A Generalization of the Néron Models of Green, Griffiths and Kerr

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Abstract. We generalize a construction of the Néron model for a family of intermediate Jacobians due to Green, Griffiths and Kerr by using the theory of mixed Hodge modules. It is a topological group defined over any partial compactification of the base space, and it 'graphs' admissible normal functions. Moreover, there is a stratification of the partial compactification such that the restriction over each stratum is a complex Lie group over the stratum.

Introduction

Let $\mathbf{H}$ be a polarizable variation of $\mathbf{Z}$-Hodge structure of weight $w < 0$ on a complex manifold S. Let $J_S(\mathbf{H})$ be the family of intermediate Jacobians over S. Its fiber at $s \in S$ is

$$J_S(\mathbf{H})_s = \mathcal{L}(s)/(F^0\mathcal{L}(s) + L_{\mathbf{Z},s}),$$

where $(\mathcal{L}, F)$ is the underlying filtered locally free sheaf of $\mathbf{H}$, and $L_{\mathbf{Z}}$ is the underlying local system of $\mathbf{H}$. Let $j : S \to \overline{S}$ be a partial compactification as a complex analytic space. Let $\{S_\alpha\}$ be a Whitney stratification of $\overline{S}$ such that S is one of the strata.

Theorem. *There is a Néron model $J_{\overline{S}}(\mathbf{H})$ over $\overline{S}$ which extends $J_S(\mathbf{H})$ and such that any admissible normal function on S uniquely extends to a section of $J_{\overline{S}}(\mathbf{H})$. Moreover, there is a short exact sequence of topological groups over $\overline{S}$*

$$0 \to J_{\overline{S}}(\mathbf{H})^0 \to J_{\overline{S}}(\mathbf{H}) \to G \to 0,$$

such that the restriction of $J_{\overline{S}}(\mathbf{H})^0$ over S_α is a complex Lie group over S_α with connected fibers and G_s is discrete and is a subgroup of $(R^1j_*L_{\mathbf{Z}})_s$ for any $s \in \overline{S}$.

In the curve case this was proved first by Clemens [3] in some special cases, and then by [10] in the general curve case where it was defined by enlarging the Zucker extension [13]. However, it was pointed out by Green, Griffiths and Kerr [7] that a subspace of it is sufficient and is more natural (still in the curve case). In this paper we show that their construction can be extended naturally to the general case using the theory of mixed Hodge modules.

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In Section 1 we recall some basics about Zucker extensions and admissible normal functions. In Section 2 we prove Theorem by constructing the Néron models.

1. Zucker Extensions and Admissible Normal Functions

1.1. Zucker extensions. Let $\mathbf{H} = ((\mathcal{L}, F), L_{\mathbf{Z}})$ be a polarizable variation of $\mathbf{Z}$-Hodge structure of weight $w < 0$ on a complex manifold S. Here $(\mathcal{L}, F)$ is the underlying filtered locally free sheaf and $L_{\mathbf{Z}}$ is the underlying $\mathbf{Z}$-local system. We assume that $L_{\mathbf{Z}}$ is torsion-free in this paper. Let $\mathcal{V}$ be the vector bundle on S corresponding to the locally free sheaf $\mathcal{L}/F^0\mathcal{L}$, and let $\Gamma \subset \mathcal{V}$ denote the subgroup over S corresponding to the subsheaf $L_{\mathbf{Z}} \subset \mathcal{L}/F^0\mathcal{L}$ where the last injectivity follows from the negativity of the weight. Set

$$J_S(\mathbf{H}) = \mathcal{V}/\Gamma.$$

Here the quotient is set-theoretically taken fiberwise. This has a structure of a complex Lie group over S.

Let $j : S \to \overline{S}$ be a smooth partial compactification of S such that $D := \overline{S} \setminus S$ is a divisor with normal crossings. Assume the local monodromies of $L_{\mathbf{Z}}$ are unipotent. Let $\widehat{\mathcal{L}}$ be the Deligne extension of $\mathcal{L}$, see [4]. By Schmid [11], the Hodge filtration F on $\mathcal{L}$ is uniquely extended to a filtration F on $\widehat{\mathcal{L}}$ such that $\mathrm{Gr}_F^p\widehat{\mathcal{L}}$ are locally free (i.e. $F^p\widehat{\mathcal{L}} = \widehat{\mathcal{L}} \cap j_*F^p\mathcal{L}$). Let $\widehat{\mathcal{V}}$ denote the vector bundle on $\overline{S}$ corresponding to the locally free sheaf $\widehat{\mathcal{L}}/F^0\widehat{\mathcal{L}}$. Let $\widehat{\Gamma}$ be the subgroup of $\widehat{\mathcal{V}}$ over $\overline{S}$ corresponding to $j_*L_{\mathbf{Z}} \subset \widehat{\mathcal{L}}/F^0\widehat{\mathcal{L}}$. Then the Zucker extension is defined by

$$J_{\overline{S}}^Z(\mathbf{H}) = \widehat{\mathcal{V}}/\widehat{\Gamma}.$$

1.2. Restriction to the diagonal curve. Let C be the diagonal curve defined by $z_i = z_j$ for $i \neq j$, where the z_i are local coordinates such that $S = \bigcap\{z_i \neq 0\}$. Since the local monodromies are unipotent, the restriction of the Deligne extension to C is again the Deligne extension. This follows from the fact that the restriction of $\exp(-\sum_i(\log z_i)N_i)u$ to the diagonal curve is $\exp(-(\log z)\sum_i N_i)u$ where u is a multivalued horizontal section. (Here $N_i = (2\pi i)^{-1}\log T_i$ with T_i the local monodromies.)

As a corollary, we see that the restriction of the Zucker extension to the diagonal curve is again the Zucker extension.

1.3. Admissible normal functions. A normal function is a holomorphic section ν of $J_S(\mathbf{H})$ satisfying the Griffiths transversality. By Carlson [2] it corresponds to an extension class of $\mathbf{Z}_S$ by $\mathbf{H}$ giving a short exact sequence

$$0 \to \mathbf{H} \to \mathbf{H}' \to \mathbf{Z}_S \to 0. \tag{1.3.1}$$

A normal function ν is called admissible with respect to a partial compactification $\overline{S}$ of S if $\mathbf{H}'$ is an admissible variation of mixed Hodge structure ([8], [12]) with respect to $\overline{S}$, see [10]. The group of such normal function will be denoted by $\mathrm{NF}(S)_{\overline{S}}^{\mathrm{ad}}$.

Assume $\overline{S}$ is as in (1.1), i.e. D is a divisor with norma crossings, and moreover the local monodromies are unipotent. By [8] (and [12] in the 1-dimensional case) the conditions for an admissible normal function are given as follows:

(i) $\mathrm{Gr}_F^p \mathrm{Gr}_k^W \widehat{\mathcal{L}}'$ are locally free.

(ii) The relative monodromy filtration exists for any local monodromy.

For $s \in \overline{S}$, we have the cohomological invariant of ν at s

$$\gamma_s(\nu) \in H^1(U_s, L_{\mathbf{Z}}|_{U_s}) = \mathrm{Ext}^1(\mathbf{Z}_{U_s}, L_{\mathbf{Z}}|_{U_s}),$$

where U_s is the intersection of S with a sufficiently small ball with center s in an ambient space. This invariant is defined by passing to the underlying short exact sequence of $\mathbf{Z}$-local systems, and restricting it over U_s, see also [1].

In the curve case it is known that the normal function extends to a section of the Zucker extension if the cohomological invariant $\gamma_s(\nu)$ vanishes, see e.g. [10], Prop. 2.3 and also [6] for the geometric case. Indeed, if $\gamma_s(\nu) = 0$, then ν is given by the class of

$$\sigma_F(1) - \sigma_{\mathbf{Z}}(1) \in \Gamma(\overline{S}, \widehat{\mathcal{L}}), \tag{1.3.2}$$

where $\sigma_{\mathbf{Z}}, \sigma_F$ are splittings of the underlying short exact sequence of locally free sheaves of (1.3.1)

$$0 \to \widehat{\mathcal{L}} \to \widehat{\mathcal{L}}' \to \mathcal{O}_{\overline{S}} \to 0,$$

such that $\sigma_{\mathbf{Z}}$ is defined over $\mathbf{Z}$ and σ_F is compatible with F. This argument can be extended to the normal crossing case of higher dimension. (In the curve case its converse is also true, see [10], Prop. 2.4.)

1.4. Relation with the limit mixed Hodge structure. In the curve case, assuming the local monodromy is unipotent, the limit mixed Hodge structure $\psi_t \mathbf{H}$ at $0 \in \overline{S} \setminus S$ is given by

$$H = ((H_{\mathbf{C}}; F, W), (H_{\mathbf{Q}}, W), H_{\mathbf{Z}}),$$

$$\text{with} \qquad H_{\mathbf{C}} = \widehat{\mathcal{L}}(0)\,(:= \widehat{\mathcal{L}}/m_0\widehat{\mathcal{L}}), \quad H_A = \psi_t L_A \;\; (A = \mathbf{Z}, \mathbf{Q}),$$

where $m_0 \subset \mathcal{O}_{\overline{S},0}$ is the maximal ideal and t is a local coordinate of $\overline{S}$. Note that the nearby cycle functor $\psi_t L_A$ can be defined in this case by

$$\psi_t L_A = \Gamma(\widetilde{\Delta}^*, \rho^* L_A),$$

where Δ is a sufficiently small disk around 0 and $\rho : \widetilde{\Delta}^* \to \Delta^*$ is a universal covering.

We have a commutative diagram of MHS

$$\begin{array}{ccccccccc}
0 & \to & \psi_t \mathbf{H} & \to & \psi_t \mathbf{H}' & \to & \mathbf{Z} & \to & 0 \\
 & & \downarrow N & & \downarrow N & & \downarrow 0 & & \\
0 & \to & \psi_t \mathbf{H}(-1) & \to & \psi_t \mathbf{H}'(-1) & \to & \mathbf{Z}(-1) & \to & 0
\end{array}$$

The horizontal exact sequence defines

$$\nu'_0 \in J(\psi_t \mathbf{H}).$$

In the case $\gamma_0(\nu) = 0$, let ν_0'' denote the value at 0 of the normal function given by (1.3.2). Then we have by the definition of the limit mixed Hodge structure (and using [2])

$$\nu_0' = \nu_0'' \in J(\psi_t\mathbf{H}).$$

Set $H_0 = \mathrm{Ker}(N : \psi_t\mathbf{H} \to \psi_t\mathbf{H}(-1))$. (This is compatible with the definition of H_s in (2.1) below.) By the snake lemma applied to the above diagram, there is

$$\nu_0 \in J(H_0),$$

whose injective image in $J(\psi_t\mathbf{H})$ coincides with ν_0', since the connecting morphism gives $\gamma_0(\nu)$. Note that this implies another proof of a theorem of Green, Griffiths and Kerr [7], Thm. (II.A.9)(i). This ν_0 coincides with the one constructed in (2.1) below in a more general situation, since the vertical arrow of the above diagram calculates the functor $i_0^*\mathbf{R}j_*$ in this case.

2. Néron Models

2.1. Identity components. Let $\nu \in \mathrm{NF}(S)^{\mathrm{ad}}_{\overline{S}}$, i.e. be an admissible normal function on S with respect to a partial compactification $j : S \to \overline{S}$. We have the corresponding short exact sequence

$$0 \to \mathbf{H} \to \mathbf{H}' \to \mathbf{Z}_S \to 0.$$

For $s \in \overline{S}$, let $i_s : \{s\} \to \overline{S}$ denote the inclusion, and set

$$H_s := H^0 i_s^*\mathbf{R}j_*\mathbf{H}, \quad J(H_s) := \mathrm{Ext}^1_{\mathrm{MHS}}(\mathbf{Z}, H_s),$$

where MHS denotes the category of mixed $\mathbf{Z}$-Hodge structures [5]. Note that the functors $H^k i_s^*\mathbf{R}j_*\mathbf{H}$ are defined in MHS (with $\mathbf{Z}$-coefficients). In fact, they are defined with $\mathbf{Q}$-coefficients in the algebraic case [9], and a similar argument works in the analytic case assuming $\overline{S} \setminus S$ is an intersection of divisors (shrinking $\overline{S}$ if necessary). Moreover, forgetting the mixed Hodge structure, they are naturally defined with $\mathbf{Z}$-coefficients so that

$$H^k i_s^*\mathbf{R}j_*L_{\mathbf{Z}} = \varinjlim_U H^k(U \cap S, L_{\mathbf{Z}}), \tag{2.1.1}$$

where U runs over open neighborhoods of s in $\overline{S}$.

We have an exact sequence of mixed $\mathbf{Z}$-Hodge structures

$$0 \to H_s \to H^0 i_s^*\mathbf{R}j_*\mathbf{H}' \to \mathbf{Z} \xrightarrow{\delta_s} H^1 i_s^*\mathbf{R}j_*\mathbf{H},$$

and the image of $1 \in \mathbf{Z}$ by δ_s is the cohomological invariant $\gamma_s(\nu)$ of the admissible normal function ν at s. So the exact sequence implies that, if $\gamma_s(\nu) = 0$, then ν defines

$$\nu_s \in J(H_s). \tag{2.1.2}$$

We define set-theoretically the identity component by

$$J_{\overline{S}}(\mathbf{H})^0 := \coprod_{s\in\overline{S}} J(H_s).$$

To define a topological structure on it, take a resolution of singularities $\pi : \overline{S}' \to \overline{S}$ such that the pull-back of $\overline{S} \setminus S$ is a divisor with normal crossings. For $s' \in \overline{S}'$, we have the canonical injection of mixed Hodge structures

$$\pi^* : H_{\pi(s')} \hookrightarrow H_{s'}. \tag{2.1.3}$$

By (2.1.1) this is induced by the restriction morphisms

$$H^0(U \cap S, L_{\mathbf{Z}}) \hookrightarrow H^0(U' \cap S, L_{\mathbf{Z}}),$$

where U, U' are open neighborhoods of s, s' in $\overline{S}, \overline{S}'$ such that $\pi(U') \subset U$. To show that this respects the mixed Hodge structures, set $Z = \pi^{-1}(s)$, $\pi_Z = \pi|_Z : Z \to \{s\}$, and let $i_Z : Z \to \overline{S}'$, $i'_{s'} : \{s'\} \to Z$, $i_{s'} : \{s'\} \to \overline{S}'$, $j' : S \to \overline{S}'$ denote the inclusion morphisms. Then (2.1.3) coincides with the composition of canonical morphisms

$$i_s^*\mathbf{R}j_*\mathbf{H} = i_s^*\mathbf{R}\pi_*\mathbf{R}j'_*\mathbf{H} = \mathbf{R}(\pi_Z)_*i_Z^*\mathbf{R}j'_*\mathbf{H} \to i'^*_{s'}i_Z^*\mathbf{R}j'_*\mathbf{H} = i_{s'}^*\mathbf{R}j'_*\mathbf{H}.$$

The above description of (2.1.3) implies that the cokernel of (2.1.3) is torsion-free since $H^0(U\cap S, L_{\mathbf{Z}})$ is identified with the monodromy invariant subspace and $L_{\mathbf{Z}}$ is torsion-free. Combining this with the negativity of the weight w, we see that (2.1.3) induces an injection

$$J(H_{\pi(s')}) \hookrightarrow J(H_{s'}).$$

Thus there is a subspace

$$J_{\overline{S}',\overline{S}}(\mathbf{H})^0 := \coprod_{s'\in\overline{S}'} J(H_{\pi(s')}) \subset J_{\overline{S}'}(\mathbf{H})^0 = \coprod_{s'\in\overline{S}'} J(H_{s'}),$$

together with the surjection

$$J_{\overline{S}',\overline{S}}(\mathbf{H})^0 \to J_{\overline{S}}(\mathbf{H})^0.$$

So the problem of constructing the Néron model is reduced to the normal crossing case using the the quotient topology. It is further reduced to the normal crossing case with unipotent monodromy by taking locally a finite covering. Then $J_{\overline{S}}(\mathbf{H})^0$ is a subset of the Zucker extension, and we have the induced topology on it. So the topology is defined. This construction is independent of the choice of the resolution.

Now let $\{S_\alpha\}$ be a Whitney stratification of $\overline{S}$ such that the $R^kj_*L_{\mathbf{Z}}$ are locally constant on each stratum. Let $i_\alpha : S_\alpha \to \overline{S}$ denote the inclusion. Then $H^ki_\alpha\mathbf{R}j_*\mathbf{H}$ are variation of mixed Hodge structures. So we have a structure of a complex Lie group on

$$J_{\overline{S}}(\mathbf{H})^0|_{S_\alpha} = \coprod_{s\in S_\alpha} J(H_s).$$

Its underlying topology coincides with the induced topology of the topology on $J_{\overline{S}}(\mathbf{H})^0$ constructed above. Indeed, we can show that an open subset V of $J_{\overline{S}}(\mathbf{H})^0|_{S_\alpha}$ as a complex Lie group over S_α is an open subset in the induced topology of the topology constructed above as follows. The assertion is reduced to the case $\overline{S}$ is smooth, $\overline{S}\setminus S$ is a divisor with normal crossings, and the local monodromies are unipotent. Then there is an open subset V' of $\widehat{\Gamma}$ such that its restriction over S_α is the inverse image of V to $\widehat{\mathcal{V}}|_{S_\alpha}$. Taking the union of V' with translates of $V'|_U$ by local sections of $\widehat{\Gamma}$ defined over open subsets U of $\overline{S}$, we may assume that V' is stable by the action of local sections of $\widehat{\Gamma}$. So the assertion follows. (The other direction is easy.)

2.2. Theorem. *Let ν be an admissible normal function with respect to a partial compactification $j : S \hookrightarrow \overline{S}$. Assume $\gamma_s(\nu) = 0$ for any $s \in \overline{S}\setminus S$. Then the ν_s in* (2.1.2) *define a continuous section of $J_{\overline{S}}(\mathbf{H})^0$, which is holomorphic over each stratum S_α.*

Proof. By the definition of the topology, the assertion is reduced to the normal crossing case with unipotent local monodromies. Then it is sufficient to show that ν_s coincides with the section of the Zucker extension over $\overline{S}$ defined by (1.3.2) using two sections σ_F and $\sigma_{\mathbf{Z}}$. So the assertion is reduced to the curve case by restricting to the diagonal curves since the restriction of the Zucker extension to the diagonal curve is again the Zucker extension, see (1.2). In the curve case, we may assume $S = \Delta^*$ and $\overline{S} = \Delta$. Then the assertion follows from (1.4).

2.3. Néron models. Let $\nu \in \mathrm{NF}(U\cap S)^{\mathrm{ad}}_U$ where U is an open subset of $\overline{S}$. It defines the ν-component $J_U(\mathbf{H}_{U\cap S})^\nu$ of the Néron model over U, which 'graphs' ν. More precisely, it has a canonical isomorphism

$$J_U(\mathbf{H}_{U\cap S})^\nu = J_U(\mathbf{H}_{U\cap S})^0, \tag{2.3.1}$$

such that ν corresponds to the zero section of the right-hand side.

For $\nu,\nu' \in \mathrm{NF}(U\cap S)^{\mathrm{ad}}_U$ such that $\gamma_s(\nu) = \gamma_s(\nu')$ for any $s \in U\setminus S$, we have a canonical isomorphism

$$J_U(\mathbf{H}_{U\cap S})^\nu = J_U(\mathbf{H}_{U\cap S})^{\nu'}, \tag{2.3.2}$$

which corresponds by the isomorphism (2.3.1) to an automorphism of $J_U(\mathbf{H}_{U\cap S})^0$ defined by $\nu-\nu'$. So the Néron model $J_{\overline{S}}(\mathbf{H})$ is defined by gluing the $J_U(\mathbf{H}_{U\cap S})^\nu$ for admissible normal functions ν defined over $U\cap S$ where U are open subsets of $\overline{S}$. This is done by induction on strata of a Whitney stratification of $\overline{S}$ such that $R^1j_*L_{\mathbf{Z}}$ is a local system on each stratum S_α. More precisely, we extend it over the open subsets $\coprod_{\dim S_\alpha\ge r} S_\alpha$ by decreasing induction on r, using the fact that $\gamma_s(\nu)$ is locally constant on each stratum S_α. Then the restriction of $J_{\overline{S}}(\mathbf{H})$ over each stratum is a complex Lie group over the stratum. Here we use the fact that if $\gamma_s(\nu) = \gamma_s(\nu')$ then $\gamma_{s'}(\nu) = \gamma_{s'}(\nu')$ for $s' \in \overline{S}\setminus S$ sufficiently near s. For $s \in \overline{S}$, set

$$G_s = \bigcup_{U\ni s} \mathrm{Im}(\mathrm{NF}(U\cap S)^{\mathrm{ad}}_U \to (R^1j_*L_{\mathbf{Z}})_s), \tag{2.3.3}$$

where U runs over open neighborhoods of s in $\overline{S}$. Then the topological group $G = \coprod_s G_s$ is identified with an open subspace of $R^1 j_* L_{\mathbf{Z}}$ viewed as an etale space associated with a sheaf over $\overline{S}$. As a conclusion, we get

2.4. Theorem. *The Néron model $J_{\overline{S}}(\mathbf{H})$ graphs any admissible normal functions ν. More precisely, ν defines a continuous section of $J_{\overline{S}}(\mathbf{H})$, which is holomorphic over each stratum S_α of a Whitney stratification. Furthermore, there is a short exact sequence of commutative topological groups over $\overline{S}$*

$$0 \to J_{\overline{S}}(\mathbf{H})^0 \to J_{\overline{S}}(\mathbf{H}) \to G \to 0.$$

2.5. Remarks. (i) The topological group G over $\overline{S}$ is locally homeomorphic to $\overline{S}$ (on a neighborhood of each point of G). However, its topology *cannot be* Hausdorff unless $|G_s| = 1$ for all points s of $\overline{S} \setminus S$. Moreover, it is unclear whether the restriction $G|_{S_\alpha}$ of G over S_α is locally trivial over S_α (on a neighborhood of each point s of S_α). It is quite difficult to determine G_s in general.

(ii) In case $\mathbf{H}$ is pure of weight -1, it is known (see e.g. [1]) that there is an inclusion

$$G_s \otimes_{\mathbf{Z}} \mathbf{Q} \hookrightarrow \mathrm{Hom}_{\mathrm{MHS}}(\mathbf{Q}, H^1 i_s^* j_{!*} \mathbf{H}_{\mathbf{Q}}), \tag{2.5.1}$$

where $j_{!*}\mathbf{H}$ denotes the intermediate direct image. If $D = \overline{S} \setminus S$ is a divisor with normal crossings, then the target is calculated by a subcomplex of the Koszul complex as is well-known. If furthermore $\mathbf{H}$ is a nilpotent orbit or if $\mathbf{H}$ corresponds to a family of Abelian varieties, then it is easy to show the surjectivity of (2.5.1) although it does not hold in general. (This subject will be treated in a forthcoming paper.)

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