Estimation of Growth Parameters in Batch Fermentation.
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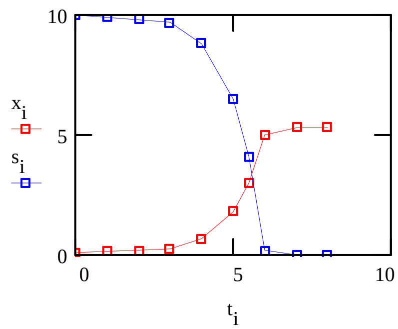
Problem Statement: Estimate the length of the lag phase, specific growth rate μ, doubling time t_d, and the yield coefficient Y from the following batch fermentation data.

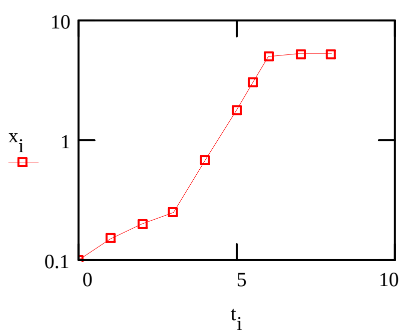
Time (hr) $t_i :=$	Biomass (g/L) $x_i :=$	Substrate (g/L) $s_i :=$
0	0.1	10
1	0.15	9.9
2	0.2	9.8
3	0.25	9.7
4	0.68	8.8
5	1.8	6.5
5.5	3.0	4.1
6	5.0	0.2
7	5.3	0
8	5.3	0

$i := 0 .. 100$

$i := 0 .. \mathrm{last}(t) \qquad \mathrm{last}(t) = 9$



Lag Phase. It is difficult to see from the biomass-versus-time plot when the lag phase ends and the exponential growth phase starts. A semi-log plot shows this much more clearly. The following plot shows that the growth curve takes off at t=3hr; thus, the lag phase lasted for 3 hours.



Equation for exponential growth:

$$\frac{d}{dt}x(t) = \mu \cdot x \qquad \text{I.C.} \qquad x(t_0) = x_0$$

When μ is constant, the above equation integrates to:

$$x(t) = x_0 \cdot \exp[\mu \cdot (t - t_0)]$$

$$\ln(x) = \ln(x_0) + \mu \cdot (t - t_0)$$

Specific Growth Rate. The specific growth rate is the slope of the exponential growth portion in the semi-log plot. Since the curve is fairly linear, we can simply take the first point at t=3hr and the second point at t=6hr to find the slope.

$$\mu := \frac{\ln(x_7) - \ln(x_3)}{t_7 - t_3} \qquad \mu = 0.999 \ (\mathrm{hr}^{-1})$$

Had the points been more scattered, we should resort to linear regression.

Extract the linear portion: $i := 3 .. 7 \qquad tt_{i-3} := t_i \qquad xx_{i-3} := \ln(x_i)$

Linear regression: $\mu := \mathrm{slope}(tt, xx) \qquad \mu = 0.996 \ (\mathrm{hr}^{-1})$

Doubling Time. $2 \cdot x_0 = x_0 \cdot \exp(\mu \cdot t_d)$ $t_d := \frac{\ln(2)}{\mu}$ $t_d = 0.696$ (hr)

Saturation Constant. At the time when substrate becomes depleted, biomass concentration is high and the cells are growing fast (exponentially). The low substrate condition is passed quickly. There is no data to calculate the saturation constant K_s in a Monod model $\mu=\mu_m \cdot s/(K_s+s)$.

Yield Coefficient.

$$Y = \frac{amount\_of\_cells\_generated}{amount\_of\_substrate\_consumed}$$

$$Y := \frac{x_{last(t)} - x_0}{s_0 - s_{last(t)}} \qquad Y = 0.52 \quad \text{(g/g)}$$