Math 406, Spring 2021
HW08, due Wednesday, April 7 at 5pm [1]

Reading: Read Chapters 8 and 9 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

`http://math.gordon.edu/ntic/`

Each problem is worth 20 points.

1 (Crisman 8.4.6)**.** In Example 8.3.2 of Crisman's text, what is the order of the group element which is rotation by ninety degrees to the left? What is the order of rotation by 180 degrees?

2. Suppose G is a group with identity element e. We say that G is *cyclic* if there exists an element $g \in G$ such that $G = \{g^n : n \in \mathbb{Z}\}$. In other words, G is cyclic if every element of G is a power of one element g. In this case, we also say that G is a *cyclic generator* of G.

Note: When the binary operation on G is called "+," the convention is that we write nx instead of x^n for the element $x + x + \cdots + x$ (n times), and we write 0 for the identity element. In this case, G is cyclic if there exists an element $g \in G$ such that $G = \{ng : n \in \mathbb{Z}\}$. By convention, we only use the notation $+$ for the binary operation on a group when the group is abelian ($=$ commutative).

(a) Is the group U_{12} cyclic? Prove or disprove.
(b) [**10 point bonus**] Is $(\mathbb{Q}, +)$ a cyclic group? Prove or disprove.

3. [Crisman 10.6.2] Suppose a is a primitive root of n, and let $\bar{a}$ denote the inverse of a modulo n. Show that $\bar{a}$ is also a primitive root modulo n.

4 (Crisman 10.6.14)**.** Suppose a, x, y and n are integers with x, y and n postive and with $x \equiv y \pmod{\phi(n)}$. Suppose further that $\gcd(a, n) = 1$. Show that $a^x \equiv a^y \pmod{n}$.

5 (Taken from Crisman 10.6.8)**.** Find the orders of all elements of U_{13}.

[1] This version created Thursday 1st April, 2021 at 19:46.