Control of Robotic Manipulators under Input/Output Communication Delays: Theory and Experiments

Yen-Chen Liu and Nikhil Chopra

***Abstract*—Input/output delays in a control system can pose significantly impediments to the stabilization problem and potentially degrade the performance of the closed loop system. In this paper, we study the classical set-point control problem for rigid robots with input-output communication delays in the closed loop system. We demonstrate that if there are transmission delays between the robotic system and the controller, then the use of the scattering variables can stabilize an otherwise unstable system for arbitrary unknown constant delays. It is also demonstrated that the proposed algorithm results in guaranteed set-point tracking. In the case of time-varying delays, scattering variables together with additional gains can be utilized to stabilize the closed loop system composed of the robotic manipulator and the controller. Furthermore, a scattering representation based design with position feedback is proposed to improve closed loop performance under time-varying delays. The proposed algorithms are validated via experiments in this paper.**

***Index Terms*—Passivity-based control, Time delays, Networked robotic systems.**

I. Introduction

The use of communication networks for interconnecting robotic systems and controllers can lead to significant advantages, such as the increased flexibility and modularity as compared to traditional wired connections. Several results in the field of closed-loop control systems over networks have been studied in [1]–[4]. A wide variety of applications have been discussed [5]–[7]. However, the communication channels are subjected to various time delays that can not only degrade the performance of the closed loop system but also render the system unstable. Therefore, in this paper, we study the problem of motion control of rigid robots in the presence of input/output communication delays, as shown in Figure 1.

Delays in a control system can pose significantly impediments to the stabilization problem and potentially degrade the performance of the closed loop system. It is well known that guaranteeing stability of a control system with time delays is a challenging problem [8]. The Smith predictor [9], an useful delay-dependent method, can be applied to stabilize the closed loop system with high performance but requires exact knowledge of time delays and is sensitive to modeling errors. The classical Smith predictor has been developed for nonlinear systems in [10], [11] and for time-varying delays in [12].

Starting with the work of [13], [14], passivity-based control [15] has emerged a fruitful methodology for control design of robotic systems. Several control design have been presented in the literature [16], [17] where the controller and the mechanical system can be represented as a negative feedback interconnection of passive systems. Invoking the fundamental passivity theorem [18], it is then possible to guarantee passivity of the closed loop system. The property that a feedback interconnection of passive systems is also passive has been utilized in the study of bilateral teleoperation system with communication delays. Under the assumption that the environment and the human operator are passive, scattering or the wave-variable representation, which was studied in [19], [20], has been proposed to ensure the passivity of the communication block.

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Recently, the scattering representation has emerged as a novel tool for studying network control systems [21]–[25]. The basic idea in these results is to use the scattering variables for guaranteeing passivity of the communication block, thereby creating a passive two-port network between a passive plant and a passive controller. The use of scattering representation for networked control systems with constant delays was proposed in [25] where the results were developed for linear time-invariant (LTI) systems. This paper demonstrated that it was possible to stabilize the closed-loop LTI system using the scattering transformation independent of the constant delay. This approach was extended in [26] for nonlinear systems with non-passive plants or controllers by using the excess of passivity from passive system to compensate the shortage of passivity in the non-passive system. A coordinated compliance control of a robot system with distributed control architecture utilized wave variables to handle the constant delays in [24]. In [23], the scattering representation was employed for the energy shaping control methodology over a constant delays communication network. It is to be noted that the aforementioned results either do not address the problem of set-point control of nonlinear robotic manipulators [21], [22], [24]–[26] in the presence of constant input-output communication delays, or the set-point convergence has not been formally demonstrated [23]. In [27], the use of scattering representation for control of robotic manipulators with constant input/output delays was studied. However, [27] only demonstrated that the state of the controller converged to the desired configuration, while the set-point control of the robotic system was not guaranteed.

As the communication delays are rarely constant in practice, the scattering representation methodology has been extended to address time-varying delays. For the problem of bilateral teleoperation, the scattering or wave variables were modified in [28]–[30] to address time-varying delays in communication channels. By sending wave variables with stamped time, [31] proposed a method to compensate the distorted wave variables with the integration of waveform errors. An energy based input/output balance monitoring method was presented in [28] to improve the drawback in [31] that the system may generate infinite energy from integration. Without the needs of integrating waveform errors or the wave variables for the sum of energy, gains dependent on the maximum rate of change of delays were utilized to scale wave variables in [30] to ensure the passivity of the communication block under time-varying delays. Although the time-varying delay problem in bilateral teleoperation was studied by [28], [30], [31] using the scattering representation, these algorithms cannot be directly utilized for studying the set-point control problem with time-varying input-output delays. The time-varying gain formalism proposed in [30] has been utilized in [32] for stabilizing the networked set-point control system. However, the proposed architecture only ensures stability of the closed loop system and does not guarantee set-point tracking in the presence of time-varying delays.

In this paper, we study the problem of set-point control in rigid robots (with revolute joints) for both constant and time-varying input/output communication delays. In the absence of precise knowledge of the time delays and the robot dynamics, the stability and performance of the closed loop system is studied. In Theorem 1, we demonstrate using Lyapunov analysis that if the scattering transformation is used to encode the input/output variables for the nonlinear robotic system and the controller, then under appropriate assumptions, stability of the closed loop is recovered independent of unknown constant time delays. Furthermore, the theorem also justifies the intuitive claim that if the initial state of the controller is equal to the initial configuration of the robotic system, then the tracking error asymptotically approaches the origin. The control architecture

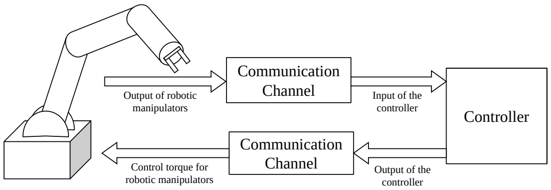


Fig. 1: The sketch of using communication networks to interconnect a robotic system with a controller.

is further validated via experiments in this paper.

The control system is extended for handling time-varying delays in Theorem 2, where in conjunction with the scattering variables, gains dependent on the maximum rate of change of delays [30] are utilized to guarantee stability of the closed loop system. However, this theorem can only ensure stability of the closed loop system and cannot guarantee set-point control in the presence of time-varying delays. Hence, in Theorem 3 delayed position feedback in conjunction with the scattering representation is proposed so as to achieve the regulation objective. The proposed control algorithm guarantees stability of the closed loop system and tracking performance under input/output time-varying delays even in the absence of innate dissipation in the robotic system [32]. Experimental results are presented to validate the efficiency for the proposed control architecture.

The rest of this paper is organized as follows. In Section II background on fundamental properties of robotic systems and passive systems is presented. This is followed by the stability result for constant input/output delays problem in Section III. Subsequently, the time-varying input/output delay problem is studied in Section IV. The proposed control algorithms are validated through experiments in Section V. The results and future work are summarized in Section VI.

II. Preliminaries

The concept of passivity is one of the most physically appealing concepts of system theory [33] and, as it is based on the input-output behavior of a system, it is equally applicable to both linear and nonlinear systems. The theorems proposed in this paper are developed based on the passivity property.

Consider a dynamical system represented by the state space model

$$\begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned} \tag{1}$$

where f: $R^n \times R^p \to R^n$ is locally Lipschitz, h:$\to R^n \to R^p$ is continuous, $f(0,0) = 0$, $h(0) = 0$ and the system has the same number of inputs and outputs.

Definition [34] The dynamical system (1) is said to be passive if there exists a continuously differentiable non-negative definite scalar function $S(x)$: $R^n \to R$ (called the storage function) such that

$$u^T y \geq \dot{S}(x), \qquad \forall (x,u) \in R^n \times R^p.$$

The robotic manipulator in the input/output delays system is modeled as a Lagrangian system. Following [35], in the absence of friction and disturbances, and assuming gravity compensation, the equations of motion for an n-degree-of-freedom robotic system are given as

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} = -\tau_s + \tau_e = \tau_t, \tag{2}$$

where $q \in R^n$ is the vector of generalized configuration coordinates, $\tau_s \in R^n$ is motor torque acting on the system, $\tau_e \in R^n$ is the external torque acting on the system, $M(q) \in R^{n\times n}$ is the positive definite inertia matrix and $C(q,\dot{q})\dot{q} \in R^n$ is the vector of Coriolis/Centrifugal forces. The above equations exhibit certain fundamental properties due to their Lagrangian dynamic structure [35].

- **Property 1:** The matrix $M(q)$ is symmetric positive definite and there exists a positive constant m such that $mI \leq M(q)$.
- **Property 2:** Under an appropriate definition of the matrix C, the matrix $\dot{M}$ - $2C$ is skew-symmetric

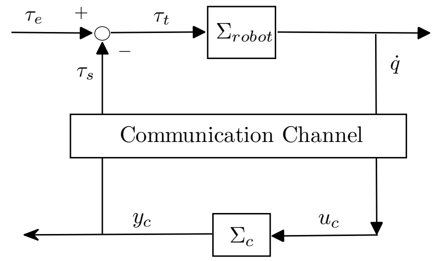


Fig. 2: A negative feedback interconnection of the robot dynamics and the controller.

Moreover, it is well known that the robot dynamics are passive [35] with $(\tau_t, \dot{q})$ as the input-output pair. The passivity property of the robot dynamics has led to constructive control designs for robot manipulators. Specifically, several robot control algorithms can be reformulated as a negative feedback interconnection of two passive systems [17]. Observing Figure 2, the controller takes in the robot velocity as the input, and the output of the controller block is fed back to the robot as the desired control input. If the controller and the communication channels are input-output passive, then by the fundamental passivity theorem [18], the closed loop system formed by the robot dynamics, the controller, and the communication channels is passive.

In this paper, we study the control problem when the communication channels between the controller and the robot are subjected to various delays. The controller dynamics are given as

$$\Sigma_c : \begin{cases} \dot{x}_c = u_c = \dot{q} \\ y_c = K_1 u_c + K_2(x_c - q_d) \end{cases} \tag{3}$$

where $K_1, K_2 > 0$ are the controller gains, q_d denotes the constant vector for the desired configuration. For simplicity, the control gains in this paper are assumed to be scalars. The stability analysis of the controller (3) and manipulators with $(\tau_t, \dot{q})$ as the input-output pair were discussed in [27].

In this paper, we assume signals are equal to zero for $t < 0$ and let $x(t) = [x_c(t) \ \ \dot{q}(t)]^T$. Denote by $\mathcal{C} = \mathcal{C}([-h, 0], R^{2n})$, the Banach space of continuous functions mapping the interval $[-h, 0]$ into R^{2n}, with the topology of uniform convergence. Define $x_t = x(t + \phi) \in \mathcal{C}, -h < \phi < 0$ as the state of the system [36]. We further assume that $x(\phi) = \eta(\phi), \eta \in \mathcal{C}$ and that all signals belong to $\mathcal{L}_{2e}$, the extended $\mathcal{L}_2$ space.

III. Constant Delays Problem

In this section, constant delays in the input/output channel are addressed. The controller dynamics are then given as in (3) with $u_c(t) = \dot{q}(t - T_1)$ and furthermore the control input to the robot is given as $\tau_s(t) = y_c(t - T_2)$, where T_1, T_2 are the constant, heterogeneous time delays between the robot and the controller. The signal $\dot{q}(t - T_1)$ (or $y_c(t - T_2)$) indicates that the output of the robotic manipulator (or the controller) was transmitted T_1 (or T_2) units of time earlier than the controller (or the robot) receives the signal at the current time instance t. It has been demonstrated via simulations (see [27]) that the closed loop system easily destabilizes even with small input/output constant delays.

With the aim of stabilizing the closed loop system, instead of transmitting the joint velocities and input torques directly, the scattering variables [19], [20] are transmitted across the communication channel

$$\begin{array}{lll} v_1 = \frac{1}{\sqrt{2b}}(\tau_s + b\dot{q}) & , & z_1 = \frac{1}{\sqrt{2b}}(\tau_s - b\dot{q}), \\ v_2 = \frac{1}{\sqrt{2b}}(y_c + bu_c) & , & z_2 = \frac{1}{\sqrt{2b}}(y_c - bu_c), \end{array} \quad (4)$$

where the wave impedance, b, is a positive constant. The proposed architecture is demonstrated in Figure 3.

The transmission equations between the robot and the controller can be written as

$$z_1(t) = z_2(t - T_2)\ ,\ v_2(t) = v_1(t - T_1). \quad (5)$$

The controller dynamics for this system are described by (3), however $u_c \neq \dot{q}(t - T_1)$ but is derived from the scattering representation (4) and the transmission equations (5).

The first claim in the paper follows.

Theorem 1. *Consider the closed loop system described by* (2)*,* (3)*,* (4) *and* (5)*. If all signals equal zero for* $t < 0$*, then*

1) *The closed loop system is input-output passive with* $(\tau_e, \dot{q})$ *as the input-output pair.*
2) *If* $\tau_e \equiv 0$ *and* $K_1 = b$*, then all signals in the closed loop system are bounded and* $\lim_{t\to\infty} \dot{q}(t) = 0$*,* $\lim_{t\to\infty}(x_c(t) - q_d) = 0$*.*
3) *If* $x_c(0) = q(0)$*, then additionally* $\lim_{t\to\infty}(q(t) - q_d) = 0$*.*

Proof: Consider a positive semi-definite storage functional for the system as

$$\begin{aligned} S(x_t) = &\frac{1}{2}\big(\dot{q}^T M(q)\dot{q} + K_2(x_c - q_d)^T(x_c - q_d)\big) \\ &+\frac{1}{2}\Big(\int_{t-T_1}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2}^{t} ||z_2(\tau)||^2 d\tau\Big). \end{aligned}$$

By substituting (2) and utilizing Property 2, $\dot{S}(x_t)$ becomes

$$\begin{aligned} \dot{S}(x_t) &= (-\tau_s + \tau_e)^T\dot{q} + y_c^T u_c - K_1 u_c^T u_c + \frac{1}{2}\big(||v_1||^2 \\ &\quad -||z_1||^2 + ||z_2||^2 - ||v_2||^2\big) \\ &= (-\tau_s + \tau_e)^T\dot{q} + y_c^T u_c - K_1 u_c^T u_c + \tau_s^T\dot{q} - u_c^T y_c \\ &= \tau_e^T\dot{q} - K_1 u_c^T u_c. \end{aligned} \quad (6)$$

From the above calculations it is evident that the closed loop system is passive with $(\tau_e, \dot{q})$ as the input-output pair.

To prove the second claim, note that with $\tau_e \equiv 0$,

$$\dot{S}(x_t) = -K_1 u_c^T u_c \leq 0.$$

Therefore, the storage function is bounded which implies that signals $\dot{q}, x_c \in \mathcal{L}_\infty$. Using the scattering variables (4) and the transmission equations (5), the relationship between the various power variables can be written as

$$y_c(t) + bu_c(t) = \tau_s(t - T_1) + b\dot{q}(t - T_1), \quad (7)$$

$$y_c(t - T_2) - bu_c(t - T_2) = \tau_s(t) - b\dot{q}(t). \quad (8)$$

Using (3) in the above equation yields

$$\begin{aligned} (b + K_1)u_c(t) + K_2(x_c - q_d) &= \tau_s(t - T_1) + b\dot{q}(t - T_1), \\ (K_1 - b)u_c(t - T_2) + K_2(x_c(t - T_2) - q_d) &= \tau_s(t) - b\dot{q}(t). \end{aligned}$$

Choosing $K_1 = b$ to avoid wave reflection [20], the above equations can be rewritten as

$$2bu_c(t) + K_2(x_c(t) - q_d) = \tau_s(t - T_1) + b\dot{q}(t - T_1), \quad (9)$$

$$K_2(x_c(t - T_2) - q_d) = \tau_s(t) - b\dot{q}(t). \quad (10)$$

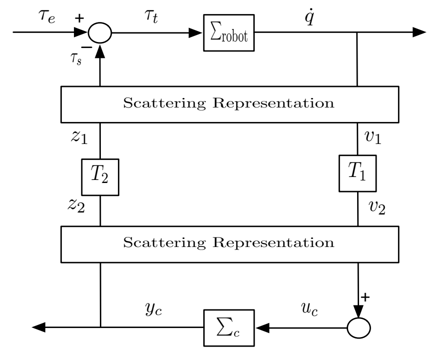


Fig. 3: A negative feedback interconnection of the robot dynamics and the controller with scattering representation.

We get that $\tau_s \in \mathcal{L}_\infty$ from (10) and the fact that $x_c,\ \dot{q}$ are bounded signals. Utilizing this result in (9) yields the boundedness of u_c. Observing the robot dynamics (2) with $\tau_e \equiv 0$ and using Property 1 give us that $\ddot{q} \in \mathcal{L}_\infty$. Differentiating (10), we then get that $\dot{\tau}_s$ is bounded and furthermore differentiating (9) we have that the signal $\dot{u}_c$ is bounded.

Integrating (6) (with $\tau_e \equiv 0$) and letting $t \to \infty$ we get that $u_c \in \mathcal{L}_2[0, \infty)$. It is well known [35] that a square integrable signal with a bounded derivative approaches the origin, and thus $\lim_{t\to\infty} u_c(t) = 0$. Delaying the transmission equation (10) by T_1 and subtracting from (9) we get that

$$2bu_c(t) + K_2\big(x_c(t) - x_c(t - T_1 - T_2)\big) = 2b\dot{q}(t - T_1).$$

Taking the limit $t \to \infty$ on both sides and using the result that $\lim_{t\to\infty} u_c(t) = 0$, we have

$$\begin{aligned} \lim_{t\to\infty} K_2\big(x_c(t) - x_c(t - T_1 - T_2)\big) &= \lim_{t\to\infty} 2b\dot{q}(t - T_1), \\ \lim_{t\to\infty} K_2 \int_{t-T_1-T_2}^{t} \dot{x}_c(\tau)d\tau &= \lim_{t\to\infty} 2b\dot{q}(t - T_1), \\ \lim_{t\to\infty} K_2 \int_{t-T_1-T_2}^{t} u_c(\tau)d\tau &= \lim_{t\to\infty} 2b\dot{q}(t - T_1). \end{aligned}$$

The last equation gives us that $\lim_{t\to\infty} \dot{q}(t) = 0$. Therefore, the robot velocity approaches the origin independent of the time delay.

Differentiating the robot dynamics (2), it can be shown that $\dddot{q}(t) \in \mathcal{L}_\infty$. This observation coupled with the fact that $\lim_{t\to\infty} \dot{q}(t) = 0$, and invoking Barbalat's lemma [34] yields that $\lim_{t\to\infty} \ddot{q}(t) = 0$. Therefore, from (2), $\lim_{t\to\infty} \tau_s(t) = 0$. Taking limits on both sides of the transmission equation (10) implies that $\lim_{t\to\infty} (x_c(t - T_2) - q_d) = 0$. As q_d is a constant reference, we have $\lim_{t\to\infty} (x_c(t) - q_d) = 0$, and hence the signal $x_c - q_d$ approaches the origin independent of the time delay.

To prove the third claim, it can be observed that as $\lim_{t\to\infty} (x_c(t) - q_d) = 0$ and $\lim_{t\to\infty} u_c(t) = 0$, from (3) $\lim_{t\to\infty} y_c(t) = 0$. Integrating (7) from 0 to time t, yields

$$\begin{aligned} &\int_0^t y_c(\tau)d\tau + b\int_0^t u_c(\tau)d\tau \\ &\quad= \int_0^t \tau_s(\tau - T_1)d\tau + b\int_0^t \dot{q}(\tau - T_1)d\tau \\ &\quad= \int_0^{t-T_1} \tau_s(\tau)d\tau + \int_{-T_1}^{0} \tau_s(\tau)d\tau + b\int_0^{t-T_1} \dot{q}(\tau)d\tau \\ &\qquad+b\int_{-T_1}^{0} \dot{q}(\tau)d\tau. \end{aligned} \quad (11)$$

Based on the assumption that all signals are zero for $t < 0$, the previous equation (11) can be rewritten as

$$\int_0^t \big(y_c(\tau) + bu_c(\tau)\big)d\tau = \int_0^{t-T_1} \big(\tau_s(\tau) + b\dot{q}(\tau)\big)d\tau. \qquad (12)$$

Similarly, the transmission equation (8) can be written as

$$\int_0^{t-T_2} \big(y_c(\tau) - bu_c(\tau)\big)d\tau = \int_0^t \big(\tau_s(\tau) - b\dot{q}(\tau)\big)d\tau. \qquad (13)$$

Subtracting (12) from (13) and letting $t \to \infty$, we get that

$$\lim_{t\to\infty} \Big(\int_{t-T_2}^t y_c(\tau)d\tau + b\int_0^t u_c(\tau)d\tau + b\int_0^{t-T_2} u_c(\tau)d\tau \Big)$$
$$= \lim_{t\to\infty} \Big(-\int_{t-T_1}^t \tau_s(\tau)d\tau + b\int_0^{t-T_1} \dot{q}(\tau)d\tau + b\int_0^t \dot{q}(\tau)d\tau \Big).$$

Since T_1 and T_2 are constant, and $\lim_{t\to\infty} y_c(t) = \lim_{t\to\infty} \tau_s(t) = \lim_{t\to\infty} u_c(t) = \lim_{t\to\infty} \dot{q}(t) = 0$, the above equation can be written as

$$\lim_{t\to\infty} 2b\int_0^t u_c(\tau)d\tau = \lim_{t\to\infty} 2b\int_0^t \dot{q}(\tau)d\tau. \qquad (14)$$

As the controller dynamics in (3) $\dot{x}_c = u_c$, the integral of u_c becomes

$$\int_0^t u_c(\tau)d\tau = \int_0^t \dot{x}_c(\tau)d\tau = x_c(t) - x_c(0). \qquad (15)$$

Letting $t \to \infty$ for the integral of u_c and noting that $\lim_{t\to\infty}(x_c(t) - q_d) = 0$ (Claim 2), (15) becomes

$$\lim_{t\to\infty} \int_0^t u_c(\tau)d\tau = \lim_{t\to\infty} x_c(t) - x_c(0) = q_d - x_c(0). \qquad (16)$$

Letting $t \to \infty$ for the integral of $\dot{q}$, we can get

$$\lim_{t\to\infty} \int_0^t \dot{q}(\tau)d\tau = \lim_{t\to\infty} q(t) - q(0). \qquad (17)$$

Substituting (16) and (17) into (14), we get that

$$2bq_d - 2bx_c(0) = 2b\lim_{t\to\infty} q(t) - 2bq(0). \qquad (18)$$

If $x_c(0) = q(0)$, then $\lim_{t\to\infty}(q(t) - q_d) = 0$. ■

Theorem 1 demonstrated that if configuration control of a robotic manipulator is subjected to unknown and constant input/output delays, then the closed loop system can be stabilized by utilizing the scattering transformation. Even though the use of scattering transformation can ensure robust stability of a class of delayed systems, the performance issues of guaranteeing position tracking have not been well studied. Theorem 1 fills this knowledge gap in the current literature. In Theorem 1, we not only prove that the $x_c - q_d$ is asymptotically stable, but also demonstrate that if $x_c(0) = q(0)$, the tracking error $q - q_d$ can eventually go to zero independent of the constant delays.

In addition to the position drift, the phenomenon of wave reflections is another issue that needs to be dealt with while using scattering transformation [20]. For the sake of avoiding wave reflections, the impedance of the wave variables (4) has to be the same for both sites of the robot and the controller. Since only the control gains on the controller side can be adjusted, in the proposed control architecture, we assume that the wave impedance is predetermined. Hence, we can modify the gain K_2 of the controller and the desired configuration q_d so as to fulfill various control demands.

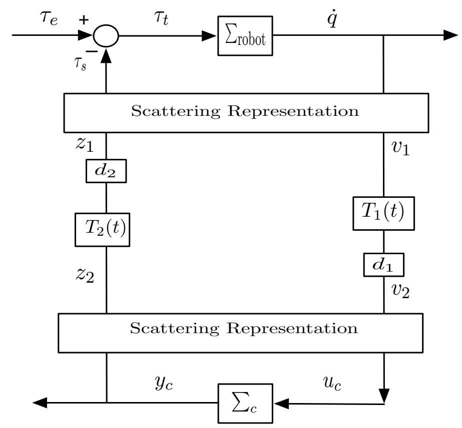


Fig. 4: The scattering transformation, together with the gains (dependent on the rate of change of delay) are used to ensure stability of the closed loop system.

IV. Time-Varying Delays Problem

As discussed in Section I, the delays in the input-output channel may be time-varying; in this section, the set point problem for robotic systems with time-varying input/output delays is studied with the use of scattering transformation.

In the first part of this section, the control of robotic manipulators under input/output time-varying delays is studied by utilizing scattering variables with gains dependent on the maximum rate of change of delays [30]. This control algorithm can guarantee the stability of the closed loop system under time-varying delays. However, this method is dependent on the maximum rate of change of delays, and additionally the control algorithm is not able to regulate the robotic system to the desired configuration. Hence, another control framework, which combines the delayed position feedback with scattering representation, is proposed in the second part of this section to achieve position regulation. Furthermore, the position feedback control algorithm can stabilize the delayed system and ensure position tracking independent of the maximum rate of change of delays.

In this section, the time-varying delays are assumed to be continuously differentiable and bounded ($0 < T_i(t) \leq T_{Mi} < \infty$), where T_{Mi} is the upper bound of $T_i(t)$. The proposed control architecture in this section does not require exact knowledge of time-varying delays.

A. Using Scattering Transformation

To passify the communication block, scattering variables, shown in (4), are used between the robotic manipulator and the controller. The time-varying delays are assumed to satisfy

$$\dot{T}_i(t) \leq \bar{T}_i < 1, \quad i = 1, 2, \qquad (19)$$

where $\bar{T}_i$ is the upper bound of $\dot{T}_i(t)$. The above condition implies that the time delays cannot grow faster than time itself. Furthermore, to address time-varying delays [30], [32], gains dependent on the maximum rate of change of delay are inserted in the communication between the robot and the controller, as shown in Figure 4. The constant gains d_1, d_2 are selected as

$$d_1^2 < (1 - \bar{T}_1) \quad , \quad d_2^2 < (1 - \bar{T}_2), \qquad (20)$$

The transmission equations between the robot and the controller can be written as

$$z_1(t) = d_2 z_2(t - T_2(t)) \ , \ v_2(t) = d_1 v_1(t - T_1(t)). \qquad (21)$$

The controller dynamics for this system are described by (3), however note that $u_c \neq \dot{q}(t-T_1(t))$ but is derived from the scattering representation (4) and the transmission equation (21).

Our next result for the time-varying input/output delay problem follows.

Theorem 2. *Consider the closed loop system described by* (2), (3), (4), *and* (21). *Then the closed loop system is input-output passive with* $(\tau_e,\ \dot{q})$ *as the input-output pair. Additionally, if* $\tau_e \equiv 0$, *then the signals* $\dot{q}$ *and* $x_c - q_d$ *are bounded.*

Proof: Consider a positive semi-definite storage functional for the system as

$$S(x_t) = \frac{1}{2}\big(\dot{q}^T M(q)\dot{q} + K_2(x_c - q_d)^T(x_c - q_d)\big) + \frac{1}{2}\Big(\int_{t-T_1(t)}^{t} ||v_1(\tau)||^2 d\tau + \int_{t-T_2(t)}^{t} ||z_2(\tau)||^2 d\tau\Big).$$

The derivative of the storage function yields

$$\begin{aligned}\dot{S}(x_t) &= \dot{q}^T(-C(q,\dot{q})\dot{q} - \tau_s + \tau_e) + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} \\ &+ K_2(x_c - q_d)^T\dot{x}_c + \frac{1}{2}\Big(||v_1||^2 - ||v_1(t-T_1(t))||^2\big(1-\dot{T}_1(t)\big) \\ &+ ||z_2||^2 - ||z_2(t-T_2(t))||^2\big(1-\dot{T}_2(t)\big)\Big).\end{aligned}$$

By applying the condition (20), the derivative of the storage function becomes

$$\begin{aligned}\dot{S}(x_t) \leq\ & \dot{q}^T(-C(q,\dot{q})\dot{q} - \tau_s + \tau_e) + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} \\ & + K_2(x_c - q_d)^T\dot{x}_c + \frac{1}{2}\big(||v_1||^2 - ||v_1(t-T_1(t))||^2 d_1^2 \\ & + ||z_2||^2 - ||z_2(t-T_2(t))||^2 d_2^2\big). \end{aligned} \tag{22}$$

By utilizing Property 2, $\dot{S}(x_t)$ becomes

$$\begin{aligned}\dot{S}(x_t) \leq\ & (-\tau_s + \tau_e)^T\dot{q} + y_c^T u_c - K_1 u_c^T u_c + \frac{1}{2}\big(||v_1||^2 \\ & - ||z_1||^2 + ||z_2||^2 - ||v_2||^2\big) \\ \leq\ & (-\tau_s + \tau_e)^T\dot{q} + y_c^T u_c - K_1 u_c^T u_c + \tau_s^T\dot{q} - u_c^T y_c \\ \leq\ & \tau_e^T\dot{q} - K_1 u_c^T u_c. \end{aligned} \tag{23}$$

Hence the closed loop system is passive with $(\tau_e, \dot{q})$ as the input-output pair. From (23) it is easy to observe that if $\tau_e \equiv 0$, then $\dot{S}(x_t) \leq 0$ and hence the signals $\dot{q}$ and $x_c - q_d$ are bounded. ■

The above result demonstrates that the closed loop system constituted by the robotic system, coupled with the PI controller, can be stabilized with the use of scattering transformation in the presence of time-varying input/output delays. Without the exact knowledge of time-varying delays, the passivity of communication channels can be guaranteed by using gains dependent on the maximum rate of change of delays. Theorem 2 provides a simple method to stabilize the control of robotic system under time-varying delays. However, the proposed control framework can only ensure that the signal $x_c - q_d$ is bounded, which implies that the robotic system is not guaranteed to be regulated to the desired configuration. The inability to achieve position tracking stems from the scaling introduced in (21), especially for rapidly varying time-varying delays. This observation is validated in the experimental results discussed in Section V.

B. Position Feedback with Scattering Transformation

In order to achieve the desired regulation goal in the presence of time-varying delays, an alternative architecture is proposed as shown in Figure 5. In contrast to the control algorithm in Section IV-A, the proposed framework does not scale the scattering or wave variables for ensuring the passivity of the communication block. In the proposed architecture, the velocity signal $\dot{q}$ is encoded by using scattering transformation and then transmitted to the controller, and the configuration of the robotic manipulator q is communicated directly to the controller. The signal u_c, which is decoded from scattering representation, and delayed position $q(t-T_1(t))$ are combined to generate a control action from the controller which is given as

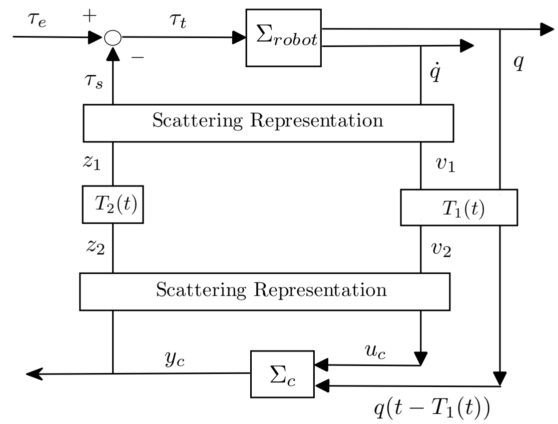


Fig. 5: A position feedback architecture with the use of scattering transformation are proposed to ensure the tracking performance and stability of the closed loop system.

$$\Sigma_c : y_c = K_1 u_c + K_2(q(t-T_1(t)) - q_d). \tag{24}$$

Then the output of the controller y_c is communicated back to the robot via the scattering transformation.

As there is no scaling in this framework, the transmission equations between the robot and the controller can be written as

$$z_1(t) = z_2(t-T_2(t))\ ,\ v_2(t) = v_1(t-T_1(t)). \tag{25}$$

Using the scattering variables z_1 and z_2 in (4) with the transmission equation $z_1(t) = z_2(t-T_2(t))$, we obtain that

$$\tau_s - b\dot{q} = y_c(t-T_2(t)) - bu_c(t-T_2(t)). \tag{26}$$

Then, the control torque to the robotic manipulator can be written as

$$\tau_s = y_c(t-T_2(t)) - bu_c(t-T_2(t)) + b\dot{q}, \tag{27}$$

where $b\dot{q}$ comes from the scattering transformation.

We next study the stability of the control system with position feedback and scattering transformation. For the sake of completeness, we provide a brief overview of a technical result developed in [37] that is utilized to finish the proof of Theorem 3.

Lemma 1. *Given signals* $x, y \in R^n$, $\forall T(t)$ *such that* $0 < T(t) \leq T_M < \infty$, *and* $\alpha > 0$ *the following inequality holds*

$$-\int_0^t x^T(\sigma)\int_{-T(\sigma)}^0 y(\sigma+\theta)d\theta d\sigma \leq \frac{\alpha}{2}||x||_2^2 + \frac{T_M^2}{2\alpha}||y||_2^2$$

where $||\cdot||_2$ *denotes the* $\mathcal{L}_2$ *norm of the enclosed signal.*

We refer the reader to [37] for a proof of the above result.

Theorem 3. *Consider the closed loop system described by* (2), (4), (24), *and* (25) *with* $\tau_e \equiv 0$. *If the time-varying delays satisfy* $0 \leq T_1(t) + T_2(t) \leq T_M < \infty$, *then for a range of the gain* $0 < K_2 < K_1/T_M$, *the signals* $\dot{q}$ *and* $q - q_d$ *are bounded for all times and asymptotically approach the origin.*

Proof: Consider a positive semi-definite storage function for the system as

$$S(\dot{q},q) = \frac{1}{2}\big(\dot{q}^T M(q)\dot{q} + K_2(q-q_d)^T(q-q_d)\big).$$

Taking the time derivative along the trajectories of the system yields

$$\dot{S} = \dot{q}^T\Big(-C(q,\dot{q})\dot{q} - \tau_s\Big) + \frac{1}{2}\dot{q}^T\dot{M}(q)\dot{q} + K_2\dot{q}^T(q-q_d). \tag{28}$$

By using Property 2, the controller (24), and the control torque (27), the derivative of the storage function becomes

$$\begin{aligned}\dot{S} &= -\dot{q}^T\Big(y_c(t-T_2(t)) - bu_c(t-T_2(t)) + b\dot{q}\Big)\\ &\quad + K_2\dot{q}^T(q-q_d)\\ &= -\dot{q}^T\Big(K_1u_c(t-T_2(t)) + K_2\big(q(t-T_1(t)-T_2(t))-q_d\big)\\ &\quad - bu_c(t-T_2(t)) + b\dot{q}\Big) + K_2\dot{q}^T(q-q_d).\end{aligned}$$

Choosing $b = K_1$ to avoid wave reflection [20], the above equation can be rewritten as

$$\begin{aligned}\dot{S} &= -\dot{q}^T K_2(q(t-T_1(t)-T_2(t)) - q_d) + \dot{q}^T K_2(q-q_d)\\ &\quad - \dot{q}^T K_1 \dot{q}^T\\ &= K_2\dot{q}^T(q - q(t-T_1(t)-T_2(t)) - K_1\dot{q}^T\dot{q}\\ &\le K_2\dot{q}^T\int_{-T_1(t)-T_2(t)}^{0}\dot{q}(t+\theta)d\theta - K_1\dot{q}^T\dot{q}.\end{aligned} \tag{29}$$

Note that as we try to upper bound the first term, the sign of the first term does not affect the subsequent calculations.

Integrating (29) from 0 to t and using Lemma 1, we get

$$\begin{aligned}&S(\dot{q}(t),q(t)) - S(\dot{q}(0),q(0))\\ &\qquad \le -K_1\|\dot{q}\|_2^2 + K_2\big(\frac{\alpha}{2}\|\dot{q}\|_2^2 + \frac{T_M^2}{2\alpha}\|\dot{q}\|_2^2\big)\\ &\qquad \le -\|\dot{q}\|_2^2(K_1 - \frac{K_2\alpha}{2} - \frac{K_2T_M^2}{2\alpha}).\end{aligned}$$

If the following inequality given by

$$K_1 - \frac{K_2\alpha}{2} - \frac{K_2T_M^2}{2\alpha} > 0 \tag{30}$$

is satisfied for $\alpha > 0$, then $S(\dot{q}(t),q(t)) - S(\dot{q}(0),q(0)) \le 0$ and hence the signal $\dot{q}(t)$ is square integrable. The above inequality has a solution $\alpha > 0$ if $K_1 > K_2T_M$. Thus, if $K_2 < K_1/T_M$, $S(\dot{q}(t),q(t)) \le S(\dot{q}(0),q(0)), \quad \forall t > 0$. Consequently, for any appropriately selected K_1 and K_2 as discussed above, the signals $\dot{q}, q-q_d \in \mathcal{L}_\infty$.

Using the scattering variables v_1 and v_2 in (4) with the transmission equations $v_2(t) = v_1(t-T_1(t))$ yields

$$\tau_s(t-T_1(t)) + b\dot{q}(t-T_1(t)) = y_c + bu_c. \tag{31}$$

Delaying the transmission equation (26) by $T_1(t)$ and subtracting from (31) we get that

$$\begin{aligned}2b\dot{q}(t-T_1(t)) &= y_c - y_c(t-T_1(t)-T_2(t)) + bu_c\\ &\quad + bu_c(t-T_1(t)-T_2(t)).\end{aligned} \tag{32}$$

Substituting the controller (24) into the equation above with $b = K_1$, we obtain that $u_c \in \mathcal{L}_\infty$. Consequently, from the controller (24) we get that $y_c \in \mathcal{L}_\infty$, hence observing (27) we have that $\tau_s \in \mathcal{L}_\infty$. Noting the system dynamics (2), this additionally implies that the robot acceleration $\ddot{q} \in \mathcal{L}_\infty$. Hence as $\dot{q} \in \mathcal{L}_2$ and its derivative is bounded, the robot velocity asymptotically approaches the origin.

To demonstrate asymptotic convergence of the tracking error, differentiating (2) yields that the signal $\dddot{q} \in \mathcal{L}_\infty$ (note that the derivative of the Coriolis term is also bounded for revolute joints [37]). Hence, the robot acceleration is uniformly continuous and $\lim_{t\to\infty}\int_0^t \ddot{q}(s)ds$ exists and is finite. Invoking Barbalat's Lemma [34], $\lim_{t\to\infty}\ddot{q}(t) = 0$. From the closed loop dynamics (2), it can be obtained that $\lim_{t\to\infty}\tau_s(t) = 0$. Delaying the transmission equation (31) by $T_2(t)$ and subtracting from (26) we get that

$$\begin{aligned}2bu_c(t-T_2(t)) &= \tau_s(t-T_1(t)-T_2(t)) - \tau_s\\ &\quad + b\dot{q}(t-T_1(t)-T_2(t)) + b\dot{q}.\end{aligned} \tag{33}$$

Taking the limit $t \to \infty$ on both sides of the above equation and using the results that $\lim_{t\to\infty}\dot{q}(t) = 0$ and $\lim_{t\to\infty}\tau_s(t) = 0$, we get $\lim_{t\to\infty}u_c(t) = 0$, which means that $\lim_{t\to\infty}y_c(t) = 0$ from (31). By observing (24), it can be obtained that $\lim_{t\to\infty}(q(t-T_1(t)) - q_d) = 0$. As q_d is a constant reference, we have that $\lim_{t\to\infty}(q(t)-q_d) = 0$ and consequently the regulation objective is achieved asymptotically.

■

Utilizing the delayed position feedback and encoding the output of the controller by scattering representation, the proposed control architecture in Figure 5 and Theorem 3 can both stabilize the robotic manipulator with input/output time-varying delays and ensure position regulation. Since the position signal is transmitted to the controller directly, in this framework the controller does not need knowledge of the initial position of the robotic manipulator. The performance of the control system can be adjusted by tuning the controller gains. Moreover, the proposed architecture is able to guarantee stability and position tracking independent of the maximum rate of change of delays.

In Theorem 2, gains d_1 and d_2 are required to satisfy the condition (20), which implies that as $\dot{T}_i(t)$ approaches one, the gains d_1 and d_2 approach zero and hence the system performance deteriorates considerably. The previous result, Theorem 2, was based on the assumption that $\dot{T}_i(t) < 1$, but this assumption is not required in Theorem 3. Hence the position feedback architecture is valid for all positive, continuously differentiable, and bounded time-varying delays even if the maximum rate of change of delays is higher than one. However, the derivative of the time-varying delays should be strictly smaller than one for control systems due to the causality implications [38]. The efficacy of the proposed control scheme when the maximum rate of change is close to one will be validated in the next two sections.

The control of robotic manipulator with time-varying input/output delays has been studied in [32] to ensure position regulation under the assumption that there exists innate dissipation in the robotic system. The proposed control scheme in Theorem 3 was developed for the robotic system without innate dissipation but can be modified for robotic systems with known internal damping. In this case, the robot dynamics are given as

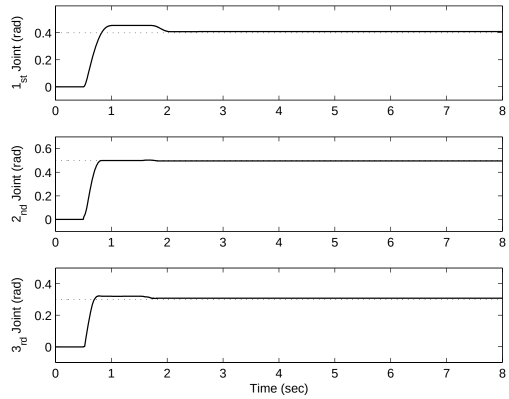
$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + B_n\dot{q} = -\tau_s + \tau_e = \tau_t, \tag{34}$$

where $B_n > 0$ is a scalar denoting the natural damping in the system. The next corollary follows from Theorem 3 for the robotic system (34).

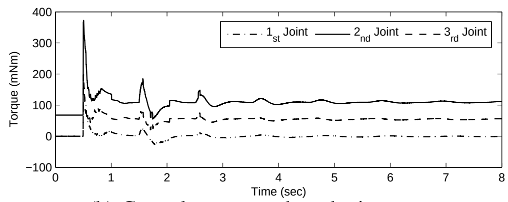
Corollary 1. *Consider the closed loop system described by* (4)*,* (24)*,* (25)*, and* (34) *with* $\tau_e \equiv 0$*. If the time-varying delays satisfy* $0 \le T_1(t) + T_2(t) \le T_M < \infty$*, then for a range of the gain* $0 < K_2 < (K_1 + B_n)/T_M$*, the signals* $\dot{q}$ *and* $q - q_d$ *are bounded and asymptotically approach the origin.*

Proof: Consider a positive semi-definite storage function for the system as

$$S(\dot{q},q) = \frac{1}{2}\big(\dot{q}^T M(q)\dot{q} + K_2(q-q_d)^T(q-q_d)\big).$$

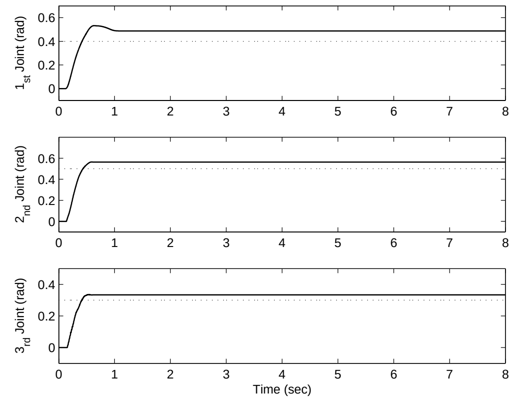


(a) Configuration of the robotic system

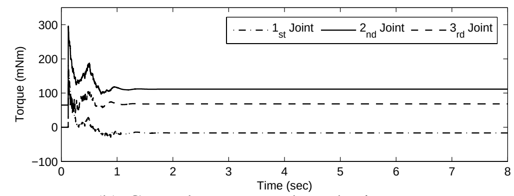


(b) Control torque to the robotic system

Fig. 6: In the constant delay case, when the scattering variables are used, the closed loop system is stable independent of the time delays.



(a) Configuration of the robotic system



(b) Control torque to the robotic system

Fig. 7: The controller in Theorem 2 can ensure the system to be stable but cannot regulate the robotic system to the desired equilibrium (dashed line).

Taking the time derivative along the trajectories of the system and following the proof of Theorem 3, the derivative of the storage function becomes

$$\begin{aligned}\dot{S} &= K_2\dot{q}(q - q(t - T_1(t) - T_2(t)) - K_1\dot{q}^T\dot{q} - B_n\dot{q}^T\dot{q} \\ &\leq K_2\int_{-T_1(t)-T_2(t)}^{0}\dot{q}(t+\theta)d\theta - (K_1 + B_n)\dot{q}^T\dot{q}. \qquad (35)\end{aligned}$$

Integrating the above equation and using Lemma 1, if $K_2 < (K_1 + B_n)/T_M$, $S(\dot{q}(t), q(t)) \leq S(\dot{q}(0), q(0))$, $\forall t > 0$. Thus, following the analysis in Theorem 3, the robot velocity asymptotically approaches the origin, and $\lim_{t\to\infty}(q(t) - q_d) = 0$. Hence, the regulation objective is achieved asymptotically. ■

V. Experiments

As it has been shown via simulation [27] that the closed loop system easily becomes unstable even with small constant input/output delays, in this paper, only the stable system with the use of the proposed schemes are demonstrated. The various controllers were validated via experiments on a PHANToM Omni haptic device. It is a cost-effective device that can be utilized to test and validate control schemes after suitable modifications [39]. In the subsequent experiments, the detachable stylus was removed and the last two joints of the manipulator were constrained for the purpose of reducing the influence of unactuated links on the robot dynamics. Consequently, the device is equivalent to a fully actuated manipulator with three revolute joints, whose joint angles are denoted by $q = [q_1\ q_2\ q_3]^T$.

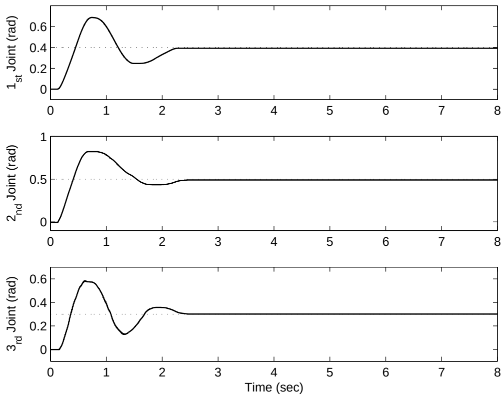
In order to implement the proposed control schemes, $g \in R^3$, the gravitational torques of the fully actuated manipulator were compensated by

$$g = \begin{bmatrix} 0 \\ \frac{1}{2}m_3gl_2s_{2,3} + \frac{1}{2}(m_2 + m_3)gl_1c_2 \\ \frac{1}{2}m_3gl_2s_{2,3} \end{bmatrix}, \qquad (36)$$

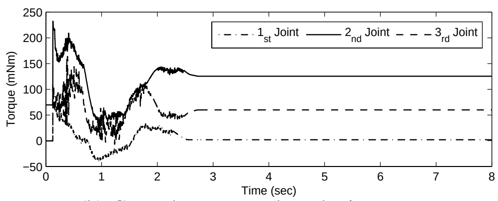
where $s_{2,3}$ denotes $\sin(q_2 + q_3)$, c_2 denotes $\cos(q_2)$, m_i is the translational inertia of link i, and l_i is the length of link i with $i = 1,\ 2,\ 3$. In the experiment, the values of $\frac{1}{2}m_3gl_2$, and $\frac{1}{2}(m_2 + m_3)gl_1$ were experimentally selected with $\frac{1}{2}m_3gl_2 = 70$mNm, and $\frac{1}{2}(m_2 + m_3)gl_1 = 85$mNm. The control program was written in C with the use of OpenHaptics API, which is due to SensAble Technologies [40]. The data collection and control input rate ran at a sampling rate of 1kHz.

Since the effect of packet loss is not considered in the theoretical results, the subsequent experiments were conducted using a single desktop computer, where no signals are transmitted through the real network. The data, transmitted between the robot and the controller, are stored in the FIFO buffers. The stored data is utilized within the computer after a certain time interval so as to imitate communication delays.

In the constant delay case, the delays were selected as $T_1 = 0.3$ sec and $T_2 = 0.2$ sec for the signals transmitting between the robot and the controller respectively. We define diag(p_1, p_2, p_3) as a 3×3 matrix whose diagonal entries starting from the upper left corner are p_1, p_2, and p_3. The desired set point was given as $q_d = [0.4,\ 0.5,\ 0.3]^T$ rad and the control parameters were chosen as $K_1 =$diag(40, 40, 40), $K_2 =$diag(400, 600, 600), and $b = 40$. The initial configuration of the robot is $q(0) = [0\ 0\ 0]^T$ rad. Under the condition that $x_c(0) = q(0)$, the experimental validation of Theorem 1, where the scattering variables are used in the control system to compensate for the time delays, is demonstrated in Figure 6 (a). As expected, the closed loop system is stable and the manipulator is successfully regulated to the desired configuration represented by the dashed lines in Figure 6 (a). The control torque for the experiment with constant delays is shown in Figure 6 (b). The torque in Figure 6 (b) takes time in settling down due to the effect of the scattering transformation. Moreover, the initial torque in the second joint is not zero due to

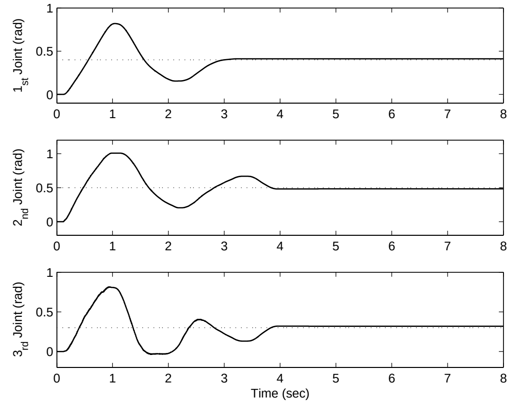


(a) Configuration of the robotic system

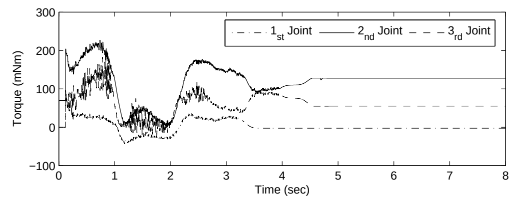


(b) Control torque to the robotic system

Fig. 8: The position feedback with the use of scattering transformation results in a stable system with better performance.



(a) Configuration of the robotic system



(b) Control torque to the robotic system

Fig. 9: Even when the derivative of time-varying delays is close to one, the stability and tracking performance are guaranteed by using the position feedback architecture.

gravity compensation (36). The initial torques in the first and third joints are zero due to the assumption that signals are zero for $t < 0$. Hence, the signals $\tau_s(t) = 0$, $v_1(t) = 0$ if $t < T_1$ and $z_1(t) = 0$ if $t < T_2$, and the control torque is transmitted to the robot after $t = T_1 + T_2$ units of time.

For the time-varying delay case, the delays were selected as

$$\begin{cases} T_1(t) = 0.15 + 0.10\sin(\frac{2}{3}\pi t) \text{ sec} \\ T_2(t) = 0.15 - 0.10\sin(\frac{2}{3}\pi t) \text{ sec} \end{cases} \tag{37}$$

which are continuously differentiable and satisfy the condition (19). Hence, the constant gains d_1 and d_2 were obtained as $d_1 = d_2 = 0.8$. In the subsequent experiments, the desired set point is the same as in the constant delay case.

The experimental results are first presented for the architecture proposed in Theorem 2. The control parameters for this case are given as $K_1 = $diag$(50,\ 50,\ 50)$, $K_2 = $diag$(450,\ 450,\ 450)$, and the wave impedance constant b is set equal to the value in the matrix K_1. As shown in Figure 7 (a), even in the presence of time-varying input/output delays, the closed loop system is stable. However, the proposed control algorithm was not able to regulate the robotic system to the desired configuration. The input torque to the robot is shown in Figure 7 (b).

Next the position feedback architecture, proposed in Theorem 3 and Figure 5, is validated in the experimental setup. In this case, the control gains is limited by the maximum value of $T_1(t)+T_2(t)$. Since there is unknown innate damping B_n in the robotic system, we utilized Corollary 1 for the following experiments. The experiment using the position feedback architecture in Section IV-B was first conducted under the time-varying delays (37), where the maximum value of the set of delays is $T_M = 0.3$ sec. Therefore, the control gains are constrained by the inequality $\frac{K_1+B_n}{K_2} > 0.3$, which implies that $K_1 > 0.3K_2 + B_n$. However, as the actual value of the natural damping in the robotic system is unknown, the control gains are experimentally selected to demonstrate the performance of the proposed control scheme. The control gains for delays (37) were selected as $K_1 = $diag$(50,\ 50,\ 50)$, $K_2 = $diag$(250,\ 330,\ 370)$, and the impedance parameter $b = 50$. Experimental results are shown in Figure 8, where the system is stable and the control system is able to regulate the robotic system to the desired equilibrium.

The proposed control architecture in Section IV-B can regulate the robotic system to the desired configuration under time-varying delays and achieve the control goal if the maximum rate of change of delays approaches one. The next experiment demonstrates the robustness of the proposed scheme under fast varying delays. We choose the set of delays

$$\begin{cases} T_1(t) = 0.15 + 0.10\sin(\frac{5}{3}\pi t) \text{ sec} \\ T_2(t) = 0.23 - 0.19\sin(\frac{5}{3}\pi t) \text{ sec} \end{cases} \tag{38}$$

which are continuously differentiable and the maximum rate of change of $T_2(t)$ is 0.9948. The maximum round-trip delays is $T_M = 0.47$ sec, so the control gains should satisfy $K_1 > 0.47K_2 + B_n$. Given $K_1 = $diag$(50,\ 50,\ 50)$, $K_2 = $diag$(200,\ 270,\ 310)$, and the impedance parameter $b = 50$, the experimental results are shown in Figure 9. It is seen that the proposed algorithm is able to ensure position regulation and the control torque is bounded independent of the maximum rate of change of time-varying delays.

VI. Conclusions

In this paper, the problem of set-point control in rigid robots with constant and time-varying input/output delays was studied. Without the precise knowledge of time delays and robot dynamics, control algorithms based on the use of scattering representation between the controller and the robotic system were proposed to ensure stability and position regulation. It was first demonstrated that using the scattering variables can stabilize an otherwise unstable system for arbitrary unknown constant delays. The tracking errors asymptotically converge to the origin if there is no initial position difference between the robot and the state of the controller.

For time-varying delays, the closed loop system was stabilized using a modified scattering representation scheme in Theorem 2. While stability was preserved by the proposed algorithm, due to scaling of the power variables in the control scheme, the regulation goal was not always achievable. Moreover, the control scheme was dependent on the maximum rate of change of delays in the communication channel. To improve the tracking performance, a new control architecture was proposed in this paper with the use of position feedback and scattering representation. Given the control gains, which are contingent on the maximum round trip delay, the architecture can guarantee the stability of the closed loop system and also the position regulation. Moreover, this algorithm works even if the maximum rate of change of delays is extremely close to one, and additionally the controller does not require knowledge of the initial position of the robotic system. Experiments were performed in this paper to validate the efficiency of the proposed control architecture.

The control architecture proposed in this paper can be applied for the system, which is subject to rapidly varying delays in the real communication network, by reformulating the signals with the use of buffers [41]–[43]. However, the actual network may also suffer from packet losses or communication blackouts. Hence, future work in this topic includes not only trajectory tracking control of robotic manipulators under input/output delays, but also possible packet loss in the communication network [44].

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