HW02, due Wednesday, February 10
Math 406, Spring 2021

Reading: Read Chapter 2 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

http://math.gordon.edu/ntic/

Each problem is worth 20 points.

1. Give proofs for facts (1), (3) and (4) from Proposition 1.2.8 in the text. In other words, suppose that a, b, c, u and v are integers. Then prove the following.

(1) [**6 points**] If $a|b$ and $b|c$, then $a|c$.
(3) [**7 points**] If $c|a$ and $c|b$, then $c|(au+bv)$.
(4) [**7 points**] If $c > 0$, then all divisors of c are less than or equal to c.

2. (Crisman, 2.5.6) Prove that

$$\gcd(a, a+2) = \begin{cases} 1, & a \text{ odd;} \\ 2, & a \text{ even.} \end{cases}$$

3. Use the Euclidean algorithm to find the gcd d of 51 and 90. Then find integers x, y such that $51x + 90y = d$. Show all your work.

4. Suppose a and b are two nonzero integers. An integer e is a *common multiple* of a and b if a and b both divide e.

(a) [**7 points**] Show that, as long as a and b are nonzero integers (as we are assuming), there always exists a positive common multiple of a and b.
(b) [**6 points**] Define the *least common multiple* of a and b to be the smallest positive common multiple of a and b. Explain why this number exists.
(c) [**7 points**] Write $\operatorname{lcm}(a, b)$ for the least common multiple of a and b. Show that $\operatorname{lcm}(a, b) = |ab|$ if and only if $\gcd(a, b) = 1$. You can assume, for simplicity, that a and b are both positive. (However, the result holds in general.)

5. Prove the second assertion of Proposition 2.4.9 in the text. In other words, assume that a and b are coprime integers and assume that $a|bc$ for some integer c. Show that $a|c$.