Practice problems for Exam 1

1. Approximate \( y = (3.5)^{1/2} \) using the Taylor polynomial \( p_2(x) \). Give an upper bound \( |y - p_2(x)| \leq \cdots \).

2. We use the following Matlab command: \( y = 1000.2 - 1000.1 \)
   Give an upper bound for the relative error of the computed result.

3. We want to compute \( y = e^{0.01} - 1 \) and use Matlab: \( y = \exp(.001) - 1 \)
   (a) Which operation will cause a large error? Analyze this operation only, and find the expected error for the computed result. Give the answer as a number like \( 10^{-8} \). Hint: Use a Taylor approximation for \( e^{0.01} \) to evaluate your expression for the error.
   (b) Can we get a more accurate result if we evaluate the Taylor approximation \( p_3(x) \) in Matlab?

4. Consider the matrix \( A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 1 & 4 \\ 4 & 1 & 2 \end{bmatrix} \)
   (a) Use Gaussian elimination WITH pivoting (use the pivot candidate with the largest absolute value) to find the matrices \( L, U \) and the vector \( p \).
   (b) Use \( L, U, p \) to solve the linear system \( Ax = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} \).
   (c) We solve the linear system \( Ax = \begin{bmatrix} 1 \\ 10 \\ 1000 \end{bmatrix} \) and find the solution vector \( x \). Then we find out that we actually need the solution vector \( \hat{x} \) for the linear system \( A\hat{x} = \begin{bmatrix} -1 \\ 10 \\ 1000 \end{bmatrix} \). Find an upper bound \( \|\hat{x} - x\|_\infty \leq \cdots \) assuming \( \|A^{-1}\|_\infty \leq 10 \).