On Synchronization of Networked Passive Systems with Time Delays and Application to Bilateral Teleoperation

Nikhil Chopra and Mark W. Spong
Coordinated Science Laboratory
University of Illinois at Urbana-Champaign
1308 W. Main St.
Urbana, IL 61801 USA
{nchopra,mspong}@uiuc.edu

Abstract: In this paper we present a result for output synchronization of dynamic agents in an arbitrary connected network. The agents are said to be output synchronized if their outputs converge asymptotically. In this note the agents are assumed to be nonlinear passive systems which are affine in the control. The main result of the paper claims that for a class of coupling controls, networked passive systems synchronize even in the presence of arbitrary constant delays in the network. The practical applicability of the result is demonstrated in the problem of bilateral teleoperation with time delay. Bilateral teleoperators, designed within the passivity framework using concepts of scattering and two-port network theory, provide robust stability against constant delay in the network and velocity tracking, but cannot guarantee position tracking in general. Also, the use of the scattering transformation can lead to wave reflections which can degrade the performance of the bilateral teleoperator. In this paper we fundamentally extend the passivity based traditional architecture to guarantee position and force tracking in the face of offset of initial conditions, environmental contacts and packet losses in the network. The novelty of the new approach lies in the fact that it guarantees delay independent exponential stability of the position and velocity tracking error without using the scattering transformation approach of [1], and is thus independent of the deleterious performance repercussions of the same.

Keywords: Synchronization, Passivity, Teleoperation

1. Introduction and Background

Collective synchronization phenomena have been observed in biological, chemical, physical and social systems and has attracted the interest of researchers for centuries. Synchronization is a key concept in understanding the phenomenon of self-organization occurring in systems of the dissipative type. Recently control theoretic methods have been used to address the synchronization phenomenon in [15, 12, 7, 6]. In [12] phase models of coupled oscillators were used to derive control laws for stabilizing collective motion of a group of self-propelled particles. In [11] consensus problems were discussed for a network of dynamic agents with fixed and switching topologies. In [6] control and graph theoretic methods were used to analyze the Kuramoto oscillators for an arbitrary graph topology.

Passivity is one of the most appealing concepts of system theory and has been widely used as a fundamental tool in the development of linear and nonlinear feedback designs. The problem that we study in this paper is that of output synchronization of N dynamic passive agents with time-delayed outputs. We also show that the main result of the paper has an important application in the problem of bilateral teleoperation.

The dynamic systems studied in this note are control affine and their evolution can be described by the following equation

$$\begin{aligned} \dot{x}_i &= f_i(x_i) + g_i(x_i)u_i \\ y_i &= h_i(x_i) \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

where $x_i \in R^n$, $f_i(.) \in R^n$, $g_i(.) \in R^{n\times m}$, $u_i \in R^m$, $h_i(.) \in R^m$, and with $f_i(0) = 0$, $h_i(0) = 0$. The admissible controls are taken to be locally square integrable. We assume here that the vector fields appearing in (1) have sufficient smoothness so that the a unique solution exists for all times. The agents are said to output synchronize if

$$y_i - y_j \to 0 \quad as \quad t \to \infty \quad \forall i, j = 1, \ldots, N$$

The system described by (1) is said to be passive with (u_i, y_i) as the input-output pair if there exists a C^1 storage function $V_i(x_i) \geq 0$, $V_i(0) = 0$, such that for all $t \geq 0$

$$V_i(x_i) - V_i(x_i(0)) \leq \int_0^t u_i^T(s) y_i(s) ds$$

At this point we recall a theorem which was originally proposed in [9] and we cite here a relaxed version from [8]. This theorem plays an important part in the subsequent analysis.

Theorem 1.1 *Consider the nonlinear system described by* (1)*. The following statements are equivalent.*

1. *There exists a C^1 storage function $V_i(x_i) \geq 0$, $V_i(0) = 0$ and a function $S_i(x_i) \geq 0$ such that for all $t \geq 0$:*

$$V_i(x_i) - V_i(x_i(0)) = \int_0^t u_i^T(s) y_i(s) ds - \int_0^t S_i(x_i(s)) ds$$

The system is Strictly Passive for $S_i(x_i) > 0$, Passive for $S_i(x_i) \geq 0$ and lossless for $S_i(x_i) = 0$.

2. *There exists a C^1 scalar function function $V_i(x_i) \geq 0$, $V_i(0) = 0$, such that*

$$L_{f_i} V_i(x_i) = -S_i(x_i)$$
$$L_{g_i} V_i(x_i) = h_i^T(x_i)$$

where $L_{f_i} V_i(x_i) = \frac{\partial V_i}{\partial x_i}^T f_i(x_i)$ and $L_{g_i} V_i(x_i) = \frac{\partial V_i}{\partial x}^T g_i(x_i)$

An interested reader is referred to [8] for the proof of this theorem.

2. Synchronization in Agents with Regular Graph Structure

The objective of this paper is to develop control strategies for synchronization of the passive agents which are networked using a general interconnection topology. The agent dynamics are assumed to be passive with positive definite storage functions given by $V_1(x_1), V_2(x_2), \ldots, V_N(x_N)$ respectively. In this paper we will assume that the agents form an m *regular* connected graph with respect to information exchange among the agents. This implies that each agent has the same number of neighbors which is given by m. We also assume that the $i-th$ agent is influenced by all its m neighbors and it in turn influences them. An example of such an topology for a system of 4 agents is illustrated in Figure 1. The agents in this case form an m

Figure 1: An example of a network of four agents

regular graph where $m = 1$. As there are time delays in the network, the agents receive a delayed version of the outputs of other agents. Let the agents be coupled together using a control which is given as

$$u_i = \sum_{j \in \mathcal{N}_i} K(y_j(t-T) - y_i) \;\; i = 1, \ldots, N \qquad (2)$$

where K is a positive constant, $\mathcal{N}_i$ is the set of m agents which are transmitting their outputs to the $i-th$ agent (thus the cardinality of each set $N_i \;\; \forall i = 1, \ldots, N$ is m), and T is the constant time-delay in the network. We are now in a position to state the main result of this paper

Theorem 2.1 *Consider the dynamical system described by* (1) *and coupled together using the control described by* (2)*. Then for all arbitrary initial conditions, all signals in the system are bounded and the nonlinear systems described by* (1) *output synchronize.*

Proof Let a positive definite Lyapunov function for N agent system be given as

$$V = mK \int_{t-T}^t (y_1^T y_1 + \ldots + y_N^T y_N) d\tau + 2(V_1 + \ldots + V_N)$$

The derivative of this Lyapunov function along trajectories of the system is given as

$$\dot{V} = mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T))$$
$$+2 \sum_{i=1}^N (L_{f_i} V_i + L_{g_i} V_i u_i)$$

Using Theorem (1.1), the derivative reduces to

$$\begin{aligned}
\dot{V} &= mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T)) \\
&+2 \sum_{i=1}^N (-S_i(x_i) + y_i^T u_i) \\
&= mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T)) - 2 \sum_{i=1}^N S_i(x_i) \\
&+2 \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i^T K(y_j(t-T) - y_i) \\
&= mK \sum_{i=1}^N (y_i^T y_i - y_i(t-T)^T y_i(t-T)) - 2 \sum_{i=1}^N S_i(x_i) \\
&-2K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i^T y_i + 2K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_j(t-T) y_i \qquad (3)
\end{aligned}$$

The term $mK \sum_{i=1}^N y_i^T y_i$ can be written as $K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i^T y_i$ as the cardinality of every set $N_i = m$. Similarly the term $-mK \sum_{i=1}^N y_i(t-T)^T y_i(t-T)$ can be written as $-K \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} y_i(t-T)^T y_i(t-T)$.

Using this in (3) we have that

$$\dot{V} = -K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_i^T y_i - K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_i(t-T)^T y_i(t-T)$$
$$+2K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} y_j(t-T)y_i - 2\sum_{i=1}^{N} S_i(x_i)$$
$$= -K\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} (y_j(t-T)-y_i)^T(y_j(t-T)-y_i)$$
$$-2\sum_{i=1}^{N} S_i(x_i)$$

As $S_i(x_i) \leq 0$ (from Theorem (1.1)), we have that that $\dot{V} \leq 0$. As the Lyapunov function is positive definite, all signals in the system are bounded. Consider the set $E = \{x_i \in R^{n\times 1}\ i = 1,\ldots,N \mid \dot{V}\equiv 0\}$. The set E is characterized by all trajectories such that $\{S_i(x_i)\equiv 0,\ \forall i = 1,\ldots,N,\ \ (y_i - y_j(t-T))^T(y_i - y_j(t-T))\equiv 0\quad \forall j \in \mathcal{N}_i\quad \forall i = 1,\ldots,N\}$. Let M be the largest invariant set contained in E. Using Extension of Lasalle's Invariance Principle for Time-Delay systems [5], all solutions of the dynamical system given by (1) and (2) converge to M as $t\to\infty$. This implies that the output of every $i-th$ agent asymptotically converges to that of its neighbors. Connectivity of the network then implies output synchronization of (1).

It is to be noted that if the output is the full state vector, i.e. $h(x) = x$, then Theorem (2.1) implies state convergence as well. In the above proof, we assumed for the sake of simplicity that there was only one constant delay in the network. The above analysis can be easily extended to the case where different agents experience different (constant) network delays.
To illustrate the result, an example of 4 agents with integrator dynamics is taken with the information topology as shown in Figure 1. The dynamics of the agents are given as

$$\dot{x}_i = u_i \qquad y_i = x_i \quad i = 1,2,3,4.$$

The agents are passive with a positive definite storage function given by $V_i = \frac{1}{2}x_i^T x_i$. From Figure 1 it is clear that $N_i = \{i+1\}\ \ i = 1,2,3$ and $N_4 = \{1\}$. Hence the cardinality of each set N_i is 1, i.e. m=1. Using (2), the system dynamics are given as

$$\begin{aligned}
\dot{x}_1 &= K(x_2(t-T) - x_1)\\
\dot{x}_2 &= K(x_3(t-T) - x_2)\\
\dot{x}_3 &= K(x_4(t-T) - x_3)\\
\dot{x}_4 &= K(x_1(t-T) - x_4)
\end{aligned} \tag{4}$$

The positive definite Lyapunov function for the system is given as

$$V = K\int_{t-T}^{t} (x_1^T x_1 + x_2^T x_2 + x_3^T x_3 + x_4^T x_4)d\tau$$
$$+(x_1^T x_1 + x_2^T x_2 + x_3^T x_3 + x_4^T x_4)$$

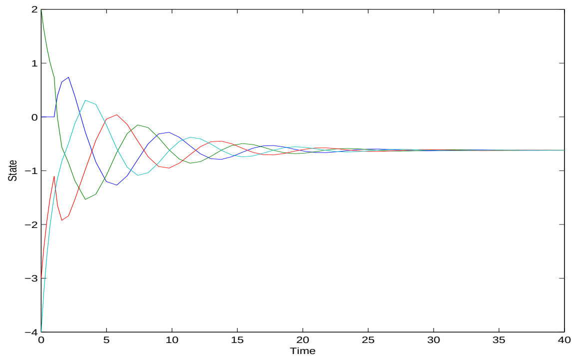

Figure 2: The state of the four agents converges asymptotically

It is easy to see using (4) that

$$\dot{V} = -\Big(||x_2(t-T) - x_1)||_2^2 + ||x_3(t-T) - x_2)||_2^2$$
$$+||x_4(t-T) - x_3)||_2^2 + ||x_1(t-T) - x_4)||_2^2\Big)$$

where the notation $||\ \cdot\ ||_2$ denotes the $\mathcal{L}_2$ norm of a signal on the interval $[0,t]$. Using Extension of Lasalle's Invariance Principle for Time-Delay systems [5], the state of the dynamical system given by (4) synchronizes. It is to be noted that state synchronization occurs as the output of each subsystem was its state. Simulating the above system with the network delay as $1s$, we see in Figure 2 that the agents synchronize.

3. Application to Bilateral Teleoperation

The results of the previous section tell us that passive systems when interconnected using (2) are robust to time-delays. This result has a very interesting ramifications for the problem of bilateral teleoperation which can be taken as a special case of the previous result with $N = 2$ and $m = 1$. It will be instructive to start with an introduction to the problem of bilateral teleoperation.

3.1 Bilateral Teloperation

In bilateral teleoperation the master and the slave manipulators are coupled via a communication network and time delay is incurred in transmission of data between the master and slave site. It is well known that the delays in a closed loop system can destabilize an otherwise stable system. The time-delay problem in bilateral teleoperation has addressed successfully using scattering theory [1], and we now provide a brief introduction to this approach.
A teleoperator consists of the following subsystems: the human operator, the master, the communication block, the slave and the environment. The human operator commands the master with force F_h to move it with velocity $\dot{q}_m$ which is sent to the slave through the communication block. A local control (F_s) on the slave side

drives the slave velocity $\dot{q}_s$ towards the master velocity. If the slave contacts a remote environment, the remote force F_e is communicated back from the slave side and received at the master side as the force F_m. The standard bilateral teleoperation architecture of [1] with the scattering transformation is shown in Figure 3. This

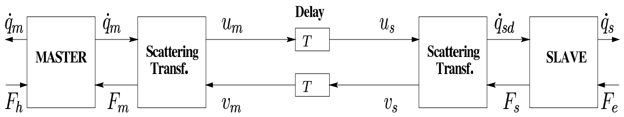

Figure 3: A Standard Bilateral Teleoperation Setup

architecture uses the passivity formalism and concepts from network theory to construct an interconnection of passive blocks which is well-known to be dissipative. The master and the slave robots are passive from force to velocity and the network block is passified by the scattering transformation. This system, when interconnected with a passive human operator and remote environment, is passive. The scattering transformation approach guarantees passivity of the network block independent of any constant delay in the network.

However, this configuration places an inherent limitation on the transparency (measure of position and force tracking) of the system. As shown in [2, 3], the architecture drives the velocity errors between the master and the slave to zero, but can only guarantee the position tracking error to be bounded. If the master and slave start with same initial position and velocity, the slave faithfully tracks the master (due to convergence of the velocities), but in the case there exists an initial offset between the master and the slave, the teleoperation configuration cannot guarantee the convergence of the position tracking error to the origin. Teleoperating devices over the Internet requires packets to be transmitted via an unreliable packet switched network. The network may induce packet drops which will lead to an unrecoverable position drift between the master and the slave robots. Additionally, lack of impedance matching when using the scattering transformation approach lead to wave reflections [10], which can seriously degrade tracking performance in bilateral teleoperation. To address the aforementioned issues, a new scheme was proposed in [4] which achieved the following goals

- A feedback control law for the master and the slave manipulator that renders the manipulator dynamics passive with respect to an output that contains both position and velocity information.
- A passive coordination control law which uses this output from the master and the slave to *kinematically lock* the motion of the two robotic systems.
- An adaptation mechanism to account for the unknown parameters

However, the proposed scheme used the scattering transformation approach to passify the communication block. We now see how Theorem (2.1) gives us an insight into extending the formulations in [4] without using the scattering transformation.

3.2 The New Architecture

Assuming absence of friction or other disturbances, the Euler-Lagrange equations of motion for a n-link master and slave robot are given as [13]

$$\begin{aligned} M_m(q_m)\ddot{q}_m + C_m(q_m, \dot{q}_m)\dot{q}_m + g_m(q_m) &= F_h + \tau_m \\ M_s(q_s)\ddot{q}_s + C_s(q_s, \dot{q}_s)\dot{q}_s + g_s(q_s) &= \tau_s - F_e \end{aligned} \tag{5}$$

where q_m, q_s are the $n \times 1$ vectors of joint displacements, $\dot{q}_m$, $\dot{q}_s$ are the $n \times 1$ vectors of joint velocities, τ_m, τ_s are the $n \times 1$ vector of applied torques, $M(q)$ is the $n \times n$ symmetric positive definite manipulator inertia matrix, $C(q, \dot{q})$ is the $n \times n$ vector of Centripetal and Coriolis torques and $g(q)$ is the $n \times 1$ vector of gravitational torques. In the analysis in this section we assume that

- The human operator and the environment can be modeled as passive systems with r_m and r_s as inputs respectively.
- The operator and the environmental force are bounded by functions of the signals r_m and r_s respectively.
- All signals belong to $\mathcal{L}_{2e}$, the extended $\mathcal{L}_2$ space.

In order to achieve the design objectives, the motor torques are given as [4]

$$\begin{aligned} \tau_m &= -F_m - \hat{M}_m(q_m)\lambda\dot{q}_m - \hat{C}_m(q_m, \dot{q}_m)\lambda q_m + \hat{g}_m(q_m) \\ \tau_s &= F_s - \hat{M}_s(q_s)\lambda\dot{q}_s - \hat{C}_s(q_s, \dot{q}_s)\lambda q_s + \hat{g}_s(q_s) \end{aligned} \tag{6}$$

where $\hat{M}_i$, $\hat{C}_i$, $\hat{g}_i \quad i = m, s$ are the estimates of the respective matrices available at that instant, F_m, F_s are the additional torques as required for coordination control and λ is a constant positive definite diagonal matrix. As the dynamics are linearly parameterizable, the motor torques can also be written as

$$\begin{aligned} \tau_m &= -F_m - Y_m(q_m, \dot{q}_m)\hat{\theta}_m \\ \tau_s &= F_s - Y_s(q_s, \dot{q}_s)\hat{\theta}_s \end{aligned}$$

where Y_m, Y_s are known functions of the generalized coordinates and $\hat{\theta}_m$, $\hat{\theta}_s$ are the time-varying estimates of the manipulators' actual constant p dimensional inertial parameters given by θ_m, θ_s respectively. The master and slave dynamics (5) reduce to

$$\begin{aligned} M_m\dot{r}_m + C_m r_m = Y_m\tilde{\theta}_m + F_h - F_m &= \tau_m^{'} \\ M_s\dot{r}_s + C_s r_s = Y_s\tilde{\theta}_s + F_s - F_e &= \tau_s^{'} \end{aligned} \tag{7}$$

where the vectors r_m and r_s are the new outputs of the bilateral teleoperator and are given as

$$\begin{aligned} r_m &= \dot{q}_m + \lambda q_m \\ r_s &= \dot{q}_s + \lambda q_s \end{aligned} \tag{8}$$

and $\tilde{\theta}_m$, $\tilde{\theta}_s$ are the estimation errors and are given as

$$\tilde{\theta}_m = \theta_m - \hat{\theta}_m$$
$$\tilde{\theta}_s = \theta_s - \hat{\theta}_s$$

It is easy to verify that the new master and slave dynamics are passive with $(\tau_m^{'}, r_m)$ and $(\tau_s^{'}, r_s)$ as the input-output pairs.

Let the coupling torques for the two bilateral teleoperator be given as

$$\begin{aligned} F_s &= K(r_m(t-T) - r_s) \\ F_m &= K(r_s(t-T) - r_m) \end{aligned} \tag{9}$$

where K is a positive definite diagonal matrix. The time varying estimates of the uncertain parameters evolve as

$$\begin{aligned} \dot{\hat{\theta}}_m &= \Gamma Y_m^T r_m \\ \dot{\hat{\theta}}_s &= \Lambda Y_s^T r_s \end{aligned} \tag{10}$$

where Γ and Λ are constant positive definite matrices. Define the coordination errors between the master and slave robots as

$$\begin{aligned} e_m(t) &= q_m(t-T) - q_s(t) \\ e_s(t) &= q_s(t-T) - q_m(t) \end{aligned} \tag{11}$$

Thus, we have two passive systems which are coupled together with a control similar to (2). Therefore, intuitively the results of Theorem (2.1) should apply to the system governed by (7), (9), and (10). To formalize this argument we state our next result

Theorem 3.1 *Consider the nonlinear bilateral teleoperator described by* (7), (9) *and* (10). *Then all signals in the system are bounded, the master and slave robots output synchronize and the coordination errors given by* (11) *are globally exponentially stable.*

Proof Define a positive definite function for the system as

$$V = \left(r_m^T M_m r_m + r_s^T M_s r_s + \tilde{\theta}_m^T \Gamma^{-1} \tilde{\theta}_m + \tilde{\theta}_s^T \Lambda^{-1} \tilde{\theta}_s\right)$$
$$+K \int_{t-T}^{t} (r_m^T r_m + r_s^T r_s) ds + 2 \int_0^t (F_e^T r_s - F_h^T r_m) ds$$

The human operator and the remote environment are passive (by assumption). Hence

$$\int_0^t F_e r_s ds \geq 0 \quad ; \quad -\int_0^t F_h r_m ds \geq 0$$

Thus the function V is positive-definite. The derivative of this function along trajectories of the system is given by

$$\dot{V} = 2r_m^T(-C_m r_m + F_h + F_m + Y_m \tilde{\theta}_m) + r_m^T \dot{M}_m r_m$$
$$+r_s^T(-C_s r_s + F_s - F_e + Y_s \tilde{\theta}_s) + 2r_s^T \dot{M}_s r_s - 2\tilde{\theta}_m^T Y_m^T r_m$$
$$-2\tilde{\theta}_m^T Y_s^T r_s + K r_m^T r_m - K r_s(t-T)^T r_s(t-T) + K r_s^T r_s$$
$$-K r_m(t-T)^T r_m(t-T) + 2F_e^T r_s - 2F_h^T r_m$$

Using the skew-symmetric property of robot dynamics, the derivative reduces to

$$\dot{V} = 2r_m^T F_m + K(r_m - r_s(t-T))^T (r_m + r_s(t-T))$$
$$+2r_s^T F_s + K(r_s - r_m(t-T))^T (r_s + r_m(t-T)$$
$$= 2r_m^T F_m - F_m^T(r_m + r_m + K^{-1} F_m) + 2r_s^T F_s$$
$$-F_s^T(r_s + r_s + K^{-1} F_s)$$
$$\dot{V} = -F_m^T K^{-1} F_m - F_s^T K^{-1} F_s$$

As $\dot{V} \leq 0$, all signals in the system are bounded. The signals F_m and F_s converge to the origin (as in Theorem (2.1) using Lasalle's principle for time-delay systems, and hence the master slave robots output synchronize (from (9)). Additionally, we that $F_m, F_s \in \mathcal{L}_2$. But,

$$F_s = \dot{e}_m + \lambda e_m$$

The above equation can be viewed as an exponentially stable linear system with the state e_m and a $\mathcal{L}_2$ input given by F_s. Hence, the coordination error is globally exponentially stable. The result similarly holds for the coordination error e_s.

It is to be noted that this approach guarantees **delay independent exponential convergence** of the tracking errors to the origin without using scattering theory. The scattering transformation approach of [1)] has been the cornerstone of bilateral teleoperator theory since its formulation some 25 years ago. The main result of this paper (Theorem (2.1)) has helped in stabilizing the bilateral teleoperator without using this transformation.

4. Simulations

In this section we simulate the schemes on a single-degree of freedom bilateral teleoperator. The master and slave dynamics are given as

$$M_m \ddot{q}_m = F_h + \tau_m$$
$$M_s \ddot{q}_s = \tau_s - F_e$$

The master robot was commanded to follow a sinusoidal trajectory (using appropriately constructed human operator torque) till time t=50s and the human operator command was shut down after this time. To obstruct the motion of the slave, a virtual wall (spring-damper system) was constructed in the simulation setup. The master motion and the resulting slave motion is shown in Figure 4. The master followed a sinusoidal trajectory till time t=50s, after which it exponentially converged to the slave position. Also, as seen in Figure 5, the environmental torque F_e on contact, represented by the dashed line, is accurately transmitted to the master as the torque F_m which is represented by the solid line in the graph.

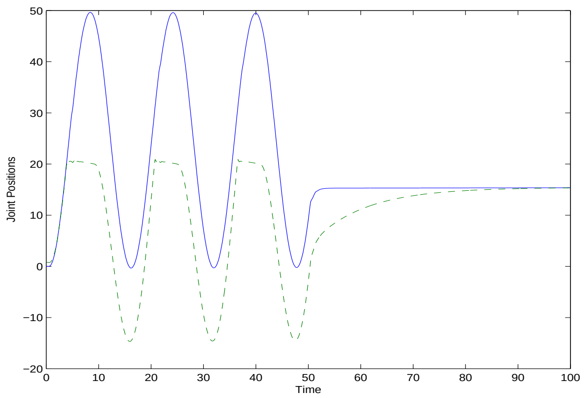

Figure 4: The master and the slave joint positions

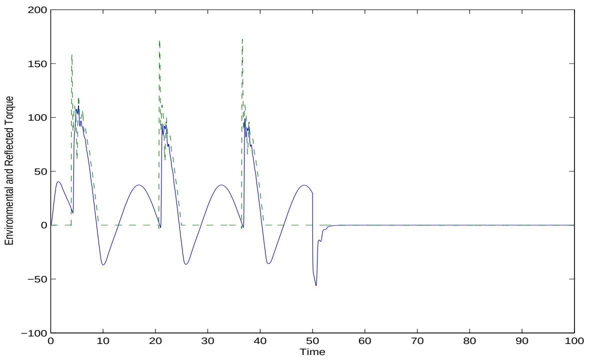

Figure 5: The reflected torque to the master F_m accurately tracks the environmental torque F_e

5. Conclusions

In this paper a new property for interconnection of passive systems was demonstrated. It was shown that agents with passive dynamics, and a regular information graph imposed on them, output synchronize even in the presence of (constant) delays in the network. It was then shown that this property has an important application in the problem of bilateral teleoperation where delay independent exponential stability of the position tracking errors was demonstrated without the use of the traditional scattering transformation approach.

6. Acknowledgements

This research was partially supported by the Office of Naval Research under Grant N00014-02-1-0011 and by the National Science Foundation under Grants ECS-0122412, Int-0128656, HS-0233314 and CCR-0209202.

References

[1] Anderson, R.J., and Spong, M.W., "Bilateral control of teleoperators with time delay," *IEEE Trans. Automat. Contr.*, AC-34, No. 5, pp. 494–501, May, 1989.

[2] Anderson, R.J., and Spong, M.W., "Asymptotic stability for force reflecting teleoperators with time delay," *The Int. J. Robot. Res.*, Vol. 11, pp. 135-149, April, 1992.

[3] Chopra, N., Spong M.W., Hirche, S., Buss, M., "Bilateral teleoperation over the Internet: the time varying delay Problem," *Proc. ACC*, Denver, CO, June 4-6, 2003.

[4] Chopra, N., Spong M.W, and Lozano, R., "Adaptive coordination control of bilateral teleoperator with time delay", *In Proc. IEEE CDC, 2004*, pp. 4540-4547, 2004.

[5] Hale, J.K. and Verduyn, S.M., *Introduction to Functional Differential Equations*, Applied Mathematical Sciences, Springer-Verlag, New York, 1993.

[6] Jadbabaie, A., Motee, N., and Barahona, M., "On the stability of the Kuramoto model of coupled nonlinear oscillators," *Proceedings of the American Control Conference*, pp. 4296 - 4301, 2004.

[7] Justh, E.W, and Krishnaprasad, P.S., "A simple control law for UAV formation flying," Technical Report 2002-38, Institute for Systems Research, 2002.

[8] Lozano, R., Brogliato, B., Egeland, O. and Maschke, B., *Dissipative System Analysis and Control: Theory and Applications*, Springer-Verlag, London, 2000.

[9] Moylan, P.J., "Implications of passivity for a class of nonlinear systems," *IEEE Tran. Automat. Contr.*, Vol. 19, pp. 373-381, Aug. 1974.

[10] Niemeyer, G. and Slotine, J., "Stable Adaptive Teleoperation," *Int. J. Oceanic Engineering*, Vol 16, No. 1, pp 152-162, 1991.

[11] R. Olfati-Saber and R.M. Murray, "Consensus problems in networks of dynamic agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, Vol.49, pp. 1520 - 1533, Sept. 2004.

[12] Sepulchre, R., Paley,D., and Leonard, N. "Collective motion and oscillator synchronization," *Cooperative Control, Lecture Notes in Control and Information Sciences*, Vol. 309, Eds: V. Kumar, N. Leonard, and A. S. Morse, Springer Verlag, 2004.

[13] Spong, M.W. and Vidyasagar, M., *Robot Dynamics and Control*, John Wiley and Sons, New York, 1989

[14] Strogatz, S.H., "From Kuramoto to Crawford: exploring the onset of synchronization in populations of coupled oscillators," *Physica D*, 143:1-20, 2000.

[15] W. Wang and J.J.E. Slotine, "On partial contraction analysis for coupled nonlinear oscillators, Technical Report, Nonlinear Systems Laboratory, MIT, 2003.