To appear in the first issue of *Meaning* (a new open access journal)

Simple Concepts and Meanings: Adequacy from Below

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Abstract

We offer a limited account of concept composition, formalized in terms of a calculus that has the computational complexity of propositional logic. We then show that small additions to this core system have dramatic effects, inviting an old idea that Pietroski (2018) develops: linguistic meanings are instructions for how to build concepts—symbols of a mental language—in constrained ways; more specifically, while some atomic concepts of the relevant mentalese are dyadic, all other concepts are monadic. In this respect, the posited generator is more Aristotelean than Fregean, and simpler than the lambda calculus of Church (1941). Icard and Moss (2023) make this vivid by establishing some results, which we review, about a precise version of the language Pietroski sketched. Here, we present the system in stages and stress a corresponding methodology: in providing theories of meaning, one can try to approach descriptive adequacy by modestly extending a model that undergenerates but captures some central phenomena, instead of starting with a powerful model that overgenerates; cp. Chomsky (1957, 1959, 1965). For example, our core system doesn't generate negations of concepts or mental correlates of relative clauses. But the final product, which is still a context free procedure, delivers conceptual analogs of 'thing that is not a cow'. Indeed, it is expressively equivalent to a monadic (one free variable) fragment of FOL. To help locate the proposed mentalese relative to more familiar models of how ideal concepts compose, we discuss some connections to modal logics and the variable-free system of Schönfinkel (1924).

1 Introduction

Humans regularly acquire languages whose expressions connect meanings of some kind with pronunciations of some kind. These expressions can be used for many purposes. But very often, they get used to express concepts. This raises questions about how meanings are related to concepts, and how semantic composition is related to conceptual composition.

In this paper, we offer a specific version of the idea that linguistic expressions are meaningful in part *because* they are systematically related to concepts that are composable symbols of a mental language that lets humans connect contents of some kind with biologically realized computations; cp. Fodor (1975, 1987, 2008). But instead of positing a powerful language of thought on the model of Church (1941) or other extensions of First Order Logic (FOL), we show how a spare concept generator can be modestly extended in ways that help explain a wide range of facts concerning composition and intuitively compelling inferences.

The posited system is surprisingly expressive. In some respects, it goes beyond FOL. But it remains simpler than FOL, in significant ways, because the generable complex concepts are all *monadic* and there is no operation of concept *negation*. Nonetheless, propositional logic is easily reconstructed in terms of concepts that apply to each thing or to nothing. The system also allows for mental analogs of 'Aristotle was not dumb', 'At midnight, a clever spy saw a nervous soldier smile', and 'Most logicians admired every spy who did not smile'; see Pietroski (2018), Icard and Moss (2023).

Linguistic meanings can be described as instructions for how to use the generator to build concepts. On this view, the meaning of 'clever spy' is an instruction that includes the meaning of 'spy' as a part, much like the concept CLEVER SPY includes the concept SPY as a part; see Pietroski (2005b, 2010, 2018), drawing on Chomsky (1995) and many others. In this sense, we offer a unified model of composition for linguistic meanings and the concepts that humans express linguistically, instead of merely saying that meanings determine semantic values that can be recursively specified. But our approach is "from below," starting with a basic system that can be supplemented in several ways.

Section two describes a possible mental language that is powerful enough to be interesting, though not remotely adequate as a metalanguage for natural language semantics. Sections three and four show how small additions can have striking effects. In section five, we show how the proposed language of thought can serve as the core of a theory of meaning for a spoken or signed language, while still leaving room to distinguish meanings from concepts.

2 A Minimal Predicate Logic

In this section, we describe a simple procedure that generates complex monadic concepts from atomic monadic concepts. Propositional logic can be reconstructed via insights from Tarski. The system also captures the intuitively compelling character of inferences—e.g., 'Miss Scarlet saw Colonel Mustard in the library, hence Miss Scarlet saw Colonel Mustard'—that are often described as instances of conjunct reduction in the scope of an existential quantifier; cp. Davidson (1967). This already raises questions about whether the apparatus of FOL, let alone Frege (1879) or Church (1941), is too powerful if the goal is to model the competence that speakers use to understand the premises/conclusions of such inferences; cp. Chomsky (1959) on the power of context-sensitive rewrite rules.

2.1 Two Simple Rules

For present purposes, think of languages as procedures that generate expressions, rather than sets of generable expressions.[1] Let **MPL** be a possible mental language that might also be described as a minimal predicate logic. Every expression of **MPL**, atomic or complex, is a monadic concept—a mental predicate with which a thinker can think about things in a certain way. These concepts have application conditions. The concept COW, with which one can think about something as a cow, applies to a thing if and only if that thing is a cow.

[1]See Chomsky (1986) on the notion of an I-language, echoing Church (1941) on functions in intension and Frege (1891) on Functions as opposed to Courses of Values; cp. Lewis (1975). A mental language can be a procedure that connects biochemical signals with contents of some kind.

There are two formation rules of **MPL**, each corresponding to a rule of interpretation.

- If Φ and Ψ are concepts of **MPL**, so is $(\Phi \wedge \Psi)$.

  For each entity e, $(\Phi \wedge \Psi)$ applies to e if and only if Φ applies to e and Ψ applies to e.

- If Φ is a concept of **MPL**, so is $\Downarrow\Phi$.

  For each entity e, $\Downarrow\Phi$ applies to e if and only if Φ doesn't apply to anything.

Any two concepts can be "joined" to form a third; and junction is conjunction. Moreover, any concept can be "polarized" to form a concept that applies to each thing (in the relevant domain) or to nothing, depending on whether or not the initial concept applies to nothing. That's it. There's no more to **MPL**. But the second rule calls for a little explication.

If Φ is a concept that doesn't apply to anything, then each thing is such that $\Downarrow\Phi$ applies to it. But if Φ applies to at least one thing, then $\Downarrow\Phi$ doesn't apply to anything. If it helps, read '$\Downarrow\Phi$' as 'is such that nothing is Φ'. If there is no cow in the relevant domain, and so each thing is such that nothing is a cow, then $\Downarrow$COW applies to each thing. But if there is a cow in the relevant domain, then $\Downarrow$COW applies to nothing.

To take another example, suppose that SQUARE and CIRCLE are atomic concepts of **MPL**; where for each entity e, SQUARE applies to e if and only if (henceforth 'iff') e is a square, and CIRCLE applies to e iff e is a circle. Then given that no square is a circle, (SQUARE $\wedge$ CIRCLE) is a concept of **MPL** that applies to nothing, and so $\Downarrow$(SQUARE $\wedge$ CIRCLE) is a concept of **MPL** that applies to each thing. By contrast, let RED be an atomic concept of **MPL** that applies to e iff e is red; and suppose that at least one thing is both a circle and red. Then (CIRCLE $\wedge$ RED) applies to something, and so $\Downarrow$(CIRCLE $\wedge$ RED) applies to nothing.

Conjoining a concept Φ with another concept is a form of restriction. The conjunction applies to each thing in some subset, perhaps improper, of the things that Φ applies to. The limiting case is a concept of the form $(\Phi\wedge\Psi)$ that applies to nothing, e.g. (SQUARE $\wedge$ CIRCLE). If a concept applies to anything, then polarization takes it "to the bottom." Given a square, $\Downarrow$SQUARE is materially equivalent to (SQUARE $\wedge$ CIRCLE); both concepts apply to nothing.

If a concept applies to nothing, polarization takes it "to the top;" $\Downarrow(\text{SQUARE} \wedge \text{CIRCLE})$ applies to each thing, since $(\text{SQUARE} \wedge \text{CIRCLE})$ applies to nothing. But apart from cases of polarizing concepts that apply to nothing, the formation rules of **MPL** yield concepts that apply to (perhaps improper) subsets of what their immediate constituents apply to.

This feature of **MPL** ensures that it provides no way of converting each generable concept Φ into a concept that applies to whatever Φ *doesn't* apply to. In this sense, **MPL** has no operation of negation/complementation. Obviously, **MPL** has no primitive symbol '$\neg$' that can attach to generable concepts. But the broader point is that there is no general recipe for using a concept Φ of **MPL** to build a concept that applies to e iff Φ does not apply to e. Icard and Moss (2023) offer a proof whose key idea is easily explained.

Consider a domain consisting of exactly two entities, El and Em. Let Φ be an atomic concept that applies to El and doesn't apply to Em. An **MPL** analog of $\neg\Phi$ would be a concept that applies to Em but not El. Adding a restriction to Φ can't do the job; any concept of the form $(\Phi \wedge \Psi)$ will either apply to nothing, or still apply to El but not Em. The only way to go from Φ to a concept that applies to Em but not El is to "go to the top" and then add a restriction that applies to Em but not El. Getting to the top is easy. For any concept Φ, $(\Phi \wedge \Downarrow\Phi)$ applies to nothing, and so $\Downarrow(\Phi \wedge \Downarrow\Phi)$ applies to everything; nothing is both Φ and such that nothing is Φ. So $\Downarrow(\Phi \wedge \Downarrow\Phi)$ applies to both El and Em. But to convert $\Downarrow(\Phi \wedge \Downarrow\Phi)$ into a concept that applies to Em but not El, one needs to restrict with a concept, Ψ, that applies to Em but not El; so using Φ to build $\Downarrow(\Phi \wedge \Downarrow\Phi)$ is irrelevant. Of course, there may *be* at least one concept Ψ that applies to Em but not El, with the result that endlessly many concepts of **MPL**—Ψ, $(\Downarrow(\Phi \wedge \Downarrow\Phi) \wedge \Psi)$, etc.—apply to Em but not El. But one can't use **MPL** to build up such a concept from Φ.

A similar argument shows that **MPL** can't be used to build up, from Φ and Ψ, a concept that applies to an entity iff *either* Φ applies to it *or* Ψ applies to it. Though as discussed in §2.2, given Φ and Ψ, **MPL** can be used to build $\Downarrow(\Downarrow\Phi \wedge \Downarrow\Psi)$, which applies to each thing iff *either something* is Φ *or something* is Ψ.

Initially, the absence of predicate negation might seem like a defect. Familiarity with FOL and set theory can make complementation seem like an obvious choice for a basic operation. But it's far from obvious that COW can be combined with an operator to form a concept, ¬COW, with which one can think about things as things that aren't cows. Phrases like 'thing which is not a cow', in which 'thing' is restricted by a relative clause that includes 'not' and a tense marker, suggest that humans can build concepts equivalent to a complement of COW. But there may not be a natural complementation operator. As discussed in section four, concepts expressed with relative clauses may be built by *abstracting* a constituent from a sentential/polarized concept that is generated in accord with a modest extension of **MPL**.

Following Aristotle and many others, we suspect that humans *don't* have a capacity to convert an atomic concept COW—as opposed to the complex thought that an indicatable thing isn't a cow—into a concept that applies to the number 17, the color blue, galaxies, and everything else that isn't a cow. Bringing this motley under a single concept may be a cognitive achievement that involves polarization *and* a kind of abstraction that requires additional computational resources, as opposed to a primitive operator that can somehow combine with a concept Φ to form a concept that applies to whatever Φ doesn't apply to. Animals seem to abhor complements of concepts they find natural; and even among humans, complement concepts seem like gruesome inventions—cp. Goodman (1983)—that we can't spontaneously think with.[2] So in our view, it's a virtue of **MPL** (as a candidate core mental language) that it doesn't provide the resources needed for predicate negation.

At a minimum, theorists can't exclude the possibility that linguistic comprehension employs an analog of '$\Downarrow$' that is more basic than any analog of complementation. One is free to not care about how humans connect sentences like 'Aristotle was not dumb' with mental representations. But our proposal belongs to the long tradition of caring; see §2.4 below.[3]

[2]See, e.g., Bruner et al. (1956). We return, briefly, to concepts like UNFRIENDLY in section three. But adding a mental analog of 'un', as an operator that can be combined with certain atomic concepts, would not significantly change the complexity or expressive power of **MPL**.

[3]And inquirers can't stipulate that theories of meaning should prescind from psychological considerations. For extended discussion, see Pietroski (ms), which summarizes experimental findings from Knowlton et al. (2023b,a, 2022, 2021); Odic et al. (2018); Lidz et al. (2011); Pietroski et al. (2009).

Once it is clear that **MPL** does not support concept negation, it should be unsurprising that **MPL** is about as simple as a language with any interesting logical structure could be. It has the same computational complexity as a standard propositional calculus. Even when **MPL** is supplemented in the ways described in section three, reasoning in the system is no harder than reasoning in propositional logic; see Icard and Moss (2023). Initially, **MPL** might have seemed to be logically trivial. But conjunction and polarization interact in interesting ways, suggesting that **MPL** merits attention, if only as a way of seeing that more powerful systems are not *needed* to do what **MPL** can do.

In what follows, it will be convenient to use '$\Uparrow$' as shorthand for '$\Downarrow\Downarrow$'. For any entity, e, a concept of the form $\Downarrow\Downarrow\Phi$ applies to e iff $\Downarrow\Phi$ applies to nothing—i.e., iff Φ applies to something. So $\Downarrow\Downarrow\Phi$, alias $\Uparrow\Phi$, applies to e iff Φ applies to something.[4] Put another way: $\Uparrow\Phi$ applies to everything or nothing, depending on whether or not Φ applies to something; $\Downarrow\Phi$ applies to nothing or everything, depending on whether or not Φ applies to something. So $\Uparrow\Phi$ can be seen as an **MPL** analog of '$\exists x\Phi x$', a familiar FOL translation of 'There is a Φ'. Likewise, $\Downarrow\Phi$ can be seen as an **MPL** analog of '$\neg\exists x\Phi x$'. Nonetheless, **MPL** has no variables and no symbol for propositional negation. We return to this important point below. But first, let's be clear about how **MPL** is related to propositional logic, which can also be characterized without a negation operator; cp. Sheffer (1913), Peirce (1980).

2.2 Propositional Logic as Polarized Predicate Logic

Note that $\Uparrow\Uparrow\Phi$ is equivalent to $\Uparrow\Phi$. For each entity, $\Uparrow\Uparrow\Phi$ applies to it iff $\Uparrow\Phi$ applies to something; and $\Uparrow\Phi$ applies to something iff Φ applies to something. So like $\Uparrow\Phi$, $\Uparrow\Uparrow\Phi$ applies to everything or nothing, depending on whether or not Φ applies to something. Moreover, $\Downarrow\Uparrow\Phi$ is equivalent to $\Uparrow\Downarrow\Phi$: $\Downarrow\Uparrow\Phi$ applies to e iff $\Uparrow\Phi$ applies to nothing (i.e., iff Φ applies to nothing); and $\Uparrow\Downarrow\Phi$ applies to e iff $\Downarrow\Phi$ applies to something (i.e., iff Φ applies to nothing).

[4] If '$\Uparrow$' is introduced as a second polarizer, the inferential relation between '$\Uparrow$' and '$\Downarrow$' has to be stipulated. But even if '$\Uparrow$' is introduced as shorthand for '$\Downarrow\Downarrow$', *endorsing* $\Uparrow$COW may be psychologically easier than *endorsing* $\Downarrow$COW, once the shorthand is used and connected with capacities to perceive cows.

As these points suggest, polarized concepts are like truth-evaluable propositions. For each instance of $\Downarrow\Phi$, there is the corresponding instance of $\Downarrow\Downarrow\Phi$. And for any such pair of concepts, either the first applies to everything while the second applies to nothing (i.e., nothing is Φ) or the first applies to nothing while the second applies to everything (i.e., something is Φ).

Similarly, given any instances of $\Downarrow\Phi$ and $\Downarrow\Psi$, there are only four logical possibilities regarding what they apply to: they both apply to everything; the first applies to everything and the second applies to nothing; the first applies to nothing and the second applies to everything; or they both apply to nothing. The first possibility is the only one in which $(\Downarrow\Phi \wedge \Downarrow\Psi)$ applies to anything; in which case, $(\Downarrow\Phi \wedge \Downarrow\Psi)$ applies to everything as does the equivalent concept $\Downarrow\Downarrow(\Downarrow\Phi \wedge \Downarrow\Psi)$, alias $\Uparrow(\Downarrow\Phi \wedge \Downarrow\Psi)$. Correlatively, the fourth possibility is the only one in which $(\Downarrow\Downarrow\Phi \wedge \Downarrow\Downarrow\Psi)$—alias $(\Uparrow\Phi \wedge \Uparrow\Psi)$—applies to anything/everything. So the fourth possibility is the only one in which $\Downarrow(\Downarrow\Downarrow\Phi \wedge \Downarrow\Downarrow\Psi)$, alias $\Downarrow(\Uparrow\Phi \wedge \Uparrow\Psi)$, applies to nothing. These facts are recorded in the "polarization table" shown in Figure 1.[5]

Figure 1: Polarization Table for $\Downarrow\Phi$ and $\Downarrow\Psi$

$\Downarrow\Phi$	$\Uparrow\Phi$ $\Downarrow\Downarrow\Phi$	$\Downarrow\Psi$	$\Uparrow\Psi$ $\Downarrow\Downarrow\Psi$	$(\Downarrow\Phi \wedge \Downarrow\Psi)$	$\Uparrow(\Downarrow\Phi \wedge \Downarrow\Psi)$ $\Downarrow\Downarrow(\Downarrow\Phi \wedge \Downarrow\Psi)$	$(\Uparrow\Phi \wedge \Uparrow\Psi)$ $(\Downarrow\Downarrow\Phi \wedge \Downarrow\Downarrow\Psi)$	$\Downarrow(\Uparrow\Phi \wedge \Uparrow\Psi)$ $\Downarrow(\Downarrow\Downarrow\Phi \wedge \Downarrow\Downarrow\Psi)$
E	N	E	N	E	E	N	E
E	N	N	E	N	N	N	E
N	E	E	N	N	N	N	E
N	E	N	E	N	N	E	N

As indicated in the penultimate column, $\Uparrow\Phi \wedge \Uparrow\Psi$ applies to everything iff both $\Downarrow\Phi$ and $\Downarrow\Psi$ apply to nothing; similarly, as the fifth column shows, $\Downarrow\Phi \wedge \Downarrow\Psi$ applies to everything iff both $\Downarrow\Phi$ and $\Downarrow\Psi$ apply to everything. Put another way: $\Uparrow(\Uparrow\Phi \wedge \Uparrow\Psi)$ applies to everything iff something is Φ *and* something is Ψ; $\Downarrow(\Downarrow\Phi \wedge \Downarrow\Psi)$ applies to everything iff something is Φ *or* something is Ψ. So while **MPL** does not support complementation or disjunction for unpolarized concepts, it does support analogs of propositional negation and disjunction.

[5]In this table, the first line corresponds to the possibility in which everything is such that $\Downarrow\Phi$ and everything is such that $\Downarrow\Psi$. This possibility can also be described as the one in which nothing is such that $\Uparrow\Phi$ and nothing is such that $\Uparrow\Psi$; or equivalently, the one in which everything is such that $\Downarrow\Downarrow(\Downarrow\Phi\wedge\Downarrow\Psi)$; etc.

The parallels to standard truth tables are not accidental. Tarski (1936, 1944) showed how to treat propositional logic as a special case of predicate logic, with propositions as predicates of adicity zero. Being explicit about this may help clarify how a minimal *predicate* logic can serve as a propositional logic, and how a *minimal* predicate logic—with no variables and no operation for negation—can have analogs of '∃xΦx' and '¬∃xΦx'.

The sentences of FOL are quite diverse. In addition to closed sentences like 'Fa & Gb' and '∃x(Fx & Gx)', the open sentences include 'Fx & Gx', 'Fx & Gy', '¬Fx', 'Rxy & ¬Gz', '∃x(Fx & Gy)', etc. Students eventually accept that the ampersand of FOL is not defined by the comforting truth table they initially learned. Rather, '&' can be flanked on the left by a sentence S_1 that has n free variables, and flanked on the right by a sentence S_2 that has m free variables (for any natural numbers n and m, $n \geq m$); where the number of free variables in 'S_1 & S_2' may be as high as $n + m$, as low as n, or any number in between. Likewise, '¬' can prefix a sentence that has any number of free variables.

Providing a semantics for such a language was no small trick. Tarski treated sentences of FOL as predicates that are satisfied by *sequences* of domain-entities; and then he stipulated that a sentence is true iff it is satisfied by *every* sequence. But each concept of **MPL** is monadic and variable-free; so appeal to sequences is unnecessary. (Or if you prefer, each **MPL**-sequence can be described as a trivial sequence consisting of a single entity.) Correlatively, one can say that a polarized concept of **MPL** is true iff it applies to everything.

Moreover, unlike a mere propositional calculus, **MPL** permits conjunction of monadic concepts. So **MPL** is not logically trivial. On the contrary, it generates variable-free analogs of '∃x(Fx & Gx)' and '¬∃x(Fx & Gx)', despite having no operation for negation. To see how this is possible, it can help to think about a Tarski-calculus that is subject to a pair of severe constraints: there is only one variable; and an atomic sentence—i.e., a sentence with no sentential constituents—can only contain one occurrence of this variable (cp. 'Rxx').

Tarski handled instances of '∃x(...x...)' by invoking sequence *variants*. A sequence σ satisfies '∃x(...x...)' iff '(...x...)' is satisfied by a sequence σ^* that is just like σ except perhaps for what σ^* assigns to 'x'. Abbreviating: σ satisfies '∃x(...x...)' iff an 'x'-cousin of σ satisfies '(...x...)'. If 'Mx' is satisfied by and only by the sequences that assign a moose to 'x', then for each sequence, it satisfies '∃x(Mx)' iff there is a moose in the domain.[6] Even if 'x' is the only variable and each atomic sentence contains exactly one occurrence of 'x', this still allows for sentences like '∃xMx', '¬∃xMx', and '∃x(¬Mx & Px)'. But then there is no need for variables, no need for appeal to sequences, and no need for the very permissive conjoiner '&', which can be flanked by sentences with any number of free variables. We could rewrite the generable sentences as '∃M', '¬∃M', and '$\exists(\neg\mathrm{M} \wedge \mathrm{P})$'. And each such sentence could be viewed as a predicate that applies to everything or to nothing. Correlatively, one wouldn't have to assume that the negation prefix in '¬M' also appears in '¬∃M'.

Instead of specifying the satisfaction/application condition for '¬∃M' in two stages—via two syncategorematic rules—one could replace '¬∃' with '∄' and say that for any predicate Φ and entity e, '$\nexists\Phi$' applies to e iff Φ doesn't apply to anything. Put another way, '$\nexists\Phi$' applies to e iff no domain-cousin of e satisfies Φ; where each thing is one of its domain-cousins. Then '∃' can be treated as shorthand for '∄∄'. At this point, replacing '∄' and '∃' with '⇓' and '⇑' is just notational variation. The only remaining difference from **MPL** is that instances of '$\nexists(\neg\Phi)$' are permitted, with '¬' as a sign for negation of open sentences. (We leave for another day the task of saying which fragment of FOL this is.)

Polarized concepts are like Tarskian closed sentences. To make this analogy explicit, let '$\mathrm{C1} \leq \mathrm{C2} \leq \mathrm{C3}$' mean that for any entity, e: if C1 applies to e, so does C2; and if C2 applies to e, so does C3. Then for any concepts Φ and Ψ, $(\Phi \wedge \Psi) \leq \Phi \leq \Downarrow(\Phi \wedge \Downarrow\Phi)$. Correlatively, the patterns indicated below hold for any concepts Φ and Ψ.

- $\Uparrow(\Phi \wedge \Psi) \leq \Uparrow(\Uparrow(\Phi) \wedge \Uparrow(\Psi)) \leq \Uparrow(\Phi)$
- $\Downarrow(\Phi) \leq \Downarrow(\Uparrow(\Phi) \wedge \Uparrow(\Psi)) \leq \Downarrow(\Phi \wedge \Psi)$

These are **MPL** analogs of valid inference patterns that are often described in terms of FOL.

[6]Sentences with multiple free variables are accommodated the same way. For example, '∃x(Mx & Rxy)' is satisfied by σ iff 'Mx & Rxy' is satisfied by some 'x'-cousin of σ; and whatever σ assigns to 'y', each 'x'-cousin of σ assigns the same thing to 'y'.

- $\exists x(Fx \ \& \ Gx) \vDash \exists x(Fx) \ \& \ \exists x(Gx) \vDash \exists x(Fx)$

- $\neg\exists x(Fx) \vDash \neg(\exists x(Fx) \ \& \ \exists x(Gx)) \vDash \neg\exists x(Fx \ \& \ Gx)$

2.3 Diamond Patterns without Variables

Now consider the implications exhibited by (1)-(9); cp. Davidson (1967), Taylor (1984).

(1) Miss Scarlet poked Colonel Mustard with a feather in the library.
(2) Miss Scarlet poked Colonel Mustard with a feather.
(3) Miss Scarlet poked Colonel Mustard in the library.
(4) Miss Scarlet poked Colonel Mustard.
(5) Miss Scarlet poked Colonel Mustard in the kitchen.
(6) Miss Scarlet poked Colonel Mustard with a spoon.
(7) Miss Scarlet poked Colonel Mustard in the kitchen with a spoon.
(8) Miss Scarlet poked Colonel Mustard with a feather in the kitchen.
(9) Miss Scarlet poked Colonel Mustard in the library with a spoon.

Sentence (1) implies (2) and (3), each of which implies (4); but the conjunction of (2) and (3) doesn't imply (1). Similarly, (7) implies (6) and (5), each of which implies (4); but the conjunction of (5) and (6) doesn't imply (7). Sentences (1)-(7) can be true while (8) and (9) are false. Yet (8) still implies the conjunction of (2) and (5); and (9) still implies the conjunction of (6) and (3). The one-way implications are summarized in Figure 2.

Figure 2: Diamond Pattern of One-Way Implications

$$\begin{tikzcd}
 & & (1) \arrow[dl] \arrow[dr] & & \\
 & (2) \arrow[dr] & & (3) \arrow[dl] & \\
(8) \arrow[ur] \arrow[dr] & & (4) & & (9) \arrow[ul] \arrow[dl] \\
 & (5) \arrow[ur] & & (6) \arrow[ul] & \\
 & & (7) \arrow[ul] \arrow[ur] & &
\end{tikzcd}$$

Sentences (10)-(18) exhibit the same pattern—ignoring readings where a second modifying clause modifies the noun in the first—as do the indicated FOL regimentations.

(10)	There was a cat on a mat that sat.	$\exists$x[Ax & Bx & Cx]
(11)	There was a cat on a mat.	$\exists$x[Ax & Bx]
(12)	There was a cat that sat.	$\exists$x[Ax & Cx]
(13)	There was a cat.	$\exists$x[Ax]
(14)	There was a cat that slept.	$\exists$x[Ax & Dx]
(15)	There was a cat on a bed.	$\exists$x[Ax & Ex]
(16)	There was a cat on a bed that slept.	$\exists$x[Ax & Dx & Ex]
(17)	There was a cat on a mat that slept.	$\exists$x[Ax & Bx & Ex]
(18)	There was a cat on a bed that sat.	$\exists$x[Ax & Dx & Cx]

The implications exhibited by the FOL sentences are instances of conjunct reduction in the scope of an (un-negated) existential quantifier. This invites the hypothesis that the implications exhibited by the English sentences are instances of a similar form.

With regard to (1)-(9), the idea is that (i) each prepositional phrase corresponds to a monadic concept that can be conjoined with others, (ii) the tensed clause 'Miss Scarlet poked Colonel Mustard' also corresponds to a monadic concept, and (iii) the complete sentence involves some way of "capping off" the tensed clause with its modifiers, corresponding to existential closure of a variable. This in turn suggests that the tensed clause corresponds to a concept of *events*, and more specifically, past events of Miss Scarlet poking Colonel Mustard. In which case, the prepositional phrases in (1)-(9) correspond to concepts of events that can be conjoined with a concept like PastPokeOfColMustardByMsScarlet.[7]

One can imagine creatures who use the resources of FOL to form mental sentences like (19); where 'e' ranges over a domain that includes events in which entities participate.

[7]Davidson (1967) provides the basic argument, elaborated by many others. As Ramsey (1927) had noted, the *fact* that Brutus stabbed Caesar is like an existential generalization that can be verified by mulitple stabbings; and see Gillon (2007) on Pāṇini's treatment of Sanskrit action reports. Taylor (1984)—drawing on Gareth Evans' example of simultaneous events with the same participants—makes it clear why type shifting accounts of adjectival/adverbial modifiers are inadequate; cp. Parsons (1970) and Kamp (1975). For further discussion, see Pietroski (2022) and references there.

(19) $\exists$e[$\textsc{PastPokeOfColMustardByMsScarlet}$(e)
& $\textsc{WithSomeFeather}$(e) & $\textsc{InTheLibrary}$(e)]

Of course, one wants to know how the conjuncts in (19) are built up from atomic constituents. And given the resources of FOL, it's easy to imagine decomposing (19) as in (20).

(20) $\exists$e[$\textsc{Past}$(e) & $\exists$x$\exists$y[$\textsc{PokeOfBy}$(e, y, x) & $\textsc{ColMustard}$(y) & $\textsc{MsScarlet}$(x)]
& $\exists$x[$\textsc{DoneWith}$(e, x) & $\textsc{Feather}$(x)] & $\exists$x[$\textsc{LocatedIn}$(e, x) & $\textsc{TheLibrary}$(x)]]

But if the goal is to capture the pattern of conjunct-reducing implications—without *yet* worrying about how the conjuncts are formed—there is no need for the power of FOL and its variables, along with the Tarski-clever operators '$\exists$' and '&'. For these purposes, **MPL** is enough. Illustrations like (19) can be replaced with sparer alternatives like (21), letting 'WSF' and 'ITL' abbreviate '$\textsc{WithSomeFeather}$' and '$\textsc{InTheLibrary}$'.

(21) $\Uparrow$(($\textsc{PastPokeOfColMustardByMsScarlet} \wedge$ WSF) $\wedge$ ITL)

Again, this highlights the question of how the conjuncts are formed; see §3.1 below. But in thinking about how to describe the relevant implications, many considerations are germane, especially if the explananda are at least partly psychological. In our view, the theoretical goal is not—or at least not merely—to specify a semantic relation that certain pairs of sentences exhibit (e.g., being such that every model of the first is a model of the second). For competent speakers, the implications summarized in Figure 2 above seem trivial. It feels like there is nothing to "work out." One just knows that deleting a prepositional phrase in (1)-(9) corresponds to an intuitively impeccable inference. This invites the hypothesis that (i) the English sentences are understood as expressing thoughts that are relevantly like existential closures of conjunctions of predicates, and (ii) competent speakers enjoy a "mental logic" that makes the implications seem as obvious as instances of 'P and Q, so P'.

Figure 3 shows the diamond pattern for the **MPL** analogs of (1)-(9), using '$\exists$' instead of '$\Uparrow$' for easy readabilty. The only inferential principle required, given this way of encoding the implications, is the obvious one: for any concepts Φ and Ψ, if the polarized concept $\Uparrow(\Phi \wedge \Psi)$ applies to something/everything, then so does the "reduced" polarized concept $\Uparrow(\Phi)$.[8]

Figure 3: Diamond Pattern for **MPL**, using '$\exists$' instead of '$\Uparrow$'

$$\begin{tikzcd}
 & & \exists((\mathrm{P} \wedge \mathrm{F}) \wedge \mathrm{L}) \arrow[dl] \arrow[dr] & & \\
 & \exists(\mathrm{P} \wedge \mathrm{F}) \arrow[dr] & & \exists(\mathrm{P} \wedge \mathrm{L}) \arrow[dl] & \\
\exists((\mathrm{P} \wedge \mathrm{F}) \wedge \mathrm{K}) \arrow[ur] \arrow[dr] & & \exists(\mathrm{P}) & & \exists((\mathrm{P} \wedge \mathrm{L}) \wedge \mathrm{S}) \arrow[ul] \arrow[dl] \\
 & \exists(\mathrm{P} \wedge \mathrm{K}) \arrow[ur] & & \exists(\mathrm{P} \wedge \mathrm{S}) \arrow[ul] & \\
 & & \exists((\mathrm{P} \wedge \mathrm{K}) \wedge \mathrm{S}) \arrow[ul] \arrow[ur] & &
\end{tikzcd}$$

2.4 Starting Small

Positing **MPL** commits us to complex monadic concepts of two kinds, conjunctive and polarized. A more familiar starting point, inspired by Frege (1891) and Church (1941), assumes endlessly many potential semantic types: two basic types, $\langle$e$\rangle$ and $\langle$t$\rangle$, exhibited by entity designators and complete sentences that designate truth values; and for any types $\langle\alpha\rangle$ and $\langle\beta\rangle$, the higher type $\langle\alpha, \beta\rangle$, corresponding to functions that map things of the sort indicated with expressions of type $\langle\alpha\rangle$ onto things of the sort indicated with expressions of type $\langle\beta\rangle$. These types can be described in terms of a hierarchy reminiscent of the iterative concepts of sets; see Boolos (1971). The basic types lie at Level-0; at Level-1, there are the four types—$\langle$e, e$\rangle$, $\langle$e, t$\rangle$, $\langle$t, e$\rangle$, and $\langle$t, t$\rangle$—that can be formed directly from the two basic types; at Level-2, there are the thirty-two types constructible from those at lower levels, including $\langle$e, $\langle$e, t$\rangle\rangle$ and $\langle\langle$e, t$\rangle$, t$\rangle$ but also odd types like $\langle\langle$e, e$\rangle$, $\langle$t, e$\rangle\rangle$ and $\langle\langle$t, e$\rangle$, e$\rangle$; 1408 more types at Level-3; another two million or so at Level-4; etc. But despite all this descriptive power, the Frege-Church typology seems inadequate for 'red square'.[9]

[8] See Icard and Moss (2023) for a completeness proof of a system that extends **MPL** as in section three.

[9] One can think about the monadic concepts of **MPL** as analogs of expressions of the types $\langle$e, t$\rangle$ and $\langle$t$\rangle$. But **MPL** generates no concepts that designate entities or truth values. Frege's interests, which concerned the foundations of arithmetic, lay with functions at Level-4; see Pietroski (2018) for discussion.

One can say that in 'That red square is a square which is red', the first occurrence of 'red' counts as an instance of the Level-2 type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$ while the other occurrence of 'red' counts as an instance of type $\langle e, t\rangle$. More specifically, one can say that core uses of 'red' indicate the function λx.Red(x)—much as 'square' indicates the function λx.Square(x)—but that modifying uses of 'red' indicate the type-*lifted* function λΦ.λx.Red(x) & Φ(x), which maps λx.Square(x) onto λx.Red(x) & Square(x), which is of type $\langle e, t\rangle$. One can then say that the modifying clause 'which is red' is also of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$ even if the embedded occurrence of 'red' counts as an instance of type $\langle e, t\rangle$. Let's not worry about 'That'. The point is that even given the descriptive power of type shifting, the semantics provides no *explanation* for why 'red square' is understood conjunctively (as if 'red' *restricts* 'square'). It is simply assumed that the type-lifted variant of λx.Red(x) is λΦ.λx.Red(x) & Φ(x) as opposed to another function. Consider λΦ.λx.Red(x) ∨ Φ(x), λΦ.λx.¬Red(x) & ¬Φ(x), etc.

To be sure, as Lewis (1970) noted, phrases like 'fake diamond' and 'alleged criminal' must also be accommodated. But this doesn't warrant generalizing to the worst case and concluding that 'A red square vanished, so a square vanished' is as invalid as 'A fake diamond vanished, so a diamond vanished'—or that the first inference is valid only because of how the modificational/type-lifted meaning of 'red' is specified. Similar remarks apply to the adverbial modifier in 'barked loudly' and endlessly many other examples; see note 7 above. So given the long history of suspecting that conjunct reduction plays an important role in natural reasoning, and that this is why modification is typically restrictive, it's no surprise that many advocates of describing meanings in terms Fregean typology follow Heim and Kratzer (1998) in positing a "predicate conjunction" rule for expressions of type $\langle e, t\rangle$. The now familiar idea is that two such expressions, Σ_1 and Σ_2, can combine to form a third that indicates a function that maps an entity e to truth iff both Σ_1 and Σ_2 indicate functions that map e to truth. But if a theory of meaning should include some such rule, in order to capture an important phenomenon, one has to consider the possibility of starting with **MPL** but then adding *less* than the Fregean hierarchy of types and a rule of Function Application.

Given the conjunctive rule of **MPL**, then as discussed above, adding the polarization rule delivers propositional logic and the Davidsonian diamond patterns. That is a reason for positing a mental language with an analog of $\Downarrow$, at least if one is willing to posit a mental language with an analog of $\wedge$.[10] Correlatively, if one is willing to adopt a theory of meaning that employs a lot of powerful apparatus *in addition to* a conjunctive rule for phrases like 'red square', then one should be happy to consider a simpler theory rooted in **MPL**. Of course, one wants see how **MPL** can be supplemented in ways that approach the levels of descriptive adequacy that can be attained in other ways. But the unfamiliarity of $\Downarrow$ should not be held against it. The issue is which technical notions earn their keep.[11]

Some semanticists may still wonder why anyone would want to replace a metalanguage that (by design) allows for unattested meanings with a more limited alternative, especially if this makes it harder to describe some attested meanings. Why not deny that psychological considerations constrain one's choice of a metalanguage? Instead of offering hypotheses about forms of mental representations, one might hope to specify truth conditions for pronounceable sentences (or perhaps associated thoughts) without worrying about what implies what, except when specifying the semantic contributions of particular words (or concepts) that belong to a limited "logical" vocabulary. We have doubts about this project; see section five and Pietroski (2018). But the point here is that *if* one is considering candidates for natural *representations of* the alleged truth conditions—say, as in Larson and Segal (1995)—then one wants to know if some modest extension of **MPL** will support a decent account of linguistic comprehension. Theorists can't stipulate that for these purposes, their preferred formalism is as good as any other. We will return to this theme after extending **MPL**.

[10] This is very different than positing a mental analog of Sheffer's stroke and suggesting that $\wedge$ is derived; see §3.4 below. One can think that for psychology, conjunct reduction is important because conjunction is irreducible. So one shouldn't assume that $\Downarrow$ is relevantly like the many inventions to which conjunction can be reduced. On the contrary, $\Downarrow$ is a simple *addition* to $\wedge$ that delivers a lot. Pietroski and Icard (2026) discuss negation in more detail, drawing on Russell (1905) and Quine (1948) to handle sentences like 'Pegasus does not exist' in terms of the variable-free mentalese offered here. They also discuss the relevance of replacing rules like "S $\Rightarrow$ NP VP" and "VP $\Rightarrow$ AUX V"—as in Chomsky (1957)—with subsequent variants like "S $\Rightarrow$ AUX VP" and "VP $\Rightarrow$ NP V", which cohere with the idea that sentences reflect polarized concepts.

[11] Horn (1989), citing Dahl (1979), casts doubt on the familiar idea that 'not' is of type $\langle t, t\rangle$. Overt sentence negation, as in [not [Aristotle was dumb]], is strikingly *absent* across languages.

3 Three Modest Additions and a Modal Interlude

In this section, we describe three ways of modifying **MPL**, starting with a very limited addition of some relational concepts. In §3.4, we note some interesting connections with a technical literature that might initially seem quite distant.

3.1 A Smidgeon of Relationlity

Every expression of **MPL** is a monadic concept. FOL allows predicates of any adicity. But let's consider a *minimal* change to **MPL** that allows for relational concepts.

Such a change would allow for dyadic concepts, but no other non-monadic concepts. A minimal change would also preserve the limitation that non-atomic concepts be monadic, thus confining relationality to atomic concepts. To make use of the (atomic) dyadic concepts, there would have to be a new rule of formation according to which a dyadic concept can be combined with a monadic concept to form another monadic concept. But otherwise, the generative procedure would be just like **MPL**. Perhaps surprisingly, this small change has a dramatic effect on expressive power.

Let's indicate dyadic concepts with underlining; and if only for simplicity, let's also say that such concepts apply to ordered pairs of things. For example, $\underline{\textsc{Above}}$ applies to an ordered pair of things iff the first member of the pair is above the second. Crucially, we assume dyadic concepts of participation relations—like $\underline{\textsc{DoneWith}}$ and $\underline{\textsc{LocatedIn}}$, but also $\underline{\textsc{DoneBy}}$ and $\underline{\textsc{DoneTo}}$—that are exhibited by event-entity pairs; see Castaneda (1966), Davidson (1985), Parsons (1990), Dowty (1991), and Schein (1993, 2002), among others.

We can now describe a language, $\underline{\textbf{MPL}}$, that extends **MPL** in the minimal way described above. Every complex concept of $\underline{\textbf{MPL}}$ is monadic, but $\underline{\textbf{MPL}}$ allows for atomic dyadic concepts. The generative procedure is that of **MPL** with an additional third rule.

- If Φ and Ψ are monadic concepts of $\underline{\textbf{MPL}}$, so is $(\Phi \wedge \Psi)$.

- If Φ is a monadic concept of $\underline{\textbf{MPL}}$, so is $\Downarrow\Phi$.

- If $\underline{\Delta}$ is an atomic dyadic concept of $\underline{\mathbf{MPL}}$, and Φ is a monadic concept of $\underline{\mathbf{MPL}}$, then $\underline{\Delta}$:Φ is a monadic concept of $\underline{\mathbf{MPL}}$.

The new formation rule, $\underline{\Delta}$-junction, allows for concepts like $\underline{\text{Above}}$:cow. The intended interpretation is that this complex monadic concept applies to an entity iff it is above a cow. The corresponding general principle is indicated below.

- $\underline{\Delta}$:Φ applies to e iff for some entity e' that Φ applies to, $\underline{\Delta}$ applies to $<e, e'>$.

So $\underline{\Delta}$:Φ applies to e iff e bears the relation that $\underline{\Delta}$ indicates to something that Φ applies to. In this sense, $\underline{\text{Above}}$:cow is an analog of '∃y[above(x, y) & cow(y)]'. But like **MPL**, $\underline{\mathbf{MPL}}$ has no variables, no existential quantifier, and no ampersand that can be flanked by expressions of arbitrary adicities. If it helps, think of $\underline{\text{Above}}$:cow as the result of applying an operator to the monadic concept. A concept of cows is converted into a concept of things that bear a certain relation to a cow. The dyadic concept serves as a kind of pivot, allowing a mind to use cow—a concept that applies to e iff e is a cow—to build a concept that applies to e iff e is above a cow.[12]

The concept $\underline{\text{DoneWith}}$ allows for pivoting from spoon to a concept of events done with a spoon. The result of $\underline{\Delta}$-junction—viz., $\underline{\text{DoneWith}}$:spoon—applies to e iff e is (an event) done with a spoon. Likewise, $\underline{\text{DoneBy}}$:spy, applies to e iff e is (an event) done by a spy; and $\underline{\text{DoneTo}}$:soldier, applies to e iff e is (an event) done to a soldier. Given a monadic concept poke that applies to events of poking, $\underline{\mathbf{MPL}}$ thus allows for monadic concepts like ($\underline{\text{DoneBy}}$:spy ∧ (Poke ∧ $\underline{\text{DoneTo}}$:soldier)). But suppose the available dyadic concepts include $\underline{\text{PokeOf}}$; where this mental analog of a passivized verb applies to an ordered pair, $<e, e'>$, iff e is an event of poking in which e' is the thing poked.[13]

Then $\underline{\mathbf{MPL}}$ allows for $\underline{\text{PokeOf}}$:soldier and polarized concepts like (22).

[12] Pietroski (2018) uses '∃[D(, __)^M(__)]' to get at the idea of linking a monadic concept to the second slot of a dyadic concept, and then closing this slot, leaving only the first slot of the dyadic concept available for a subsequent composition. Icard and Moss (2023) use '∃D.Φ', as in description logics; cp. Liang (2013).

[13] This coheres with a cluster of developments discussed in Pietroski (2018); see (Baker, 1988, 1997), Larson (1988), Schein (1993), Kratzer (1996), Borer (2005), Svenonius (2007), and Ramchand (2008) among others.

(22) $\Uparrow(\textsc{past} \wedge (\underline{\textsc{DoneBy}}{:}\textsc{spy} \wedge (\underline{\textsc{PokeOf}}{:}\textsc{soldier} \wedge \underline{\textsc{DoneWith}}{:}\textsc{spoon})))$

This concept applies to everything or nothing, depending on whether or not something was done by a spy, a poke of a soldier, and done with a spoon. Replacing concepts like SPY and SOLDIER with more "definite" concepts—e.g., THATSPY and THESOLDIER, or MISSSCARLET and COLMUSTARD—introduces complications. But let's assume that all such concepts are monadic, allowing for the possibility of conjunctions with constituents whose application conditions are deictic and context sensitive; cp. Burge (1973), Katz (1994), Fara (2015). The important point here is that $\underline{\textbf{MPL}}$ allows for concepts like (23)-(26).

(23) $\Uparrow(\underline{\textsc{DoneBy}}{:}\textsc{MissScarlet} \wedge ((\underline{\textsc{PokeOf}}{:}\textsc{ColMustard} \wedge \underline{\textsc{DoneWith}}{:}\textsc{feather}) \wedge \underline{\textsc{LocatedIn}}{:}\textsc{TheLibrary}))$

(24) $\Uparrow(\underline{\textsc{DoneBy}}{:}\textsc{MissScarlet} \wedge (\underline{\textsc{PokeOf}}{:}\textsc{ColMustard} \wedge \underline{\textsc{DoneWith}}{:}\textsc{feather}))$

(25) $\Uparrow(\underline{\textsc{DoneBy}}{:}\textsc{MissScarlet} \wedge (\underline{\textsc{PokeOf}}{:}\textsc{ColMustard} \wedge \underline{\textsc{LocatedIn}}{:}\textsc{TheLibrary}))$

(26) $\Uparrow(\underline{\textsc{DoneBy}}{:}\textsc{MissScarlet} \wedge \underline{\textsc{PokeOf}}{:}\textsc{ColMustard})$

As these examples show, polarizing a conjunction of $\underline{\Delta}$-junctions can be like saturating a polyadic concept with multiple arguments; cp. Davidson (1967, 1985). We assume that humans and other animals have polyadic concepts, just not as concepts of $\underline{\textbf{MPL}}$. But given a concept that applies to $<e, x, y>$ iff e is an event of x poking y, a mind might be able to use this concept—along with $\underline{\textsc{DoneBy}}$ and some other resources—to introduce a dyadic concept like $\underline{\textsc{PokeOf}}$. Likewise, a singular concept might be used to introduce a monadic concept that applies (only) to what the singular concept applies to; cp. Quine (1948). Introducing concepts in these ways might be useful for a mind that can use $\underline{\textbf{MPL}}$ to combine the introduced concepts with others in systematic ways.

Such a mind might also be able to stich together two (or more) concepts that can be used to think about a single causal process from different perspectives. An episode of a vase breaking might be represented as the terminating event of a process that was initiated by an action that some agent performed. Let the concept BreakOf apply to an ordered pair iff the first element is an event of the second element breaking. Let TerminatesIn apply to an ordered pair iff the first element is a process that ends with the second element. Then **MPL** generates TerminatesIn:BreakOf:vase, which applies to e iff e is a process that ends in an event of a vase breaking. But despite the embedding—cp. Above:Above:cow—every application of $\underline{\Delta}$-junction yields a monadic concept. So TerminatesIn:BreakOf:vase is a monadic concept that can be conjoined with others as in (27), which applies to e iff e is done by a child and a process that ends in an event of a vase breaking.

(27) DoneBy:child ∧ TerminatesIn:BreakOf:vase

Perceptual reports are interestingly related. Suppose that in response to being poked, Colonel Mustard shrieked, and this shriek was heard by Professor Plum. One can describe Plum's auditory experience by saying that Plum heard Mustard, or by saying that Plum heard Mustard shriek. The tensed verb 'heard' can take 'Mustard' as its direct object. But 'heard' can also combine with the untensed clause 'Mustard shriek.' Correlatively, given the relevant atomic concepts, **MPL** generates both HearingOf:Mustard and HearingOf:ShriekBy:Mustard. Moreover, the latter concept can be modified by a $\underline{\Delta}$-junction (e.g., LocatedIn:TheKitchen) in the two ways indicated in (28) and (29).

(28) HearingOf:(ShriekBy:Mustard ∧ LocatedIn:TheKitchen)

(29) HearingOf:(ShriekBy:Mustard)∧LocatedIn:TheKitchen

The concept indicated in (28) applies to events of hearing kitchen-located shrieks by Mustard. The concept indicated in (29) applies to kitchen-located events of hearing shrieks by Mustard. And (30) is ambiguous in this way; see Higginbotham (1983a), Vlach (1983).

(30) Plum heard Mustard shriek in the kitchen.

It doesn't follow that **MPL** is an adequate starting point for capturing the thoughts that humans can linguistically express. Pietroski (2018) discusses more examples of the sort covered in a typical first course in semantics. But our aim here has been to show how far $\underline{\mathbf{MPL}}$ can go beyond **MPL** by adding some dyadic concepts and the operation of $\underline{\Delta}$-junction.

This is not a place to discuss words like ‘unfriendly’ and ‘disobey’ in any detail. But as Horn (1989) and many others have noted, negative morphemes of this sort do not convert a predicate Φ into a predicate that applies to whatever Φ doesn’t apply to. Someone can fail to be friendly without being unfriendly; and they can, in various ways, fail to obey in order without disobeying it. Perhaps ‘unfriendly’ can be analyzed as if it were a phrase. But specifying a meaning for ‘un’ is not easy. So perhaps the systematic contribution of ‘un’ (and ‘ly’) to many words reflects a capacity to create morphologically complex expressions that correspond to complex mental representations that are atomic symbols of $\underline{\mathbf{MPL}}$ (i.e., concepts with no parts that are symbols of $\underline{\mathbf{MPL}}$). And even if ‘unfriendly’ can be analyzed as an instance of Δ:$\textsc{friendly}$, or a more complicated monadic concept, other examples may be more resistant to analysis. The details depend on the right conception of semantic composition; and fit with morphology is only one consideration among many.

Uses of ‘not’ in sentences like (31) deserve separate attention, especially given the many discussions of so-called negative events in philosophy and linguistics.[14]

(31) While attending a meeting, I watched a costumed busker not move a muscle.

But it’s a familiar point that *refraining from moving* takes work. Agents who do this work differ importantly from things that merely fail to move. And **SMPL** permits concepts like $\underline{\textsc{WatchOf}}{:}(\underline{\textsc{AgentOf}}{:}(\textsc{busker}{\wedge}\textsc{costumed}){\wedge}\underline{\textsc{RefrainFrom}}{:}(\underline{\textsc{MovingOf}}{:}\textsc{muscle}))$. This remark is far from a theory. But our hope and suspicion is that getting the details right for examples like (31) will be a matter of correctly specifying the relevant syntax and atomic concepts, rather than adopting a semantics with more sophisticated composition operations.

[14] For a short review with many references, see section 2.5 of Casati and Varzi (2025). For a recent proposal that appeals to events that don’t occur, see Bernard and Champollion (2024).

3.2 Universal Quantification via Concept Equivalence

Given concepts Φ and Ψ, **MPL** yields $\Uparrow(\Phi \wedge \Psi)$ and $\Downarrow(\Phi \wedge \Psi)$, analogs of '$\exists$x(Fx & Gx)' and '$\neg\exists$x(Fx & Gx)'. But since **MPL** doesn't support negation or disjunction, it doesn't generate analogs of '$\neg\exists$x($\neg$Fx)' or '$\neg\exists$x($\neg$Fx $\vee$ Gx)'—a.k.a. '$\forall$xFx' and '$\forall$x(Fx $\supset$ Gx)'. This expressive gap can be filled, however, without adding a complementation operation.

Consider a simplified form of the traditional square of opposition, depicted in Figure 4 below. **MPL** provides unary analogs of the northeast and southwest corners, but no analogs of the other two. The northwest corner corresponds to the affirmative universal quantifier. The southeast corner corresponds to a coherent but unattested quantifier 'Sumaint'.[15] This raises the question of whether **MPL** can be supplemented in a way that allows for an analog of 'Every' without allowing for negation or 'Sumaint'. If so, then **MPL** could be seen as intriguingly extendable, as opposed to hopelessly impoverished.

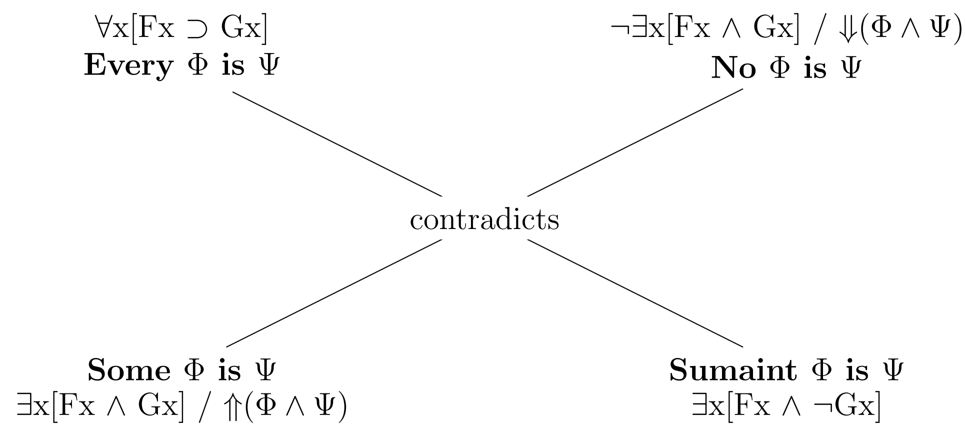

Figure 4: Square of Opposition

The trick is to accommodate both affirmative and negative universal quantification without analyzing any form of quantification in terms of negation. To this end, let's imagine

[15]Sometimes called 'Nall' to suggest 'Not all'. According to Aristotle (as usually read) and many others, the universal quantifiers are used with a presupposition that the predicate they combine with is not empty. See Horn (1989) for helpful discussion; see Read (2015) for an alternative reading of Aristotle. Horn also offers a plausible account of why 'Sumaint' would be pragmatically defective if it was cognitively available. So the mere fact that 'Sumaint' is unattested doesn't show that it is cognitively unavailable. Though it might be unavailable even if it would be pragmatically defective. We address examples like 'Some frog is not green'—in which the *copula* is negative, and the quantificational subject has arguably *raised*—in section four; cp. the resumptive construction 'Some frog is such that it is not green'.

a mind that can treat monadic concepts as interchangeable (i.e., materially equivalent) in reasoning: if the mind can generate Φ and Ψ, it can build premises of the form $\approx(\Phi,\ \Psi)$, which license replacement of either conceptual constituent with the other. The posited symbol, $\approx$, can be described as an operator that combines with two monadic concepts to form a sentential concept. So let's say that $\approx(\Phi,\ \Psi)$ applies to each thing or nothing—like a polarized concept—depending on whether or not for each entity *e*, Φ applies to *e* iff Ψ does. But let's *not* say that instances of $\approx(\Phi,\ \Psi)$ are concepts that can be polarized with $\Downarrow$. That would support the fourth corner quantifier via $\Downarrow\approx(\Phi,\ (\Phi \wedge \Psi))$; cp. $\neg\forall x(\Phi x \equiv (\Phi x \wedge \Psi x))$.

Think of $\approx$ as providing a distinctive way of linking two monadic concepts in a thought. This addition to **<u>MPL</u>** doesn't support predicate negation. But it permits premises of the form $\approx(\Phi,\ (\Phi \wedge \Psi))$, according to which the Φs *are* the Φs that are also Ψs. So instances of EVERY-$\Phi(\Psi)$ can be introduced as "inference tickets" for treating Φ and $(\Phi \wedge \Psi)$ as equivalent.

This idea can be encoded as a syncategorematic rule: EVERY-$\Phi(\Psi)$ iff $\approx(\Phi,\ (\Phi \wedge \Psi))$; cp. the classical *dictum de omni*. One doesn't need a special license to think that whatever holds of the Φs (e.g., the cows) holds of the Φs that are also Ψs (e.g. the brown cows). But if every Φ is Ψ, the Φs that are Ψs *are* the Φs—and hence, whatever holds of the Φs that are Ψs holds of the Φs. So if every Φ is Ψ, Φ is equivalent to $(\Phi \wedge \Psi)$.[16]

Instances of NO-$\Phi(\Psi)$ and SOME-$\Phi(\Psi)$ can be introduced similarly. Let '$\perp$' stand for some instance of $(\Phi \wedge \Downarrow\Phi)$. Let '$\top$' stand for the corresponding instance of $\Downarrow(\Phi \wedge \Downarrow\Phi)$. Then as a matter of logic, $\perp$ applies to nothing, and $\top$ applies to each thing. Now say that for any concepts Φ and Ψ, NO-$\Phi(\Psi)$ iff $\approx((\Phi \wedge \Psi),\ \perp)$, and SOME-$\Phi(\Psi)$ iff $\approx(\Uparrow(\Phi \wedge \Psi),\ \top)$. So the Aristotelean quantifiers can be described as devices that combine with monadic concepts to form concepts according to which certain equivalences hold. For each thing, EVERY-$\Phi(\Psi)$ applies to it iff $\approx(\Phi,\ (\Phi \wedge \Psi))$ does; NO-$\Phi(\Psi)$ applies to it iff $\approx((\Phi \wedge \Psi),\ \perp)$ does; and SOME-$\Phi(\Psi)$ applies to it iff $\approx(\Uparrow(\Phi \wedge \Psi),\ \top)$ does.

[16] This is the key feature of affirmative universal generalizations, at least in Aristotelian logic; see Valencia (1991, 1994), van Benthem (2008), Ludlow (2002), Parsons (2014), Ludlow and Živanović (2022). So one hopes to avoid positing a primitive quantifier, EVERY, and stipulating its inferential character; cp. note 4.

The resulting language, $\underline{\mathbf{SMPL}}$, is in some respects like a second-order monadic predicate logic. And unsurprisingly, the following inference forms are valid.

- $\textsc{some}\text{-}(\Phi \wedge \Upsilon)(\Psi \wedge \Omega) \Rightarrow \textsc{some}\text{-}\Phi(\Psi)$
- $\textsc{no}\text{-}\Phi(\Psi) \Rightarrow \textsc{no}\text{-}(\Phi \wedge \Upsilon)(\Psi \wedge \Omega)$
- $\textsc{every}\text{-}\Phi(\Psi \wedge \Omega) \Rightarrow \textsc{every}\text{-}(\Phi \wedge \Upsilon)(\Psi)$

If some brown cow is a clever logician, then some cow is a logician. If no cow is a logician, then no brown cow is a clever logician. And if every cow is a clever logician, then every brown cow is a logician. Aristotelian logic can be reconstructed in these terms.

Nonetheless, $\underline{\mathbf{SMPL}}$ still has the same computational complexity as a propositional calculus; see Icard and Moss (2023). At first, this might seem surprising. But extending **MPL** to $\underline{\mathbf{MPL}}$ just added finitely many atomic dyadic concepts, each of which can be described as an operator that converts one monadic concept into another. And from a formal perspective, $\approx$ is just one more dyadic notion, albeit one that combines with a pair of monadic concepts to form an interestingly complex monadic concept. Put another way, while the extension from **MPL** to $\underline{\mathbf{SMPL}}$ requires a nontrivial addition to the stock of atomic mental symbols, the result is still a fundamentally monadic system with finitely many bells and one important whistle. Correlatively, merely adding $\approx$ does not allow for predicate negation or any way of introducing a fourth-corner quantifier $\textsc{sumaint}\text{-}\Phi(\Psi)$.

Given the concepts $\textsc{raven}$ and $\textsc{black}$, $\underline{\mathbf{SMPL}}$ generates $\approx((\textsc{raven}, (\textsc{raven}\wedge\textsc{black}))$ but not $\approx(\neg\textsc{black}, (\neg\textsc{black} \wedge \neg\textsc{raven}))$; cp. 'all ravens are black' and 'all nonblack things are nonravens'. Nor can $\underline{\mathbf{SMPL}}$ be used to define $\textsc{grack}$ as follows: $(\textsc{black}\wedge\Phi)$ *or* $(\textsc{green} \wedge \neg\Phi)$; where Φ imposes some condition—e.g., having been observed—satisfied by all observed ravens. So a mind that can form $\approx((\textsc{raven}, (\textsc{raven}\wedge\textsc{black}))$ might be unable to form the Goodmanian analog $\approx((\textsc{raven}, (\textsc{raven}\wedge\textsc{grack}))$.

3.3 Adding Subtraction and MOST

One might wonder how any similarly modest extension of **SMPL** could allow for mental analogs of sentences like 'Most of the circles are blue'. For one might think that instances of MOST-$\Phi(\Psi)$ will have to be introduced in terms of concept complementation, as suggested by the equivalence indicated below.

- MOST-$\Phi(\Psi)$ iff $\#(\Phi \wedge \Psi) \succ \#(\Phi \wedge \neg\Psi)$

The familiar idea is that most of the Φs are Ψs iff the number of Φs that are also Ψs exceeds the number of Φs that are *not* Ψs.[17] And if adding MOST to **SMPL** requires an operation of negation or complementation, then one might think that the mental language used to introduce other quantificational determiners—including 'every'—makes some such operation available. But there is independent evidence that 'most' is understood in terms of cardinalty subtraction, as in the equivalence shown below, and not in terms of negation; see Pietroski et al. (2009), Lidz et al. (2011), Tomaszewicz (2013), Knowlton et al. (2022).

- MOST-$\Phi(\Psi)$ iff $[\#(\Phi \wedge \Psi) \succ \#\Phi - \#(\Phi \wedge \Psi)]$

Given a scene with circles, most of them are blue iff the number of blue circles exceeds the result of subtracting that number from the number of circles. So instead of positing concept negation, one can posit a primitive operation of cardinality subtraction along with an operator, #, that can combine with a monadic count-concept Φ to yield another monadic concept, $\#\Phi$, that applies to e iff e is the number of things that Φ applies to. Likewise, one can posit a primitive notion of one cardinality exceeding another.[18] Then MOST—a somewhat quirky concept that isn't lexicalized in most spoken languages—can be introduced

[17] Or one might replace appeal to cardinalities with appeal to one-to-one correspondence and leftovers: OneToOnePlus$[(\Phi \wedge \Psi), (\Phi \wedge \neg\Psi)]$; where a concept C bears the OneToOnePlus-relation to a concept D iff for some concept C- that applies to a proper subset of the things that C applies to, the things that D applies to correspond one to one with the things that C- applies to (but not with the things that C applies to).

[18] For simplicity, let's restrict attention to count-concepts and not worry yet about concepts like WATER, AIR, and HONESTY. Once **SMPL** has been extended to allow for mental analogs of 'Most of the circles are blue', we can think about generalizing the extension to allow for mental analogs of 'Most of the paint is blue'. But see Odic et al. (2018) for an argument against analyzing mass concepts in terms of count concepts.

along with the Aristotelian quantificational concepts, albeit via different primitives. In short, MOST-$\Phi(\Psi)$ applies to everything iff $\#(\Phi \wedge \Psi) \succ [\#\Phi - \#(\Phi \wedge \Psi)]$.

This makes it explicit that acquiring MOST requires a kind of conceptual sophistication that isn't required for the Aristotelian quantificational concepts. It is well known that the truth-theoretic content of MOST is not first-orderizable; see Rescher (1962); Wiggins (1980). In this respect, MOST is like ONE-TO-ONE. But atomic concepts of **SMPL**, which already includes $\approx$, need not have first-orderizable contents. Moreover, as with the addition of $\approx$, adding a few atomic arithmetic operators can leave the combinatorics and computational complexity unchanged.[19] For instance, Endrullis and Moss (2019) show that the result of adding MOST to the classical syllogistic (with SOME and ALL) remains of remarkably low complexity: like the classical syllogistic itself, the extension is even simpler than propositional logic. In this context, it's also worth noting that even if arithmetic cannot be reduced to logic in any scientifically interesting way, a "psychologically natural logic" may not segregate formal validity from good arithmetic reasoning; see van Benthem and Icard (2023).

Restrictable quantifiers correspond to set-theoretic relations that are conservative in the sense described by Barwise and Cooper (1981): $\mathbf{R}(\mathrm{s}_E,\ \mathrm{s}_I) \equiv \mathbf{R}(\mathrm{s}_E \cap \mathrm{s}_I,\ \mathrm{s}_I)$. For example, every circle is blue iff the blue things include the circles. In this sense, EVERY corresponds to *inclusion*—or more precisely, the relation *being a perhaps improper superset of*. And this relation is conservative: $\mathbf{SupersetOf}(\mathrm{s}_E,\ \mathrm{s}_I) \equiv \mathbf{SupersetOf}(\mathrm{s}_E \cap \mathrm{s}_I,\ \mathrm{s}_I)$. Correlatively, every circle is blue iff every circle is a blue circle. Repeating 'circle', the internal argument of 'every', as a restrictor of the external argument 'is blue' makes no logical difference.

Given the Fregean hierarchy of types (see §2.4), it is tempting to think that quantificational determiners *express* conservative relations; see Westerståhl (2024) for helpful discussion of the formal background. One might then describe the absence of determiners that express non-conservative relations, like $\mathbf{Identical}(\mathrm{s}_E,\ \mathrm{s}_I)$ and $\mathbf{Equinumerous}(\mathrm{s}_E,\ \mathrm{s}_I)$, via

[19] One is free to explore more reductive options; see note 17. Though given cardinality representations, THE-$\Phi(\Psi)$ can be defined as follows: $\approx(\Phi\ (\Phi \wedge \Psi)) \wedge \#\Phi = 1$; or equivalently, $\approx(\Uparrow(\Phi \wedge \Psi),\ \top) \wedge \#\Phi = 1$.

Barwise and Cooper's metaphor that determiners "live on" their internal arguments.[20] But instead of describing a filter on expressible set-theoretic relations, one can return to the more Aristotelean idea that natural quantifiers include a restricting predicate.

Higginbotham and May (1981) develop this idea in the context of transformational grammars and the idea that phrases like 'every philosopher' get displaced, yielding sentences in which a restricted quantifier has the analog of an open sentence as its external argument. From this perspective, the conservativity (or *intersectivity*) constraint reflects a mapping from a grammatical asymmetry to a restrictor/scope contrast, as opposed to a mere ordering of the argument positions associated with a second-order relation. Knowlton et al. (2023b) provide psycholinguistic evidence that competent speakers *don't* understand 'every' as indicating a relation, but instead construe 'every circle is blue' as a claim about the circles, which get represented as a group in a way that the blue things do not.[21]

So we doubt that 'every' expresses inclusion as opposed to the restrictable concept EVERY- (characterized in the **SMPL** terms of equivalence and conjunction). But we don't conclude that second-order relations are inexpressible. Perhaps numeric words like 'four' and 'five' are understood in terms of concepts whose constituents include a concept of equinumerosity. The "conservativity constraint" on determiner meanings may reflect factors other than the limits imposed by the relevant language of thought; see §4.3 and Pietroski (2018).

3.4 Interlude: A Modal Logical Perspective

From a logical point of view, there is a way of understanding the systems presented so far as restricted fragments of *modal logics*, which in turn can be viewed as sitting properly inside of First Order Logic; see van Benthem (1986).

[20] Each asymmetric conservative relation corresponds to at least one non-conservative relation that isn't expressed by an attested determiner. Consider: **Intersects**(s_E, s_I) & $\#(s_E) = 1$; **SubsetOf**(s_E, s_I); or even **ProperSupersetOf**(s_E, s_I). We assume that 'only', which appears in many positions not available to a determiner, isn't a determiner that expresses the subset relation.

[21] Even when speakers correctly judge that (in a presented scene) every circle is blue, they don't associate 'blue circle' with a group representation—despite representing the circles, all of which are blue, as a group.

Consider the base system **MPL**. As a purely formal exercise, we might think of atomic monadic concepts as "sentence symbols" in a modal language. Concept junction $(\Phi \wedge \Psi)$ is just conjunction of modal formulas. The upward polarization operator, $\Uparrow$, we could think of as a kind of modality: instead of writing $\Uparrow\Phi$, we could write $\Diamond\Phi$.

Note that this way of reading of $\Diamond$ is rather different from the common alethic reading in terms of metaphysical possibility. Modal logic captures the technical sense in which different flavors of modality may behave in structurally similar ways. A useful tool here is the modal ("Kripke") model, consisting of a set of points with an "accessibility" relation between points. In the present setting, we can think of the points as the possible things to which a concept could apply; in which case our formal system becomes a fragment of modal logic. For instance, the equivalence between $\Uparrow\Uparrow\Phi$ and $\Uparrow\Phi$, noted at the beginning of §2.2, is analogous to the modal law $\Diamond\Diamond\Phi \leftrightarrow \Diamond\Phi$. Indeed, full modal logic **S5** results as soon as we add propositional negation $\neg\Phi$ to the language. This allows us to express the modal "box" $\Box\Phi$ as $\neg\Diamond\neg\Phi$, which is a (global) universal modality, true just in case *all* points are Φ points. We can likewise express the analog of $\Downarrow\Phi$ as $\neg\Diamond\Phi$.

Like **MPL**, modal **S5** is also relatively simple among logical systems, with reasoning in the system being again no harder than that of propositional logic. However, the two behave differently when we consider extensions. This is another way of seeing that the presence of (Boolean) negation matters.

As mentioned above in §3.1, extending **MPL** to the minimally relational $\underline{\textbf{MPL}}$ does not result in greater computational complexity. Technically, the way dyadic relations are added to **MPL** is tantamount to adding a set of unary modalities. Recall that for a dyadic concept $\underline{R}$, the complex concept $\underline{R}{:}\Phi$ applies to any object that is related via $\underline{R}$ to *some* object that is Φ. In modal logic, we would write $\Diamond_{\underline{R}}\Phi$ for a formula that will be true at any point that is related via $\underline{R}$ to some Φ point. Adding this to the version of **S5** described above (where $\Box$ is a "global" universal modality) results in a well studied system—*multimodal logic with a universal modality*; see Goranko and Passy (1992)—call it **mKU**. In contrast to $\underline{\textbf{MPL}}$, which omits negation, **mKU** is known to be highly complex.[22]

To illustrate these connections more concretely, return to the example introduced earlier of the monadic concept $\underline{\textsc{Above}}{:}\textsc{cow}$, built out of the monadic $\textsc{cow}$ and the dyadic $\underline{\textsc{Above}}$. A typical situation might appear as in Figure 5. In modal logic we would say, e.g., that the modal formula $\Diamond_{\underline{\textsc{Above}}}\textsc{cow}$ is "true" at the point v (because it can "access" a $\textsc{cow}$ point, namely w, via the relation $\underline{\textsc{Above}}$), but false at z, w, and u. The **MPL** way of saying this is simply that $\underline{\textsc{Above}}{:}\textsc{cow}$ applies to v but not to z, w, or u.

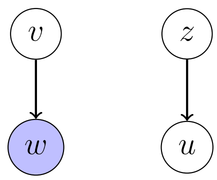

Figure 5: A modal logical interpretation of **MPL**.
($\textsc{cow}$ applies only to the node shaded blue, while v is above w and z is above u.)

Meanwhile, the fact that $\Diamond_{\underline{\textsc{Above}}}\textsc{cow}$ is true "somewhere" means the modal formula $\Diamond(\Diamond_{\underline{\textsc{Above}}}\textsc{cow})$ is true everywhere (at all points). This is the modal analog of the fact that $\Uparrow\underline{\textsc{Above}}{:}\textsc{cow}$ applies to everything. Yet while there is a modal expression that is true only at z, namely $\Diamond_{\underline{\textsc{Above}}}\neg\textsc{cow}$—the property of being above a non-cow—this is not expressible in **MPL**. And in fact there is *no* **MPL** expression that uniquely identifies z.

The extension to **SMPL** described in §3.2, for encompassing quantifiers, was substantive in our formalization of concept operations. But it is already part of the usual understanding of a (Boolean) logical system. Given two expressions Φ and Ψ, an equivalence $\approx(\Phi,\Psi)$ is tantamount to the modal *validity* of the formula $\Phi \leftrightarrow \Psi$, where the biconditional $\leftrightarrow$ is of course definable in the presence of negation. Thus, from this logical point of view, all of the systems described so far can technically be embedded into FOL, simply by virtue of being fragments of **mKU**, itself embeddable into FOL.

[22]Specifically, the satisfiability problem is decidable in exponential time—see Hemaspaandra (1996)—which could be end up being more difficult than propositional logic (whose satisfiability problem is decidable in *non-deterministic polynomial* (NP) time). This is an open question in computational complexity theory.

This connection helps clarify what makes these formal systems so tractable. Unlike FOL, modal logic is often said to be *robustly decidable* (see Vardi 1996; Marx and Venema 2007), in the sense that one can add a wide variety of logical devices while still retaining decidability. The common wisdom is that this robustness is partly due to a "tree model property." Roughly speaking, if one wants to know whether a given expression is satisfied by some entity (that is, at some "point" in a model), then one need only check the status of other points that are "accessible" from that point via one of the binary relations. For instance, returning to Figure 5, to know whether v satisfies $\underline{\text{ABOVE}}$:COW, one only needs to check whether among the things v is above, there is a cow. In this case, we need only check that w is a cow; the other objects u and z are totally irrelevant.

Decidability is inherited via the tree model property, even for quite strong extensions of basic modal logic such as the "guarded" fragments of first-order logic; see Andréka et al. (1998); Bárány et al. (2015); Pratt-Hartmann (2023). This suggests that many other extensions may remain relatively tractable, once we take into account the further simplifications (e.g., undefinability of negation) enjoyed by the **MPL**-languages discussed above,

4 Minimal Abstraction

SMPL doesn't generate concepts that correspond to relative clauses like 'who Scarlet poked' or 'who Scarlet did not poke'. In terms of Figure 5, we can't yet express that something above u isn't a cow. Adding the requisite form of abstraction delivers an analog of predicate negation at the cost of increased computational complexity. But the resulting generative system can, unlike FOL, be described as a context-free rewrite procedure. In this sense, one can add abstraction to **SMPL** in a relatively modest way.

4.1 Deixis and Displacement

MPL already generated concepts like (32) via conjunction, $\underline{\text{D}}$-junction, and polarization.

(32) $\Uparrow$(PAST $\wedge$ (<u>DoneBy</u>:SPY $\wedge$ <u>PokeOf</u>:SOLDIER))

Each of these operations is constructive in the sense of combining a concept Φ with some independent element of the resulting concept. But the concept expressed with 'who Scarlet poked with a feather' seems to be formed by *abstracting* a constituent from a sentential concept like the one expressed with 'Scarlet poked that person with a feather'; and the result is a complex subsentential concept that wasn't a building block in the constructive process. Using a sentential concept to create a new subsentential concept, which can then be used as a building block, seems like a major cognitive achievement; and perhaps it is confined to humans. But adding a *limited* form of abstraction to **<u>SMPL</u>**, permitting use of a polarized concept to form a "depolarized" concept, can still be a modest supplement.

Let's assume a mind whose monadic concepts include indexed instances of a deictic concept: THAT_1, THAT_2, and so on; where for each of these concepts, its application condition can be specified relative to contexts that assign values to the indices. For example, relative to each context/assignment **A**, THAT_1 applies to *e* iff **A** assigns *e* to the first index. This allows for conjunctive concepts like $\text{THAT}_1 \wedge \text{SPY}$ and $\text{THAT}_2 \wedge \text{SOLDIER}$; where a concept of the form $\text{THAT}_1 \wedge \Phi$ applies to *e* (relative to **A**) iff both THAT_1 and Φ apply to *e*. So if $\text{THAT}_1 \wedge \Phi$ applies to anything, the value of the index is a thing that Φ applies to.

A concept can contain multiple indices as in (33), which applies to each thing or to nothing, relative to any assignment of values to indices.

(33) $\Uparrow$(PAST $\wedge$ (<u>DoneBy</u>:($\text{THAT}_1 \wedge$ SPY) $\wedge$
(<u>PokeOf</u>:($\text{THAT}_2 \wedge$ SOLDIER) $\wedge$ <u>DoneWith</u>:($\text{THAT}_3 \wedge$ SPOON))))

But the indices, which are never bound by quantifiers, are not variables. And (33) is not a concept *of* contexts or assignments; it is simply a concept whose application condition is assignment-relative. Relative to any assignment, (33) applies to *e* iff *e* was (i) done by a spy who is assigned to the first index, (ii) a poke of a soldier who is assigned to the second index, and (iii) done with a spoon that is assigned to the third index.

Finally, let's assume that certain monadic concepts—including PERSON, PLACE, TIME, and an unrestricted concept THING—are possible targets for abstraction; cp. 'who', 'where', 'when', and 'which'.[23] We can now state a new rule of formation for the mental language **A-<u>SMPL</u>**, which extends **<u>SMPL</u>**. This rule, μ-**Abstraction**, converts a polarized concept that contains an indexed target for abstraction into a depolarized monadic concept.

- For any index i: if $\Downarrow\ldots(\text{THAT}_i \wedge \Phi)\ldots$ is a concept of **A-<u>SMPL</u>** and Φ is a possible target for abstraction, then $(\text{THAT}_i \wedge \Phi)[\Downarrow\ldots(\text{THAT}_i \wedge \Phi)\ldots]$ is a concept of **A-<u>SMPL</u>**.

The corresponding rule of interpretation is relativized to assignments; cp. §2.2 above.

- Relative to any assignment **A**:
  $(\text{THAT}_i \wedge \Phi)[\Downarrow\ldots(\text{THAT}_i \wedge \Phi)\ldots]$ applies to e iff $(\text{THAT}_i \wedge \Phi)$ and $[\Downarrow\ldots(\text{THAT}_i \wedge \Phi)\ldots]$ apply to e relative to an assignment that assigns e to index i and is otherwise like **A**.

Trivially, THAT_i applies to e relative to any "i-cousin" of **A** that assigns e to the index. (And $\Uparrow$ abbreviates $\Downarrow\Downarrow$.) So relative to any assignment **A**, (34) applies to e iff: e is a person; and (35) applies to e relative to the 1-cousin of **A** that assigns e to the first index.

(34) $(\text{THAT}_1 \wedge \text{PERSON})\Uparrow(\text{PAST} \wedge (\underline{\text{DoneBy}}{:}(\text{THAT}_1 \wedge \text{PERSON}) \wedge \underline{\text{PokeOf}}{:}(\text{THAT}_2 \wedge \text{SOLDIER})))$

(35) $\Uparrow(\text{PAST} \wedge (\underline{\text{DoneBy}}{:}(\text{THAT}_1 \wedge \text{PERSON}) \wedge \underline{\text{PokeOf}}{:}(\text{THAT}_2 \wedge \text{SOLDIER})))$

Hence, (34) applies to e iff e is a person who poked the soldier assigned to the second index. Likewise, (36) applies to e iff: e is a person and (37) applies to e relative to the 2-cousin of **A** that assigns e to '2'; that is, e is someone who was poked by the spy assigned to '1'.

(36) $(\text{THAT}_2 \wedge \text{PERSON})\Uparrow(\text{PAST} \wedge (\underline{\text{DoneBy}}{:}(\text{THAT}_1 \wedge \text{SPY}) \wedge \underline{\text{PokeOf}}{:}(\text{THAT}_2 \wedge \text{PERSON})))$

(37) $\Uparrow(\text{PAST} \wedge (\underline{\text{DoneBy}}{:}(\text{THAT}_1 \wedge \text{SPY}) \wedge \underline{\text{PokeOf}}{:}2))$

[23] A mind with indices might also have concepts like **A**-SPEAKER, **A**-PLACE, and **A**-TIME; cp. Kaplan (1989). Mental analogs of anaphora (e.g., 'poke herself') can be coded as in $(\underline{\text{DoneBy}}{:}(\text{THAT}_1 \wedge \text{SPY}) \wedge \underline{\text{PokeOf}}{:}(\text{THAT}_{2\to1} \wedge \text{PERSON}))$; where $_{2\to1}$ indicates that the second index is dependent on the first, and not associated with an independent act of reference; cp. Higginbotham (1983b).

The abstracted concept applies to e iff the polarized concept applies to e (and everything elese) when e is the value of the targeted index. So μ-abstraction just reflects the Tarskian assignment variants—like Church-style lambda abstraction, but in a more limited way.

Given a polarized concept, μ-**Abstraction** lets a thinker *copy* a suitable constituent and recombine it with the polarized concept. This is a direct analog of Chomsky's notion of "internal merge," which is the basic operation of merging two items (lexical or phrasal) applied to the special cases in which one of the merged items is a copy of a constituent of the other; see Chomsky (1995, 1980); Hornstein (1995). With regard to semantic composition, μ-**Abstraction** obviously differs from mere conjunction of monadic concepts. But the difference lies with the relativization to assignment variants. The significance assigned to *combining* a polarized concept with one of its monadic constituents is still conjunctive.[24]

Truth, for a polarized concept, is a matter of applying to everything; and if such a concept applies to e, then it applies to everything. So an abstracted concept of **A-<u>SMPL</u>** applies to e iff the basic polarized concept applies to e/everything when e is assigned to the targeted index. If it helps, read '$(\textsc{that}_i \wedge \Phi)[\Downarrow \ldots (\textsc{that}_i \wedge \Phi) \ldots]$' in either of the following ways: is both Φ and such that $\Downarrow \ldots i \ldots$ applies to it (and everything else) when it is the value of the index; i.e., is a Φ such that when assigned to the index, $\Downarrow \ldots i \ldots$ is true of it.

4.2 Predicate Negation via Abstraction

The operation of μ-**Abstraction** allows for a complex analog of predicate negation. Relative to any assignment **A**, the concept shown in (38) applies to e iff the concept shown in (39) applies to e relative to the 1-cousin of **A** that assigns e to the first index.

(38) $(\textsc{that}_1 \wedge \textsc{thing}) \Downarrow ((\textsc{that}_1 \wedge \textsc{thing}) \wedge \textsc{cow})$

(39) $\Downarrow ((\textsc{that}_1 \wedge \textsc{thing}) \wedge \textsc{cow})$

[24] It's logically redundant to impose and reimpose the condition that e is a person. But psychologically, the copy might presuppose a condition that the copied constituent imposes; cp. Herburger (2000).

And if e is assigned to the first index, then (39) applies to e iff nothing is such that e is a thing that is a cow. So (38) applies to e iff e isn't a cow. But as we suggested in §2.1, while this generable concept is *equivalent to* a complement of COW, it gets formed in a complex way that involves abstraction on a polarized concept—and not by simply combining COW with a primitive operator. Generating (38) requires distinctive resources.

Pietroski and Icard (2026) discuss this point in the context of examples like 'Pegasus does not exist' and Russellian analyses in terms of variables and sentential negation. Here, we simply note that even **MPL** allows for both $\Uparrow$(PEGASUS$\wedge$EXISTS)—a concept that applies to nothing, given that PEGASUS applies to nothing—and $\Downarrow$(PEGASUS$\wedge$EXISTS), which applies to each thing; cp. Quine (1948). Adding abstraction allows for the contrast between (40)—according to which something is both Pegasus and a thing that doesn't exist—and the more plausible (41), according to which nothing is both Pegasus and a thing that exists.

(40) $\Uparrow$(PEGASUS $\wedge$ (THAT$_1$$\wedge$THING)$\Downarrow$((THAT$_1$$\wedge$THING)$\wedge$EXISTS))

(41) $\Downarrow$(PEGASUS $\wedge$ (THAT$_1$$\wedge$THING)$\Uparrow$((THAT$_1$$\wedge$THING)$\wedge$EXISTS))

Let $\overline{\text{COW}}$ abbreviate (38); and more generally, for any concept Φ, let $\overline{\Phi}$ be the concept THING-THAT-ISN'T-Φ. Given this indirect route to an analog of complementation, via polarization and abstraction, there is an obvious way of introducing concept disjunction in a form that isn't limited to polarized concepts: $\underline{\text{DISJOIN}}(\Phi, \Psi) \equiv \overline{(\overline{\Phi} \wedge \overline{\Psi})}$. For example, $\underline{\text{DISJOIN}}$(COW, HORSE) applies to e iff e isn't a thing that both isn't a cow and isn't a horse. Or a mind might introduce disjunction via quantification over alternatives. Let Ω be a concept such that every cow falls under it, and so does every horse, but nothing else falls under it: $(\approx(\text{COW}, (\text{COW} \wedge \Omega)) \wedge (\approx(\text{HORSE}, (\text{HORSE} \wedge \Omega))) \wedge \Downarrow(\Omega \wedge (\overline{\text{COW}} \wedge \overline{\text{HORSE}})))$. Then Ω is equivalent to the result of disjoining COW with HORSE.[25]

[25]The complexity might help explain why disjunction emerges late, only in children who have significant grammatical competence. But we take no stand on the details, in part because an interesting cluster of facts merits extended discussion. Feiman et al. (2022) review evidence that human infants and non-human primates can represent a list of options—typically contraries—without representing the corresponding disjunction (or appreciating disjunctive syllogism in the sense of taking P or Q and ¬P as evidence for Q). Focusing on *assertion* makes disjunction seem cautious. But it's quite bold to move from *representing*

Given an indirect route to predicate negation, a second form of universal qualification can be introduced. Each circle is green iff no circle (is a thing that) isn't green. So instances of $\textsc{each}$-$\Phi(\Psi)$ could be viewed as abbreviations, in **A-SMPL**, for instances of $\Downarrow(\Phi \wedge \overline{\Psi})$. Knowlton et al. (2023b) offer experimental evidence that 'each' and 'every' are understood differently, as suggested by Vendler (1962) and many others, because 'each' is linked to a representational system that is good for tracking features of individual objects (e.g., color) while 'every' is linked to a different system that is good for representing properties of ensembles (e.g., numerosity). In short, 'each' and 'every' are associated with object-focused and group-focused conceptions of universal generalization. So it is interesting that **A-SMPL** allows for both $\Downarrow(\textsc{circle} \wedge \textsc{that-thing-isn't-green})$ and $\approx(\textsc{circle}, (\textsc{circle} \wedge \textsc{green}))$; see Pietroski (ms) for discussion and a proposed implementation of the semantic contrast.

4.3 Still Monadic and Context-Free

As one might expect, **A-SMPL** is like FOL in being undecidable: there is no decision procedure for answering questions like whether a given concept is equivalent to $\top$ (or $\bot$); see Icard and Moss (2023). The payoff is that **A-SMPL** permits an analog of one very special case of Lambda Abstraction; cp. Church (1941). Polarized concepts are like Church-style denoters of type $\langle$t$\rangle$, and de-polarized products of μ-**Abstraction** are like Church-style denoters of type $\langle$e,t$\rangle$. But there are no denoters in the mental languages we have posited.

More importantly, Church reconstructed Frege (1879, 1891), allowing for abstraction on expressions of the endlessly many types described in §2.4 above. By contrast, **A-SMPL** does not even permit abstraction on $\underline{\textsc{Above}}$ in (42).

(42) $\Uparrow((\textsc{that}_1 \wedge \textsc{circle}) \wedge \underline{\textsc{Above}}{:}(\textsc{that}_2 \wedge \textsc{square}))$

some options (e.g., about the location of a treat) to representing them as exhaustive. Moreover, if natural notions of disjunction are rooted in capacities to consider options, it's unclear why a concept of disjunction should be logically inclusive. But if $\textsc{or}(\textsc{dog}, \textsc{cat})$ is introduced in a way which ensures that it applies to everything $\textsc{dog}$ applies to *and* everything $\textsc{cat}$ applies to, the connection between options and inclusivity is less mysterious. Relatedly, one can be unsure about whether something is a cow or a horse. But it's not clear that one can think about something *as* a cow or a horse; cp. thinking about some things as the cows or horses.

Combining $\underline{\text{Above}}$ with THAT_3 would yield, via D-junction, $\underline{\text{Above}}$:THAT_3. So **A-<u>SMPL</u>** can't be used to form an analog of $\text{THAT}_3 \wedge \text{ABOVE}^*$, with ABOVE^* as a monadic target of abstraction that applies to (and only to) the corresponding dyadic *relation*. Likewise, **A-<u>SMPL</u>** does not permit abstraction on concepts like $\underline{\text{DoneBy}}$—or unindexed monadic concepts like RED in $\Uparrow$(RED $\wedge$ CIRCLE). Given a circle, one can think about the color the circle exhibits. But it doesn't follow that RED can be abstracted from $\Uparrow$(RED $\wedge$ CIRCLE).

We won't show here how the **MPL** family can be extended to allow for mental analogs of *plural* noun phrases like 'spies who poked that person' and 'transitive relations'. One obvious suggestion is to draw on Boolos (1998), allow for plural predicates that can apply to some things collectively without applying to any one of them, and let assignments assign *one or more* values to an index; see Schein (1993, 2002), Pietroski (2005a, 2018). But our aim is not to defend an account of plurality. We just want to note that thinking about relations, say those that a certain circle bears to a certain square it is above, doesn't require abstraction on the relational concept in a thought like (42).[26]

To be sure, the unavailabilty of expressions like 'whonk the circle the square' raises the question of why relative clause formation is so limited in natural languages; see Chierchia (1984). But if grammatical abstraction reflects a relatively small modification of **<u>SMPL</u>**, associated with the grammatical possibility of displacing certain indexed expressions, then the severe "type limitation" on relative clauses is to be expected. That said, μ-**Abstraction** is still a powerful operation. It can be used to generate mental analogs of pronounceable sentences like (44), which is roughly synonymous with (43).

(43) That spy poked every soldier.

(44) Every soldier is one who was poked by that spy.

Let POKED-BY-THAT-SPY abbreviate (45), which can formed by abstraction from (46).

[26] Much less abstraction on higher-order relations—e.g., from TRANSITS(LESSTHAN, PREDECESSOROF) to WHUNK(LESSTHAN, PREDECESSOROF), which is of type $\langle\langle\langle e, \langle e, t\rangle\rangle, \langle\langle e, \langle e, t\rangle\rangle, t\rangle\rangle, t\rangle$.

(45) $(\textsc{that}_2 \land \textsc{person})\Uparrow(\textsc{past} \land (\underline{\textsc{DoneBy}}{:}(\textsc{that}_1 \land \textsc{spy}) \land \underline{\textsc{PokeOf}}{:}(\textsc{that}_2 \land \textsc{person})))$

(46) $\Uparrow(\textsc{past} \land (\underline{\textsc{DoneBy}}{:}(\textsc{that}_1 \land \textsc{spy}) \land \underline{\textsc{PokeOf}}{:}(\textsc{that}_2 \land \textsc{person})))$

Then since **A-<u>SMPL</u>** allows for EVERY and abstraction, (47) is a generable analog of (44).

(47) $\approx(\textsc{soldier}, \textsc{soldier} \land \textsc{poked-by-that-spy})$

Pietroski (2018) shows how to combine this idea about how 'every' is understood with the now common hypothesis that the grammatical form of (43) is roughly as shown in (48), with the quantified direct object copied and recombined with the sentential expression.

(48) [[every soldier] [[that spy] [poked [~~every soldier~~]]]]

The strikeout indicates that the lower/original copy of 'every soldier' is to be treated like an index for purposes of interpretation; see May (1985), Chomsky (1975, 1995).

Similar remarks apply to (49), which has the paraphrase (49-a), assuming that 'Some frog' raises as indicated with the simplified grammatical form (49-b).[27]

(49) Some frog isn't green.

a. Some frog (is such that it) isn't green.

b. [[some frog]$_1$ [t$_1$ isn't green]]

The familiar idea is that the quantificational subject gets displaced to a position higher than that of tense/negation. So while there is no lexical analog of 'Sumaint' (see §3.2 above), (49) can be used to express an instance of $\Uparrow(\Phi \land \Downarrow(\Phi \land \Psi))$, which is a mental analog of the FOL sentence $\exists$x(Fx & $\neg$Gx); cp. the incoherent reading of 'Pegasus does not exist' noted in §4.2 above. But on this view, the thought expressed with (49) *isn't* of the form '$\exists$x(Fx & $\neg$Gx)'. The analog of predicate negation involves quantifier raising and abstraction.

In comparing **A-<u>SMPL</u>** to FOL, it is useful to focus on expressions like '$\exists$y(Rxy)' and

[27] See May (1985), Chomsky (1975, 1995), Huang (1995). Note that 'A frog is not green' sounds a bit odd—and suggests a negative *generic* claim—presumably because the indefinite article 'a', which marks 'frog' as a count noun, is not a quantificational determiner that raises.

'∀x∃y(Sxyz & My)', in which quantifiers bind all but one of the variables (each of which can have arbitrarily many occurrences). This "monadic fragment" of FOL cannot be described as a context-free language; see van Benthem (1987), Marsh and Partee (1987). Yet **A-<u>SMPL</u>** defines the same concepts; see Icard and Moss (2023) for a proof. This striking fact coheres with an attractive conjecture offered in Idsardi (2018): while the syntax for a natural spoken language may be transformational in the sense of Chomsky (1957, 1995), phonology and semantics each make do with *sparer* computational resources.[28]

Linking pronunciations to meanings may be harder, in some sense, than generating the pronunciations and meanings. In the last thirty years, it has become pretty clear that phonological rules can be characterized in very simple terms. It's plausible that these rules are naturally implemented by finite-state systems of a restricted kind. Heinz and Idsardi (2013) make the point explicitly and concisely. "Phonological generalizations appear to require much less than context-free power, apparently staying within a small class of subregular patterns." In this respect, phonology seems to be importantly different—and strikingly simpler—than syntax.[29] Moreover, as Heinz and Idsardi discuss, many regular patterns are phonologically unattested. So one wants to know if there is some interesting subclass of regular (finite-state) patterns that includes the attested ones. Or in an overtly psychological mode: are acquired PGs mentally formulated in a way that limits generalizations to those

[28]The full range of FOL sentences, with no restriction on the number of variables, can be specified in context-free terms. But any such generative procedure overgenerates wildly if viewed as a model of linguistic expressibility. Chomsky (1957, 1959) made the general point vivid: generating is easy; generating in accord with a constraint can be harder. A simple finite state procedure generates (all and only) the strings consisting of 'a' and/or 'b'; another such procedure generates the strings of the form '$a^n b^m$', in which one or more occurrences of 'b' follow one or more occurrences of 'a'. But context-free resources are needed to specify the '$a^n b^n$'-strings in which the occurrences of 'a' and 'b' are balanced. And to specify the '$a^n b^n c^n$'-strings, one needs computational resources of a kind that go beyond those employed by the grammars that the human children naturally acquire. Similarly, given just one atomic sentence with one free variable, FOL generates complex sentences of adicity n for every natural number n. Initially, one might hope to filter out complex sentences that have more than one free variable (and then perhaps add some back in if necessary). But if we start with all of FOL, filtering out unwanted overgeneration requires resources beyond the context-free.

[29]Kaplan and Kay (1994) showed that any "phonological grammar" (PG) formulated in the vocabulary of Chomsky and Halle (1968) can be seen as describing a regular relation. If A, B, C, and D are regular expressions, any rule **R** of the context-*sensitive* form "A → B/C ___ D" describes a regular relation so long as **R** meets a certain further condition (limiting its reapplication) that is satisfied by rules permitted by the Chomsky-Halle system: a PG takes the form of an ordered list of such rules that apply in succession; and regular relations are closed under this kind of rule composition. See §3.5 of Beesley (2003).

that can be realized by finite-state machines of a certain restricted type?[30]

Given a posited Universal Grammar that permits transformations but no context-sensitive rewriting while imposing *tighter* restrictions on phonological computations, one would want to see reasons for abandoning these constraints in favor of formulating proposals about specific languages in a far more liberal metalanguage. Likewise, given a decent theory of meaning formulated in terms of a mental language that generates its concepts *without* employing the computational resources needed for context-sensitive rewriting, one would want to see the payoff for formulating the theory in a more permissive idiom. If describing what expressions mean (and how they compose) requires powerful resources, then semanticists should be able to *find out* that this is so, in part by showing how proposals like ours are inadequate even if they are supplemented in modest ways. Theorists don't get to assume that computability is the only real constraint on composition—or that lexical meanings can have any type in the Fregean hierarchy—and that any attempt to make do with less has to be justified by showing that it leads to a *more* adequate theory, especially not if considerations of simplicity and overgeneration are set aside as not relevant to theory choice.

4.4 Schönfinkel Naturalized?

We conclude this section by comparing **A-<u>SMPL</u>** to Schönfinkel (1924); Pietroski and Icard (2026) offer more details. Schönfinkel described a notion U such that for any unary functions f and g, Ufg is equivalent to $\neg\exists \mathrm{x}[f(\mathrm{x})\ \&\ g(\mathrm{x})]$.[31] His other primitives, now called the combinators K and S, indicate functions not specific to logic: K$fg = f$; and S$fgh = (fh)(gh)$. Combinators correspond to instructions for how to rewrite subsequent symbols in a string, in ways that make variables dispensible; see Curry et al. (1958), Quine (1960).

The **MPL**-languages have no variables or combinators. The key compositional resource

[30] Heinz et al. (2011) posited "tiered" representations; see Graf (2022) for a recent review of relevant work.

[31] And as he notes, all functions can be described as unary, since functions can map functions onto functions. Instead of saying that subtraction maps pairs of numbers onto numbers, one can say that it maps numbers onto functions (from numbers to numbers); cp. Frege (1950, 1891). And for these purposes, let's focus on the functions Frege called Concepts, which map (ordered n-tuples of) things onto truth values. Given other functions, U invites delicate questions regarding typology.

is that two monadic concepts, Φ and Ψ, can be used to create an instance of $\Downarrow(\Phi \wedge \Psi)$. This complex mode of combination, involving polarization and a limited form of conjunction, is similar to U. But U combines with function-expressions, allowing for relationality in ways that **MPL**-languages don't. Relatedly, even with regard to (50)-(53), the Schönfinkel-reductions have very different forms than the **MPL** counterparts.

(50) $\exists x(fx)$

(51) $\exists x(fx \;\&\; gx)$

(52) $\exists x((fx \;\&\; gx) \;\&\; hx)$

(53) $\exists x(fx \;\&\; (gx \;\&\; hx))$

While (50) and (51) can be recoded as $\mathrm{U}(\mathrm{U}ff)(\mathrm{U}ff)$ and $\mathrm{U}(\mathrm{U}fg)(\mathrm{U}fg)$, neither $\mathrm{U}fg$ nor $\mathrm{U}(\mathrm{U}fg)$ corresponds to $(fx \;\&\; gx)$. And recoding this constituent of (51) and (52) make the implications far less obvious. For example, if 'B' indicates a bracketing function—$\mathrm{B}xyz = x(yz)$—we can let '$*$' abbreviate '$\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}f)g))(\mathrm{S}(\mathrm{BU}f)g)$'. Then (51) and (52) can be rewritten as $\mathrm{U}(\mathrm{U}{*}{*})(\mathrm{U}{*}{*})$ and $\mathrm{U}(\mathrm{U}{*}h)(\mathrm{U}{*}h)$. Figure 6 shows the diamond headed by (53) in terms of **MPL** on the left with Schönfinkel-reductions on the right. The details illustrate the contrast between extending **MPL** modestly and reducing FOL to a variable-free core.

$$\begin{tikzcd}
& \Uparrow(\Phi \wedge (\Psi \wedge \Omega)) \arrow[dl] \arrow[dr] & \\
\Uparrow(\Phi \wedge \Psi) \arrow[dr] & & \Uparrow(\Phi \wedge \Omega) \arrow[dl] \\
& \Uparrow(\Phi) &
\end{tikzcd}
\qquad
\begin{tikzcd}
& \mathrm{U}(\mathrm{U}f[\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}g)h))(\mathrm{S}(\mathrm{BU}g)h)])(\mathrm{U}f[\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}g)h))(\mathrm{S}(\mathrm{BU}g)h)]) \arrow[dl] \arrow[dr] & \\
\mathrm{U}(\mathrm{U}fg)(\mathrm{U}fg) \arrow[dr] & & \mathrm{U}(\mathrm{U}fh)(\mathrm{U}fh) \arrow[dl] \\
& \mathrm{U}(\mathrm{U}ff)(\mathrm{U}ff) &
\end{tikzcd}$$

Figure 6: Recoding the Diamond headed by $\exists x(fx \;\&\; (gx \;\&\; hx))$

Schönfinkel began by introducing a variable-ized version of the stroke Sheffer (1913) used to signify the truth-function NAND, which can be used to define the other truth functions:

$(P|Q) \equiv \neg(P \,\&\, Q)$; $(P|P) \equiv \neg P$; $(P|P)|(Q|Q) \equiv (P \vee Q)$; etc. Schönfinkel defined '$f\mathrm{x} \,|^{\mathrm{x}}\, g\mathrm{x}$' as a generalization equivalent to '$\forall \mathrm{x}(f\mathrm{x}|g\mathrm{x})$', which is equivalent to '$\forall \mathrm{x}\neg(f\mathrm{x} \,\&\, g\mathrm{x})$' and hence '$\neg\exists \mathrm{x}(f\mathrm{x} \,\&\, g\mathrm{x})$'. Schönfinkel's stroke can also connect sentences that have multiple variables as in (54), and the quantification can be vacuous as in (55).

(54) $f\mathrm{x} \,|^{\mathrm{x}}\, r\mathrm{xy}$

(55) $\exists \mathrm{y}(f\mathrm{y}) \,|^{\mathrm{x}}\, r\mathrm{wz}$

So each sentence of FOL is equivalent to one in which the variable-ized stroke is the only connective. If the stroke variables are fresh, $\neg\alpha$ is equivalent to $(\alpha \,|^{\mathrm{w}} \alpha)$, and $(\alpha \vee \beta)$ is equivalent to $((\alpha \,|^{\mathrm{z}} \alpha) \,|^{\mathrm{w}} (\beta \,|^{\mathrm{z}} \beta))$. Universal quantification can be characterized as follows: $\forall \mathrm{x}(f\mathrm{x}) \equiv (f\mathrm{x} \,|^{\mathrm{w}} f\mathrm{x}) \,|^{\mathrm{x}} (f\mathrm{x} \,|^{\mathrm{w}} f\mathrm{x})$. This set up the main task of eliminating the variables.

A formula like $(r\mathrm{bx} \,|^{\mathrm{x}}\, r\mathrm{cx})$—with the stroke-variable in the last position of each connected sentence—can be rewritten as $\mathrm{U}(r\mathrm{b})(r\mathrm{c})$, which can be unpacked as $\neg\exists \mathrm{w}(r\mathrm{bw} \,\&\, r\mathrm{cw})$ if the variable is fresh. The stroke-variable may fail to appear in one or both of the connected sentences, as in (55). But given the combinator that indicates "constancy", K*fg*=*f*, formula (55) can be recoded as '$\mathrm{K}(\exists \mathrm{y}(f\mathrm{y}))\mathrm{x} \,|^{\mathrm{x}}\, \mathrm{K}(r\mathrm{wz})\mathrm{x}$' and then '$\mathrm{U}(\mathrm{K}(\exists \mathrm{y}(f\mathrm{y})))(\mathrm{K}(r\mathrm{wz}))$'.

For cases like (54), where the stroke-variable appears but not always in a final position, K won't help. But given an "interchange" combinator ($\mathrm{C}fxy = fyx$), (54) can be rewritten as '$f\mathrm{x} \,|^{\mathrm{x}} \mathrm{C}r\mathrm{yx}$' and then '$\mathrm{U}f(\mathrm{C}r\mathrm{y})$'. Similarly, (56) can be rewritten as shown below.[32]

(56) $f\mathrm{y} \,|^{\mathrm{y}} (g\mathrm{x} \,|^{\mathrm{x}} r\mathrm{xy})$

a. $f\mathrm{y} \,|^{\mathrm{y}} (g\mathrm{x} \,|^{\mathrm{x}} \mathrm{C}r\mathrm{yx})$

b. $f\mathrm{y} \,|^{\mathrm{y}} \mathrm{U}g(\mathrm{C}r\mathrm{y})$

c. $f\mathrm{y} \,|^{\mathrm{y}} \mathrm{B}(\mathrm{U}g)(\mathrm{C}r)\mathrm{y}$

d. $\mathrm{U}f(\mathrm{B}(\mathrm{U}g)(\mathrm{C}r))$

[32]Note that '$\mathrm{C}r$' indicates a function that maps each entity to a function, even though '$\mathrm{C}r$' is followed by only one variable in '$\mathrm{C}ry$'; see Bimbó (2008). Historical aside: Schönfinkel used T for interchange, C for constancy, and Z instead of B (which he called composition).

Schönfinkel also introduced an identity function: I(x) = x. And his "fusion" function, S, lets one "reduce the number of occurrences of a variable" in a string: Sfgh = $(fh)(gh)$. While he didn't *prove* that his combinators suffice for rewriting any sentence FOL without variables, he offered a compelling proof-sketch that can be filled out; cp. the first few pages of Church (1941). He also showed that his combinators can be reduced to S and K.[33]

Given combinators, (fx & gx) can be recoded as an instance of the explicitly unary form μx. Correlatively, (52) can be recoded as (57), and (53) can be recoded as (58).

(52) $\exists \mathrm{x}((f\mathrm{x} \ \& \ g\mathrm{x}) \ \& \ h\mathrm{x})$

(57) $\mathrm{U}(\mathrm{U}(\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}f)g))(\mathrm{S}(\mathrm{BU}f)g))h)(\mathrm{U}(\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}f)g))(\mathrm{S}(\mathrm{BU}f)g))h)$

(53) $\exists \mathrm{x}(f\mathrm{x} \ \& \ (g\mathrm{x} \ \& \ h\mathrm{x}))$

(58) $\mathrm{U}(\mathrm{U}f(\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}g)h))(\mathrm{S}(\mathrm{BU}g)h)))(\mathrm{U}f(\mathrm{S}(\mathrm{BU}(\mathrm{S}(\mathrm{BU}g)h))(\mathrm{S}(\mathrm{BU}g)h)))$

The inferences to (59), from (57) or (58), are valid but not trivial; cp. (60) and (61).

(59) $\mathrm{U}(\mathrm{U}fg)(\mathrm{U}fg)$

(60) $\Uparrow((\textsc{frog} \wedge \textsc{green}) \wedge \textsc{happy})$; so $\Uparrow(\textsc{frog} \wedge \textsc{green})$

(61) $\Uparrow(\textsc{frog} \wedge (\textsc{green} \wedge \textsc{happy}))$; so $\Uparrow(\textsc{frog} \wedge \textsc{green})$

The inference from (59) to $\mathrm{U}(\mathrm{U}ff)(\mathrm{U}ff)$, which recodes $\exists\mathrm{x}(f\mathrm{x})$, may be pretty obvious for those familiar with U. But it's not an instance of junct *reduction*. Note that Uff implies Ufg.

Our point is not that (57) and (58) are unduly complicated. Schönfinkel made *explicit* the work done by occurrences of 'x' in sentences like (52) and (53), which are not pellucid

[33] B is equivalent to S(KS)K, and I is equivalent to SKK. Reducing C is easy given B(SI)K, which converts fg to gf: $\mathrm{B(SI)K}fg = (\mathrm{SI})(\mathrm{K}f)g = \mathrm{I}g((\mathrm{K}f)g) = g((\mathrm{K}f)g) = gf$; so $\mathrm{C}fgh = f(\mathrm{B(SI)K}gh)$. S and K suffice for describing the recursive functions, and hence the Turing-computable procedures; see Curry et al. (1958). There are, however, other Turing-complete sets of combinators—e.g., [K, B, C, W], where Wfg = fgg. Akhlaghpour (2022) argues that known biological mechanisms, at the level of RNA, would suffice for implementing [K, B, C, W] but not S; and while S can be defined as B(BW)(BBC)—see Smullyan (1985)—the "variable-fusing" power of S might not be practically available to a biologically instantiated KBCW system. So again, one might hope for a computationally simple variable-free language of thought.

with regard to *their* logical forms; see Pietroski and Icard (2026). But given U as the only primitive logical operator, conjunct reduction gets coded in a complex way. Note that (53)/(58) is the *base* of the diamond indicated in Figure 7, whose top corresponds to (62);

(62) $\exists$x(fx & (((gx & hx) & ix) & jx))

where the function symbols correspond to concepts like Past, DoneByScarlet, etc.

$$\begin{tikzcd}[column sep=tiny] & \Uparrow(\Phi \wedge (((\Psi \wedge \Sigma) \wedge \Gamma) \wedge \Omega) \arrow[dl] \arrow[dr] & \\ \Uparrow(\Phi \wedge ((\Psi \wedge \Sigma) \wedge \Omega)) \arrow[dr] & & \Uparrow(\Phi \wedge ((\Psi \wedge \Sigma) \wedge \Gamma)) \arrow[dl] \\ & \Uparrow(\Phi \wedge ((\Psi \wedge \Sigma)) & \end{tikzcd} \qquad \begin{tikzcd}[column sep=tiny] & \text{SOLONG-}ij \arrow[dl] \arrow[dr] & \\ \text{SOLONG-}i \arrow[dr] & & \text{SOLONG-}j \arrow[dl] \\ & (62) & \end{tikzcd}$$

Figure 7: Recoding the Diamond whose base is $\exists$x(fx & (gx & hx))

Spelling out 'SOLONG-i' and 'SOLONG-j' would be unhelpful, even given wider pages. But 'SOLONG-i' corresponds to (63), which can be partly recoded as indicated, letting '$(g \cap h)$' abbreviate 'S(BU(S(BUg)h))(S(BUg)h'. Formula (62) adds a further layer of complexity.

(63) $\exists$x(fx & ((gx & hx) & ix))

a. $\exists$x(fx & [($g \cap h$)x & ix])

b. $\exists$x(fx & U[S(BU($g \cap h$))ix][S(BU($g \cap h$))ix)]

c. $\exists$x(fx & S(BU(S(BU($g \cap h$))i))S(BU($g \cap h$))ix)

Like Ufg, $\Downarrow(\Phi \wedge \Psi)$ provides a variable-free way of recoding the negation of (51). But eschewing variables in the **MPL** way differs from eliminating them via combinators. This matters if one wants to account for the compellingness of diamond patterns by positing a mental language that generates analogs of (51)-(53), along with a natural logic that treats the valid inferences as analogs of conjunct reduction; see note 10 above.

5 Less than Logic, More than Grammar

In this final section, we address some connections to other discussions of "natural logic." We then conclude by saying how linguistic meanings can be described in terms of—but also distinguished from—concepts of **A-SMPL**.

5.1 Natural Fragments of a *Begriffsschrift*

Saying what Logic is, in a way that distinguishes it from Mathematics and Psychology, is hard. It's surely more than **A-SMPL**. But even **MPL** isn't logically trivial. Given modern logic, **A-SMPL** is (like Aristotle's syllogistic) relatively boring as a statement of some laws governing truth and falsity. But it may be more interesting as a fragment of Logic that can be naturally recognized, at least by certain animals who can also acquire a suitable grammar. Sommers (1970, 1984) advocated a version of this idea, which has been developed in many ways, with at least two distinguishable but overlapping motivations.

One is to provide some account of why syllogistic logic has seemed so intuitive and compelling to many thinkers, while modern logic seems to require more instruction and "mathematical" thinking that seems less elementary. Many authors have focused on polarity and monotonicity; see Valencia (1991, 1994), van Benthem (1986, 2008), Ludlow (2002), Ludlow and Živanović (2022), among others. A related project has been to explore extensions of classical syllogistic systems, in search of a precise "Aristotle boundary" between traditional logic and modern mathematical logic; see Pratt-Hartmann and Moss (2009), Pratt-Hartmann (2009), Moss (2015). One can view **A-SMPL**, as we do, as a contribution to this larger program of "natural logic." We have stressed connections to Language of Thought hypotheses—see Fodor (1975), Quilty-Dunn et al. (2023)—in part because we suspect that some of the resistance to such hypotheses is due to the mistaken idea that any mentalese worth positing would have to be sophisticated (and perhaps variable-laden) in ways that would make it hard to implement biologically; cp. note 33.

A distinct goal has been to provide computationally reasonable ways of classifying strings (of phonemes or text) as instances of valid inference patterns, specified in terms of what is sometimes called "surface syntax;" cp. Montague (1974). We suspect that success will still require considerable abstraction from the strings, given the ubiquitous mismatches between phonological forms and any interesting notion of logical form, as illustrated with the contrast between (64) and (65); cp. Chomsky (1964).

(64) That duck is eager to eat; so it is eager to eat something.

(65) That duck is easy to eat; so it can easily eat something.

But one can grant the need for substantive syntactic analysis of such strings while resisting the idea of regimenting them as sentences of some preferred version (or extension) of FOL *before* deciding whether or not the regimented inferences are valid.

Automating inference is not our goal.[34] But one might have thought that appeal to monadic concept conjunction *has* to be supplemented with a Stoic/Fregean negater, or some operation of complementation, in order to provide a basis for a propositional calculus. So it's relevant that polarization is enough, and that adding a notion of concept equivalence suffices for Aristotelian logic. Instead of starting with a mental analog of a propositional calculus and following the order of a standard introductory course in Logic—next allowing that the sentences can have parts as in 'Fa', and then adding quantifiers/variables as in '∃xFx'—one might borrow from Schönfinkel the idea of a variable-free predicate calculus that can treat propositions as special predicates, and then borrow from Tarski the idea that truth can be described as satisfaction by everything. Though as discussed in §4.3 above, variable-free systems can differ with regard to how instances of conjunct reduction are described.

[34] Though one might hope that given a plausible syntax, a language like **A-<u>SMPL</u>** might be a basis for a decent "semantic parser;" see Buder-Gröndahl (2023) for interesting suggestions regarding automated translation. And we don't rule out the possibility that some animals have a "propositional calculus module" (as opposed to mere IF-gates and NOT-gates, along with AND-gates and perhaps OR-gates) that can treat mental sentences as atomic constituents of premises in instances of modus ponens (P, Q if P; so Q) or disjunctive syllogism (P or Q, not-P; so Q). These are topics of current debate; see Quilty-Dunn et al. (2023) and references there.

5.2 Meanings as Instructions that Intail Instructions

If humans enjoy a concept generator like **A-SMPL**, with variation across individuals in the atomic concepts acquired, this invites at least three suggestions about how linguistic meanings are related to the generable concepts. Perhaps each speaker of a spoken/signed language has a grammar that connects each generable expression to a particular concept generated by that speaker's version of **A-SMPL**, and each expression meaning just *is* the corresponding concept. A second proposal would be that for each speaker, each of her generable expressions has a "semantic value" that is the extension of the corresponding concept generated by her version of **A-SMPL**; cp. Larson and Segal (1995). This makes room for conceptual variation among individuals who seem to "speak the same language." So one might say that meanings are classes of extensionally equivalent concepts, or that theories of meaning just need to specify the relevant extensions, which might be mappings (i.e., functions in extension) from contexts to sets of some sort.

Pietroski (2018) extends and connects various reasons for being skeptical of these two familiar suggestions. The proposed third alternative is a view according to which a lexical item can be linked, polysemously, to a family of two or more concepts that are not extensionally equivalent. Think of 'window', which be used to talk about certain openings or fillers; 'book', which be used to talk about certain vehicles or contents; 'clear', which be used to modify 'window', 'book', or 'sky'; etc. Such words seem to have multiple senses that correspond to distinct ways of thinking about distinct things. Thinking about something as a window that is a hole differs from thinking about something as a window that is a hole-filler. Thinking about something as a book-vehicle differs from thinking about something as a book-content, even if one refuses to draw an ontological distinction here.

Correlatively, words can often be used "co-predicationally" as in (66) and (67).

(66) The window that they cut in this wall was nicer than the one they installed.

(67) The book was too heavy to pack, so I finished it online.

The pronoun 'one' has 'window' as its antecedent. But it seems that any concept or extension plausibly expressed by using 'window' in (66) differs from any concept or extension plausibly expressed by using 'one' in the same sentence.[35]

For present purposes, we don't need to *reject* the first two options in favor of the third. But prima facie, a lexical item can be(come) conceptually equivocal; and it's not hard to allow for this possibility. One can say that the meaning of a lexical item, L, is an instruction for how to access a concept of **A-<u>SMPL</u>** from an address that is shared by various concepts that can be expressed with L. In which case, the meaning of 'window' can be executed in slightly different ways (on different occasions) to access distinct but related concepts, WINDOW-HOLE and WINDOW-FILLER. Likewise for the noun 'book' and the adjective 'clear'. Compare an instruction, issued at a well-stocked bar, to fetch a bottle of gin.

On this view, the meaning of a phrase like 'clear window' or 'clear book' is not a concept formed by conjoining two concepts expressed with the constituent words. But we can say that the phrasal meaning is an instruction for how to build a concept: conjoin monadic concepts accessed via the words. Let μ(X) be the meaning of expression X, and let JOIN(I, I*) be a complex instruction, executed by conjoining results of executing the sub-instructions I and I*. Then while the proposal shown below on the right is a simple modification of the one shown on the left, this modification allows for polysemous lexical items.

$\mu(\text{window}_N) = \textsc{window}$	$\mu(\text{window}_N) = \textit{fetch}@\text{window}_N$
$\mu(\text{book}_N) = \textsc{book}$	$\mu(\text{book}_N) = \textit{fetch}@\text{book}_N$
$\mu(\text{clear}_A) = \textsc{clear}$	$\mu(\text{clear}_A) = \textit{fetch}@\text{clear}_A$
$\mu([[..._A][..._N]]) = \mu([..._A])\wedge\mu([..._N])$	$\mu([[..._A][..._N]]) = \text{JOIN}[\mu([..._A]), \mu([..._N])]$
$\mu([[\text{clear}_A][\text{window}_N]]) = \textsc{clear}\wedge\textsc{window}$	$\mu([[\text{clear}_A][\text{window}_N]]) =$ $\text{JOIN}[\textit{fetch}@\text{clear}_A,\ \textit{fetch}@\text{window}_A]$
$\mu([[\text{clear}_A][\text{book}_N]]) = \textsc{clear}\wedge\textsc{book}$	$\mu([[\textit{clear}_A][\textit{book}_N]]) =$ $\text{JOIN}[\textit{fetch}@\text{clear}_A,\ \textit{fetch}@\text{book}_A]$

[35]Parallel remarks apply to (67) and endlessly many other examples. Consider ideal circles that can't be perceived, perceptible circles drawn on a board, circles with red areas, circles with red perimeters, etc.

As an analogy, consider a recipe like the following: pour—into a glass with ice—an ounce of gin, an ounce of sweet vermouth, and an ounce of Campari; stir and add a twist of orange. This leaves room for discretion regarding the choice of gin and vermouth.[36]

Instructions, unlike polarized concepts, don't exhibit relations of validity. But it's easy to define a notion of "intailment" for instructions. Let 'UP:I' and 'Down:I' indicate the obvious instructions for how to build polarized concepts from a result of executing the embedded instruction. Then the valid inferences on the left correspond to the "instruction inferences" on the right, in which the analogs of premises/conclusions are potential expression meanings.

$\underline{\Uparrow(\text{CLEAR}\wedge\text{BOOK})}$	$\underline{\text{UP:JOIN}[\textit{fetch}@\text{clear}_A,\ \textit{fetch}@\text{book}_N]}$
$\Uparrow(\text{BOOK})$	$\text{UP:}\textit{fetch}@\text{book}_N$
$\underline{\Downarrow(\text{BOOK})}$	$\underline{\text{DOWN:}\textit{fetch}@\text{book}_N}$
$\Downarrow(\text{CLEAR}\wedge\text{BOOK})$	$\text{DOWN:JOIN}[\textit{fetch}@\text{clear}_A,\ \textit{fetch}@\text{book}_N]$

More generally, the valid patterns "$\Uparrow(\Phi\wedge\Psi)$, so $\Uparrow(\Phi)$" and "$\Downarrow(\Phi)$, so $\Downarrow\Phi\wedge\Psi$)" correspond to the intailment patterns "UP:JOIN(I, I*), so UP:I" and "DOWN:I, so DOWN:JOIN(I, I*)."

This account of meaning doesn't connect pronounceable expressions with Tarskian satisfaction conditions, and it says nothing about how concepts have their contents. But our proposal can be supplemented with a theory of content for concepts. And if one wants a truth-theoretic semantics for some uninvented language, it may be a mistake to focus on spoken/signed languages, as opposed to more tractable mental languages; see, e.g., Field (1978) and Fodor (1987). If only because of polysemy, it's hard to see how the prospects for recursively specifying satisfaction conditions for concepts of **A-<u>SMPL</u>** could be dimmer than the prospects for the parallel project applied directly to spoken/signed expressions.

[36] A theory of meaning should allow for cases of co-predication; and specifying meanings as flexible instructions is an obvious way of doing this. But we are not offering an account of the many *contrasts* in acceptability across cases of co-predication—e.g., from Asher and Lascarides (2011), 'The city has 500.000 inhabitants and outlawed smoking in bars last year' vs. 'The city outlawed smoking in bars last year and has 500,000 inhabitants. The details presumably depend on various cognitive factors interact with grammar. For further discussion, see Murphy (2021, 2024), Michel and Löhr (2024), Collins and Vincente (ms).

Moreover, our proposal doesn't merely connect pronounceable expressions with mere representations of their meanings. The hypothesized meanings are specified in a substantive way that indicates (at least some of) their logical properties. Indeed, the proposal describes how the hypothesized meanings compose in a robust part-whole way that matters for relations of validity: the meaning of 'brown cow' is an instruction—JOIN(*fetch*@'brown', *fetch*@'cow')—with parts that are the meanings of 'brown' and 'cow', much as a concept formed by executing this instruction has simpler concepts as parts. In this sense, describing meanings as instructions for how to build concepts (generated by an independently specified language of thought) is more ambitious than the project of specifying alleged satisfaction conditions for pronounceable expressions, while remaining neutral about how and why the satisfaction condition of a complex expression E is determined by the grammatical structure of E along with the satisfaction conditions of its constituents.

Lewis (1970) famously said, in reply to Katz and Postal (1964), "Semantics with no truth conditions is no semantics." This combined a fair point with a misleading stipulation. The fair point is that a theory of meaning needs to do more than merely supplement a posited expression-generator with an algorithm that links each generable expression E with a representation of what E means.[37] But in discussions of natural languages like spoken English, it is dangerous to insist on using 'semantics' in the technical sense introduced by Tarski (1936, 1944); see Harman (1974) and Burgess (2008). This stipulation greases an illicit slide from a truism—viz., that a theory of meaning with no truth conditions is not a Tarskian semantics—to the conjecture that good theories of linguistic meaning will specify Tarskian satisfaction conditions for sentences, as opposed to instructions for assembling concepts of a special sort.[38]

[37] See also Pietroski (2000) in reply to Horwich (1998). An advocate of Lewis (1970) might respond to Pietroski (2018) by saying that whatever its merits a piece of speculative psychology, it wasn't backed by an explicit formal system of the sort that many semanticists crave. Icard and Moss (2023) provide the backing; but one might wonder *how* their results connect to issues that cognitive scientists, including many linguists, care about. In this paper, we hope to have made some relevant connections.

[38] For helpful comments, advice, and questions after talks, we thank the reviewers, editors, and many members of many audiences.

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