HW8, due Friday, November 18
Math 403, Fall 2011
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Reading Assignment

Begin reading about rings in Chapter 3.1-2. Also read the material in Herstein about the symmetric group.

Problem 1. (40 points) Let G be a group and $a \in G$. An element b in G is said to be *conjugate* to a if there exists a $g \in G$ such that $gag^{-1} = b$.

(a) Show that the relation "a is conjugate to b" is an equivalence relation in G. The equivalence classes are called *conjugacy classes* and the conjugacy class of a is the set of all elements of G conjugate to a.
(b) What are the conjugacy classes in each of the three non-abelian groups of order ≤ 8?

Problem 2. (40 points) Let n and k be positive integers with $k \leq n$ and let $i_1, \ldots i_k$ be distinct integers in $\{1, 2, \ldots, n\}$.

(a) Suppose $1 \notin \{i_1, \ldots i_k\}$. Compute $(1i_1)(1i_2 \ldots i_k)(1i_1)$.
(b) Show that $(i_1 i_2 \ldots i_k)$ is conjugate in S_n to $(12 \ldots k)$.

Problem 3. (20 points) Suppose R is a ring. The *center* $Z(R)$ of R is the set of all $r \in R$ such that, for all $x \in R$, $rx = xr$.

(a) Show that the center of a ring is a subring. That is show that 0 and 1 are in $Z(R)$ and, for any $x, y \in Z(R)$, $x - y$ and xy are in $Z(R)$.
(b) Let $M_2(\mathbf{R})$ denote the set of all 2×2 matrices with real coefficients. What is the center of $M_2(\mathbf{R})$?