1. We are given the following information about $f(x)$:

\[
    f(0) = 2, \quad f(1) = 1, \quad f(3) = 0, \quad f(4) = 1
\]

(a) Write down the divided difference table. Find interpolating polynomial in Newton form (i) for the nodes in the order 0, 1, 3, 4, (ii) for the nodes in the order 4, 3, 1, 0.

(b) Assume we know that the 4th derivative satisfies $|f^{(4)}(x)| \leq 10$ for $x \in [0, 4]$. Find an upper bound for $|f(2) - p(2)|$.

2. Consider the $(x, y)$ data points $(-1, 2), (1, 1), (2, 0)$. We want to fit the data with a function $g(x) = c_1 + c_2 x^2$

(a) Find the best least squares fit by hand.

(b) Write a Matlab program which uses the backslash command (without computing $A^T A$) to solve this problem.

3. We want to find $x$ such that $x + x^5 = 3$.

(a) Perform one step of the bisection method with $a_0 = 1, b_0 = 2$. Find $k$ such that $|b_k - a_k| \leq 10^{-6}$.

(b) Perform one step of the secant method with $x_0 = 1, x_1 = 2$ to find $x_2$.

(c) Will the Newton method converge if we start with $x_0$ sufficiently close to the solution $x^*$? Explain.

4. Consider the nonlinear system

\[
    x_1 + x_1 x_2 + x_2 = 2, \quad x_1 - x_2 - x_1 x_2^2 = 0
\]

(a) Perform one step of the Newton method starting with initial guess $x^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(b) Write a Matlab program which uses the Newton method to find a solution, starting with initial guess $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The program should print out the approximation for $x$ after each iteration.