Math 406, Exam 1

Directions: Work over Zoom and turn in answers to each problem on one page (answer sheet). Turn in answers on Gradescope.

This is a closed book exam. No notes, calculators, or cell phones allowed.

Please answer Question 1 on Answer Sheet 1. (20 points)

a. Compute $d = \gcd(51, 84)$.

b. Find integers x and y such that $51x + 84y = d$.

Please answer Question 2 on Answer Sheet 2. (20 points)

Consider the linear diophantine equation

$$17x + 13y = 100.$$

a. Find a solution $(x_0, y_0) \in \mathbb{Z}^2$ or show that the equation has no solutions with x and y integers.

b. Write down the general solution.

Please answer Question 3 on Answer Sheet 3. (20 points)

Suppose p is an odd prime number. Show that $(p-2)! \equiv 1 \pmod{p}$.

Please answer Question 4 on Answer Sheet 4. (20 points)

Suppose a, b and c are integers with $c > 0$ and with $a \equiv b \pmod{c}$. Show that $\gcd(a, c) = \gcd(b, c)$.

Please answer Question 5 on Answer Sheet 5. (20 points)

A positive integer n is *palindromic* if its decimal expansion $b_k b_{k-1} \dots b_0$ is the same backwards and forwards. In other words, n is palindromic if $b_i = b_{k-i}$ for $i = 0, \dots k$. The integers 11, 121 and 5335 are examples of palindromic integers.

Show that any palindromic integer with an even number of digits is divisible by 11.