1. True or False

1. (5 points) Suppose n is an integer with exactly 3 positive divisors. Then $n = p^2$ for some prime p.

A. True, B. False.

2. (5 points) Suppose that $a \equiv b \pmod{m}$ and $c \equiv d \pmod{m}$ with $c|a$ and $d|b$. Then

$$\frac{a}{c} \equiv \frac{b}{d} \pmod{m}.$$

A. True, B. False.

2. Multiple Choice

3. (5 points) The number of primitive roots of 98 is

A. 0, B. 12, C. 34, D. 42, E. none of the above.

4. (5 points) The number of primitive roots of 99 is

A. 0, B. 8, C. 34, D. 66, E. none of the above.

5. (5 points) The number of zeros at the end of the decimal representation of $153!$ is

A. 28, B. 33, C. 37, D. 62, E. none of the above.

6. (5 points) $10^{200,000,000,000,000,000}$ days from today it will be

A. Sunday, B. Monday, C. Tuesday, D. Wednesday, E. none of the above.

7. (10 points) Let n be the solution to the following ancient Indian problem (taken from Rosen):

> If eggs are removed from a basket $2, 3, 4, 5$ and 6 at a time, there remain respectively, $1, 2, 3, 4$ and 5 eggs. But if the eggs are removed 7 at a time, no eggs remain. What is the least number of eggs that could have been in the basket?

The number n is congruent to which of the following modulo 13?

A. 1, B. 2, C. 3, D. 4, E. none of the above.

3. Prove or Disprove

For this problem, *clearly indicate* whether the statement is *true or false* then prove or disprove it.

8. (12 points) If $\phi(n)|n-1$ then n is squarefree. (Here ϕ is the Euler ϕ-function.)

4. PROVE

In this section, prove the statement given to you.

9. (12 points) Suppose r is a primitive root modulo p and $p \equiv 1 \pmod 4$. Show that $-r$ is also a primitive root modulo p.

10. (12 points) Suppose a and b are positive integers. Show that

$$(a,b)[a,b] = ab.$$

11. (12 points) Suppose a and N are integers with $N \geq 0$. Show that

$$(1+a)^N \equiv 1+Na \pmod{a^2}.$$

12. (12 points) Show that a positive integer n is composite if and only if

$$\phi(n) \leq n - \sqrt{n}.$$

Math 312 Final Fall 2012

Patrick Brosnan, Instructor

First Name/Last Name:______________________________

Student ID Number:______________________________

Section/Professor:______________________________

Signature:

By signing here, you confirm you are the person identified above and that all the work herein is solely your own.

Instructions:

- You are allowed to use pencil, pen and eraser only. **No notes, index cards or calculators**
- You may use the back of a sheet for calculations.
- Put your name on **all** sheets in the alloted space.
- **Box any final answers.**

Problem	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	10	
8	12	
9	12	
10	12	
11	12	
12	12	
Total	100	