

HW8, due Monday, April 8
Math 403, Spring 2017
Patrick Brosnan, Instructor

Practice Problems: Do the following problems from Herstein for practice, but do not turn them in. The format below is that **H4.5** means “Chapter 4, Section 5 of Herstein.”

H4.3: 1, 4, 6, 20

H4.4: 1, 2

H4.5 1, 3, 10

H4.6 1, 2, 5

Graded Problems: Work the following problems for a grade.

1. Let \mathbb{F}_5 denote the field with 5 elements. Show that the polynomial $p = x^2 + 2$ is irreducible in \mathbb{F}_5 . Using this, conclude that $L := \mathbb{F}_5[x]/p\mathbb{F}_5[x]$ is a field with 25 elements.
2. Let $\varphi : \mathbb{F}_5[x] \rightarrow L$ denote the quotient map for the ring in Problem 1 and write $\bar{x} := \varphi(x)$.
 - (a) Show that $(\bar{x})^2 = -2$.
 - (b) Find $a, b \in \mathbb{F}_5$ such that $(\bar{x} + 1)^{-1} = a\bar{x} + b$.