Math 406, Spring 2021
HW06, due Wednesday, March 24 at 5pm [1]

Reading: Read Chapter 7 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

`http://math.gordon.edu/ntic/`

Each problem is worth 20 points.

1. Solve the simultaneous system of congruences:

$$\begin{aligned} x &\equiv 2 \pmod{3} \\ x &\equiv 3 \pmod{5} \\ x &\equiv 7 \pmod{13}. \end{aligned}$$

Be sure to show your work.

2 (Crisman 5.6.17)**.** When eggs in a basket are removed two, three, four, five, or six at a time, there remain, respectively, one, two, three, four, or five eggs. When they are taken out seven at a time, none are left over. Find the smallest number of eggs that could have been contained in the basket. (Brahmagupta, 7th century AD)

3. Using Fermat's Little Theorem compute the following quickly. (Show your work, and write your answers as integers k satisfying $0 \leq k < n$ where n is the modulus.)

(a) $512^{351} \pmod{13}$.
(b) $3444^{4518} \pmod{17}$.

4. Suppose $n > 1$ is a composite number. Show that $(n-1)! \not\equiv -1 \pmod{n}$. (See the hint in Crisman 7.7.11 if you get stuck.)

5. Suppose b is a positive integer and n is a composite number. Then n is a called a *pseudoprime to the base b* if $b^n \equiv b \pmod{n}$.

Show that 91 is a pseudoprime to the base 3.

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[1]This version created Wednesday 24$^{\text{th}}$ March, 2021 at 19:37.