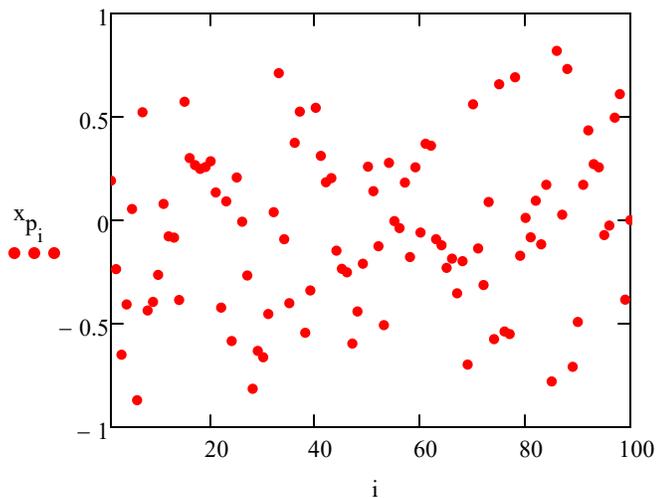
Given

$$\frac{d^2}{dz^2}x(z) = -k_0{}^2\,x(z) - k_0{}^4 \cdot x(z)^3 - k_0{}^2 \cdot \left(\frac{d}{dz}x(z)\right)^2 \cdot x(z)$$

$$x(0) = x0_i \qquad x'(0) = xp_j$$

$$x(i,j) := \text{Odesolve}(z,5)$$

$$X(i,j,z) := \begin{vmatrix} X1 \leftarrow x(i,j) \\ \overrightarrow{X1(z)} \end{vmatrix} \qquad\qquad XP(i,j,z) := \frac{d}{dz}X(i,j,z)$$

$X(1,10,z)$ (red solid line)

$X(50,50,z)$ (blue dashed line)

$XP(1,10,z)$ (red solid line)

$$X2(N,z) := \begin{vmatrix} \text{for } i \in 1..N \\ \quad \text{for } j \in 1..N \\ \qquad \begin{vmatrix} X1 \leftarrow X(i,j) \\ X2_{i,j} \leftarrow \overrightarrow{X1(z)} \end{vmatrix} \\ X2 \end{vmatrix}$$
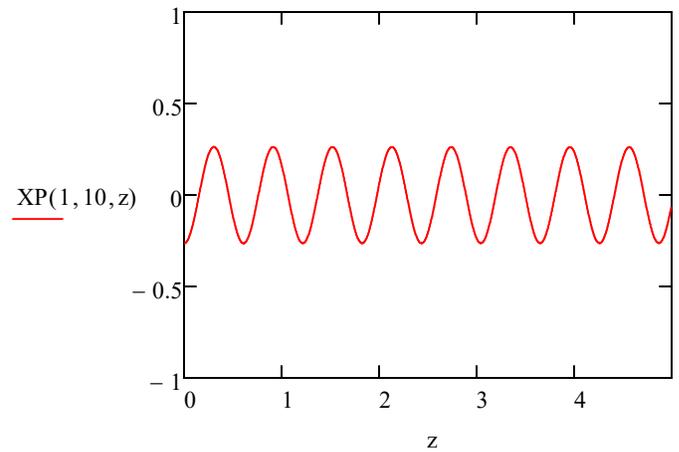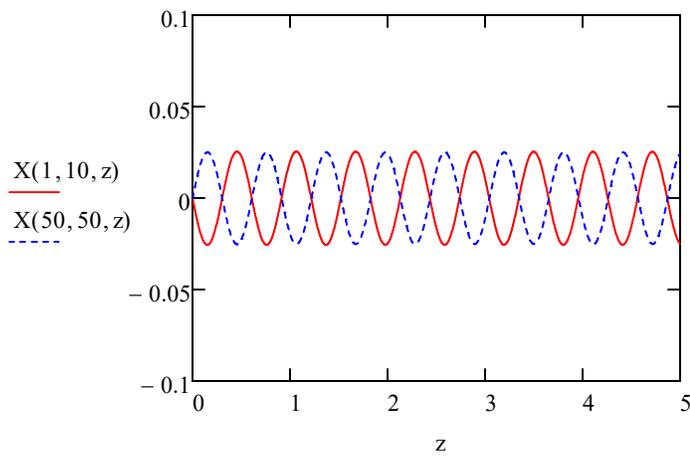
$$k := 1..10 \qquad z_k := \frac{k-1}{10} \qquad \text{Xresults} := X2(N,z)$$

$\text{Xresults}_{3,5}$
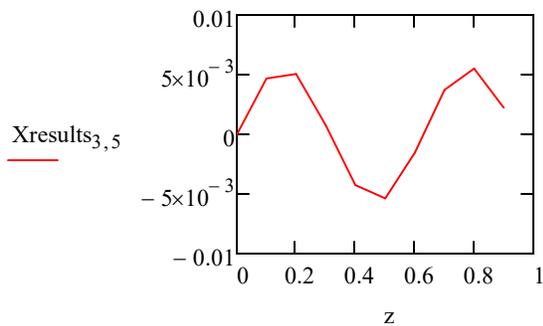
$$\lambda := \frac{2\cdot\pi}{k_0} = 0.6283 \qquad \frac{\lambda}{4} = 0.1571$$

$$\frac{\lambda}{2} = 0.3142 \qquad \frac{3\cdot\lambda}{4} = 0.4712 \qquad 5\cdot\lambda = 3.142$$

Plot 1: XP(1,j,0.05) vs X(1,j,0.05)

Plot 2: XP(1,j,0.1) vs X(1,j,0.1)

Plot 3: XP(1,j,0.1571) vs X(1,j,0.1571)

Plot 4: XP(1,j,0.2) vs X(1,j,0.2)

Plot 5: XP(1,j,0.25) vs X(1,j,0.25)

Plot 6: XP(1,j,0.3142) vs X(1,j,0.3142)

Plot 7: x_p vs x_0

Plot 8: XP(1,j,3.142) vs X(1,j,3.142)
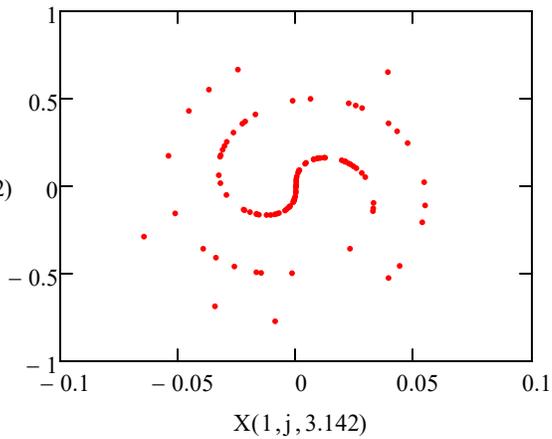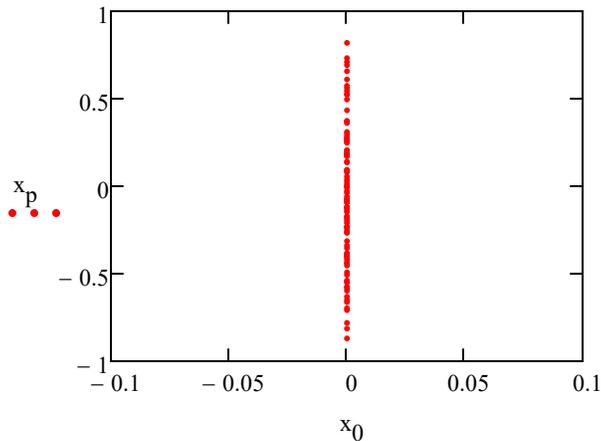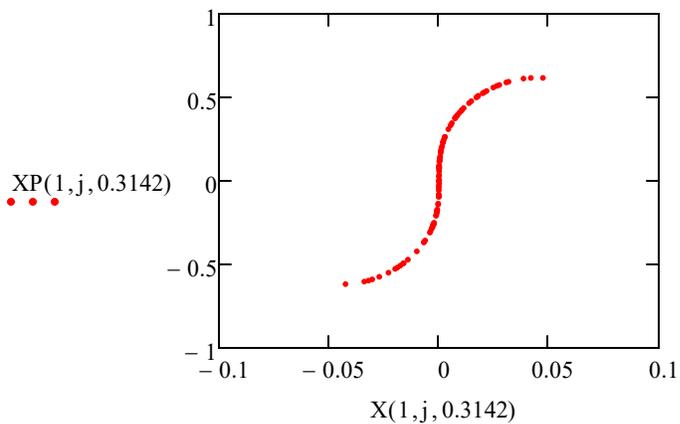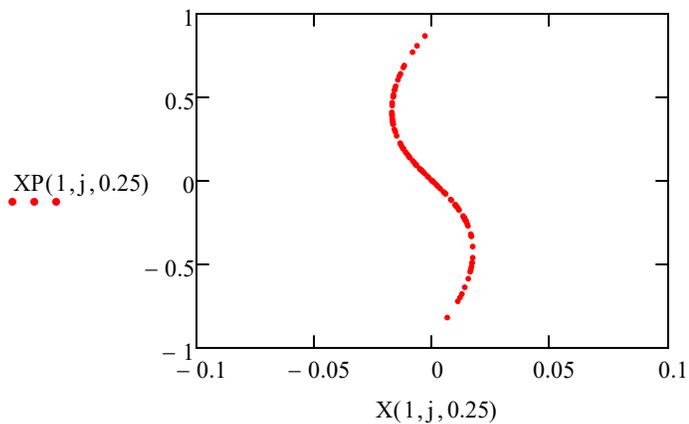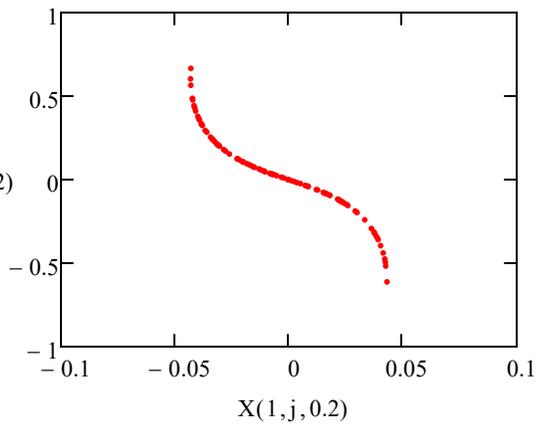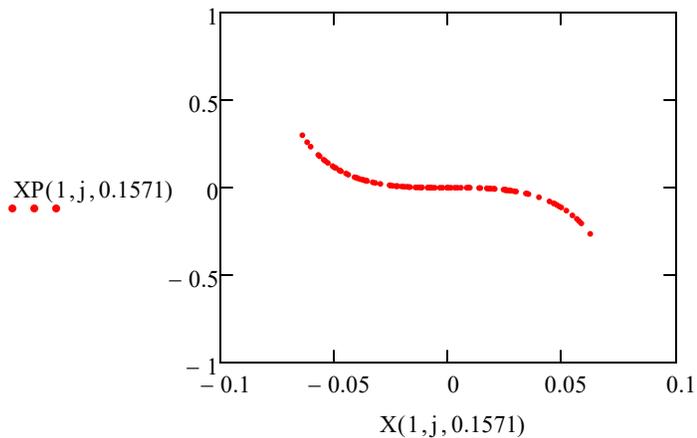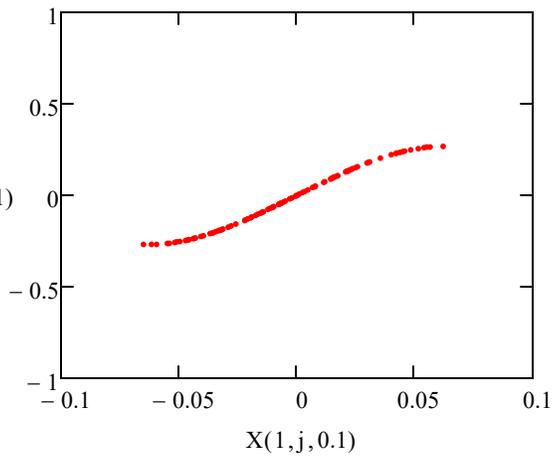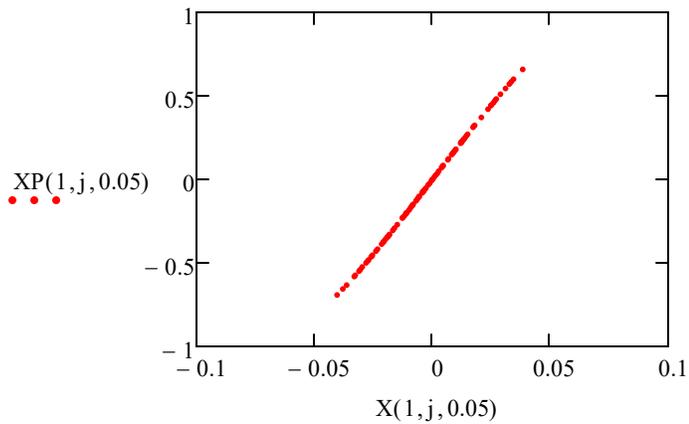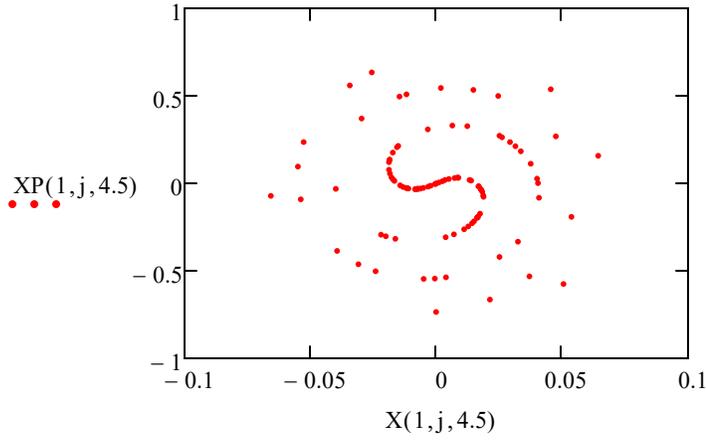
$$X_{ave} := \frac{1}{N} \cdot \sum_j X\left(1, j, \frac{\lambda}{2}\right) = 1.648 \times 10^{-3}$$

$$XP_{ave} := \frac{1}{N} \cdot \sum_j XP\left(1, j, \frac{\lambda}{2}\right) = 0.04$$

$$X_{rms} := \sqrt{\frac{1}{N} \cdot \sum_j \left(X\left(1, j, \frac{\lambda}{2}\right) - X_{ave}\right)^2} = 0.014$$

$$XP_{rms} := \sqrt{\frac{1}{N} \cdot \sum_j \left(XP\left(1, j, \frac{\lambda}{2}\right) - XP_{ave}\right)^2} = 0.353$$

$$XXPcorr := \frac{1}{N} \cdot \sum_i \left[\sum_j \left[\left(X\left(1, i, \frac{\lambda}{2}\right) - X_{ave}\right) \cdot \left(XP\left(1, j, \frac{\lambda}{2}\right) - XP_{ave}\right)\right]\right] = \blacksquare$$

$$XXPcorr := 0$$

$$RMS\_Emitt := \sqrt{X_{rms}^2 \cdot XP_{rms}^2 - XXPcorr^2} = 5.107 \times 10^{-3}$$

$$Eff\_Emitt := 4 \cdot RMS\_Emitt = 0.0204$$

$$X_{ave2} := \frac{1}{N} \cdot \sum_j X(1,j,5 \cdot \lambda) = -2.188 \times 10^{-4}$$

$$XP_{ave2} := \frac{1}{N} \cdot \sum_j XP(1,j,5 \cdot \lambda) = 0.02$$

$$X_{rms2} := \sqrt{\frac{1}{N} \cdot \sum_j \left(X(1,j,5 \cdot \lambda) - X_{ave2}\right)^2} = 0.025$$

$$XP_{rms2} := \sqrt{\frac{1}{N} \cdot \sum_j \left(XP(1,j,5 \cdot \lambda) - XP_{ave2}\right)^2} = 0.273$$

$$XXPcorr2 := \frac{1}{N} \cdot \sum_i \left[\sum_j \left[\left(X(1,i,5 \cdot \lambda) - X_{ave2}\right) \cdot \left(XP(1,j,5 \cdot \lambda) - XP_{ave2}\right) \right] \right] =$$

$$XXPcorr2 := 0$$

$$RMS\_Emitt2 := \sqrt{X_{rms2}^2 \cdot XP_{rms2}^2 - XXPcorr2^2} = 6.861 \times 10^{-3}$$

$$Eff\_Emitt2 := 4 \cdot RMS\_Emitt2 = 0.027$$