HW4, due Friday, March 10
Math 403, Spring 2017
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Practice Problems: Do the following problems from Herstein for practice, but do not turn them in. The format below is that **H4.5** means "Chapter 4, Section 5 of Herstein."

H2.4: 1, 3, 7,
H2.5: 3, 12, 15
H2.6: 1, 2, 7, 8

Graded Problems: Work the following problems for a grade.

1. Suppose n and m are non-zero integers. The *least common multiple* of n and m is $nm/(n,m)$. We write either $\operatorname{lcm}(n,m)$ or $[n,m]$ for the least common multiple. Show the following

(1) n and m both divide $[n,m]$.
(2) If e is an integer divisible by both n and m, then $[n,m]|e$.

2. Suppose G is a group and $x \in G$. Then *centralizer* of x is the set $Z(x) = \{y \in G : yx = xy\}$. In other words, the centralizer of x is the set of all elements of G which commute with x. Show that $Z(x) \leq G$.

3. Suppose G is a group and x, y are two elements of G which commute with each other: $xy = yx$.

(1) Show that, for any integer i, $(xy)^i = x^i y^i$.
(2) Suppose $|x| = n, |y| = m$ with n and m finite. Show that $|xy|$ divides $[n,m]$.
(3) Suppose n and m above are relatively prime. Show that $|xy|$ is equal to $[n,m]$.
(4) Given an example to show that the conclusion of (3) is false without the assumption that $(n,m) = 1$.

4. Write the cycle decomposition of the element

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$$

of S_5.