

HW9, due Wednesday, May 7
Math 403, Spring 2014
Patrick Brosnan, Instructor

1. Suppose F is a field. Show that F has characteristic either equal to 0 or a prime number.
2. Suppose A is a commutative ring, and let N denote the set of nilpotent elements of A . Show that N is an ideal in A .
3. What are the idempotent elements in the ring $\mathbb{Z}/12\mathbb{Z}$? What is the ideal of nilpotent elements? What are the prime ideals?
4. Suppose x is a nilpotent element of a commutative ring A . Show that x is in every prime ideal of A .
5. Show that the polynomial $x^4 + 1$ is irreducible over \mathbb{Q} .