HW2, due Wednesday, February 12
Math 403, Spring 2014
Patrick Brosnan, Instructor

1. Let $M_2(\mathbb{R})$ denote the set of 2×2-matrices with coefficients in the real numbers, and let

$$* : M_2(\mathbb{R}) \times M_2(\mathbb{R})$$

denote the binary operation $X * Y = XY - YX$ where XY denotes the matrix multiplication of X and Y. Show that $*$ is not associative. The operation $*$ is known as the *Lie bracket* operation. Usually $X * Y$ is written as $[X,Y]$.

2. Suppose G is a group with identity element e. If $g^2 = e$ for all $g \in G$, show that G is abelian.

3. Suppose M is a monoid with binary operation $*$ and identity element e. We say that an element $m \in M$ is *central* if, for all $n \in M$, $m * n = n * m$. The center of M is the set $Z(M)$ of all central elements of M. Show that $Z(M)$ is a submonoid of M. That is, show that $e \in Z(M)$ and that, if $m, n \in Z(M)$ then $m * n \in Z(M)$.

4. Let M denote the monoid $M_2(\mathbb{R})$. What is $Z(M)$? Prove your answer.