Math 406, Spring 2021
HW04, due Wednesday, February 24 [1]

Reading: Read Chapter 5 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

`http://math.gordon.edu/ntic/`

Each problem is worth 20 points.

1. (Crisman 4.7.2) Prove that (a) 13 divides 145^6+1 and (b) 431 divides $2^{43}-1$ without a computer.

2. Suppose n is a positive integer with decimal expansion $b_k b_{k-1}\cdots b_0$.

(a) Show that $\sum_{i=0}^{k} b_i \equiv n \pmod 9$.

(b) Show that $\sum_{i=0}^{k} (-1)^i b_i \equiv n \pmod{11}$.

These two facts are the bases of the tricks *casting out nines* and *casting out elevens*, which can be used to check arithemetic.

3. Suppose S is a set with 2 elements. How many partitions are there of S? Similarly, how many partitions are there of a set with 3 elements? In each case, write down all the partitions.

4. Suppose R is an equivalence relation on a set S, and x and y are elements of S. Show that $[x]=[y]$ if and only if xRy holds.

Hint: This is very closely related to the fact (proved in class) that the set of equivalence classes is a partition of S. You can use that fact to answer this question. Of you can use the ideas from the proof if you want.

5. For each of the following linear congruences, find all solutions x (if any).

(a) $3x \equiv 5 \pmod 7$.

(b) $17x \equiv 14 \pmod{21}$.

(c) $6x \equiv 3 \pmod 9$.

(d) $15x \equiv 9 \pmod{25}$.

---

[1] This version created Wednesday 24th March, 2021 at 19:37.