HW7, due Tuesday, December 10
Math 403, Fall 2013
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Practice Problems: Do the following problems from Gallian for practice, but do not turn them in. The format below is that **G4** means "Chapter 4 of Gallian."

G12: 9, 19, 27
G13: 5, 11, 15
G14: 9
G15: 13
G16: 3, 15

Graded Problemes: Work the following problems for a grade.

1. Let M be an abelian group and write $\operatorname{End} M$ for the set of group homomorphisms $\phi : M \to M$. In class, I mentioned that $\operatorname{End} M$ has two binary operations: $(f,g) \mapsto f+g$ and $(f,g) \mapsto f \circ g$ given on $m \in M$ by the formulas

$$(f+g)(m) = f(m) + g(m)$$
$$(f \circ g)(m) = f(g(m)).$$

Show that $\operatorname{End} M$ with these operations is a ring with unity.

2. Suppose a and b are elements of a commutative ring R. We say that a divides b and write $a|b$ if there is an element c of R such that $ac = b$. Let R be the ring consisting of real numbers of the form $a + b\sqrt{3}$ with $a, b \in \mathbb{Z}$. Show that $1+\sqrt{3}$ does not divide $5+2\sqrt{3}$.

3. Suppose R is an integral domain and r is an element of R satisfying $r^2 = r$. Show that r is either 0 or 1.

4. Suppose p is a prime number and k is an integer satisfying $1 < k < p$. Show that p divides $\binom{p}{k}$. Using this, show that, if R is a commutative ring of characteristic p and $x, y \in R$, then $(x+y)^p = x^p + y^p$.