Exercise 8 <u>Plant Wide Process Control Design. Assessing Interaction Using The RGA On A Linearized State Space Dynamic Model</u>

I. OBJECTIVE

The objective of this exercise is to demonstrate how a steady state gain matrix, K can be calculated from a linearized state space dynamic process model. The approach used has been published by Arkun and Downs [1], and it is aimed at handling cases where integrating elements, e.g. liquid levels, are present. The dynamic approach is applied to the Tennessee Eastman process, and the relative gain (RGA), and Niederlinski index (NI) are used on the resulting gain matrix as screening tools[1]. This exercise demonstrates the importance of a plant's level control structure on the overall plantwide control system performance.

II. CONTROL TECHNOLOGY

In previous exercises it was assumed that a steady state process simulation was available. If a dynamic simulation is available, then it can be used to calculate steady state gain information. If one only had access to a steady state model, then the approximate approach of Mc Avoy and Miller [2] can be used to estimate K. **A major advantage of using a gain matrix generated in this manner is that it contains complete information about the process under study without the necessity of first specifying level pairings.** If there are integrating measurements in the model, y_l, then one can calculate gain elements for the rate of change of these elements, $\dot{y}_l$. The approach used is based on a paper by Arkun and Downs [1]. Assume that a nonlinear dynamic model is available as:

$$\dot{x} = f(x,u) \tag{1}$$

$$y = g(x,u) \tag{2}$$

where x is the state vector, u is the vector of manipulated variables, and y is the vector of process measurements. The nonlinear functions f and g in Eqns. 1 and 2 can be linearized around the steady state operating point using numerical differentiation to give:

$$\dot{x} = Ax + Bu \tag{3}$$

$$y = Cx + Du \tag{4}$$

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[1] Although SVD could be used as well, it is not. The purpose of the exercise is to focus on the use of the RGA and NI.

where A,B,C,D are constant matrices. Equations 3 and 4 are the starting point for Arkun and Downs analysis. If Equations 3 and 4 do not contain integrating terms, then it is straight forward to use the Laplace transform to determine the transfer function between y and u as:

$$\frac{y(s)}{u(s)} = C(sI - A)^{-1}B + D \qquad (5)$$

From Eqn. 5 the steady state gain matrix between y and u can be obtained by setting s=0 to give:

$$G(0) = -CA^{-1}B + D \qquad (6)$$

However, if there are integrators in the process model, then the rate of change of some process states does not depend on the state itself. For example, for some liquid levels the rate of change of the level is independent of the level itself. When integrators are present the A matrix is singular and A^{-1} does not exist. Arkun and Downs give expressions for the individual process transfer functions when integrating elements are present as:

$$G_{i,j}(s) = \frac{G_{\mathrm{I}_{i,j}}(s)}{s} + G_{0_{i,j}}(s) \qquad (7)$$

The measurements, y, that are non-integrating have 0. values for their G_{I} terms. Their steady state gain can be calculated from their G_0 terms as $G_0(0)$. For measurements, y_{I}, that have integrating terms Eqn. 7 can be rewritten to give:

$$\frac{\dot{y}_{\mathrm{I}}(s)}{u(s)} = G_{\mathrm{I}} + sG_0 \qquad (8)$$

If s is set to 0. in Eqn. 8 the gains for $\dot{y}_{\mathrm{I}}$ can be calculated as $G_{\mathrm{I}}(0)$. The final gain matrix to be used for interaction analysis consists of the appropriate rows from $G_0(0)$ and $G_{\mathrm{I}}(0)$. The controlled variables for this matrix are y and $\dot{y}_{\mathrm{I}}$, and the manipulated variables are u. A recent paper [2] discusses how one can obtain a very good approximation to the same gain matrix (median error 6.1% for the Tennessee Eastman process) using only a steady state simulation. Having an approach that makes use of only steady state models greatly expands the number of processes to which the methods in the exercise can be applied.

To calculate values for $G_0(0)$ and $G_{\mathrm{I}}(0)$, Arkun and Downs start with the singular value decomposition of A:

$$A = U\Sigma V' \tag{9}$$

where

$$\Sigma = \begin{bmatrix} \Sigma_r & 0 \\ 0 & 0 \end{bmatrix} \tag{10}$$

and Σ_r is an rxr diagonal matrix of the non-zero singular values of A. If A is an nxn matrix, then n-r is equal to the number of integrating elements in the model. The integrating gains, $G_1(0)$, can be calculated from the expressions given by Arkun and Downs as:

$$G_1(0) = CV \begin{bmatrix} 0 & 0 \\ 0 & Z_{22} - Z_{21} Z_{11}^{-1} Z_{12} \end{bmatrix} U' B \tag{11}$$

where

$$V' U \overset{\Delta}{=} \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \; with \; Z_{11} \in R^{rxr} \tag{12}$$

The non-integrating gains can be calculated as:

$$G_0(0) = -CV \begin{bmatrix} \Sigma_r^{-1} & \Sigma_r^{-1} Z_{11}^{-1} Z_{12} \\ Z_{21} Z_{11}^{-1} \Sigma_r^{-1} & Z_{21} Z_{11}^{-1} \Sigma_r^{-1} Z_{11}^{-1} Z_{12} \end{bmatrix} U' B + D \tag{13}$$

Equations 11 and 13 are applied to the Tennessee Eastman plant.

If a gain matrix, K, is assembled from $G_0(0)$ and $G_1(0)$ using the approach discussed above, then it can be used to screen process alternatives. In particular, one can assess interaction among level loops and cascade loops using the RGA and Niederlinski index. The Tennessee Eastman process is used to illustrate these points. It can be noted that the Arkun and Downs approach can be applied to systems that are open loop unstable in order to extract information about the integrating elements in the process. Open loop stability can be ascertained by checking the eigen values of the A matrix. For stability **all** eigen values must be negative or have negative real parts.

III. PROCESS DESCRIPTION

A description of the Tennessee Eastman process has been given earlier. It is possible to enumerate all of the

possible level control pairings for this process that are reasonable to consider. These pairings are given in Table 1. From the original nonlinear dynamic process model A,B,C, and D can be calculated using numerical differentiation. For the Eastman plant D=0. since the process measurements are not directly affected by the manipulated variables. The manipulated variables only affect the process states, x. In the model there are 50 process states, and as a result the A matrix is 50x50. The B matrix is 50x12, and the C matrix is 41x50.

Table 1. - Possible Level Control Pairings

Pairing Number	Reactor Level	Separator Level	Stripper Level
1	E-Feed mv-2	Condens CW mv-11	Product mv-8
2	React CW mv-10	Condens CW mv-11	Product mv-8
3	Condens CW mv-11	Sep Exit mv-7	Product mv-8
4	React CW mv-10	Sep Exit mv-7	Product mv-8
5	E-Feed mv-2	React CW mv-10	Product mv-8
6	E-Feed mv-2	Condens CW mv-11	Steam mv-9
7	React CW mv-10	Condens CW mv-11	Steam mv-9
8	Condens CW mv-11	Sep Exit mv-7	Steam mv-9
9	React CW mv-10	Sep Exit mv-7	Steam mv-9
10	E-Feed mv-2	React CW mv-10	Steam mv-9
11	E-Feed mv-2	Sep Exit mv-7	Steam mv-9
12	E-Feed mv-2	React CW mv-10	Sep Exit mv-7
13	E-Feed mv-2	Condens CW mv-11	Sep Exit mv-7
14	React CW mv-10	Condens CW mv-11	Sep Exit mv-7
15	E-Feed mv-2	Sep Exit mv-7	Product mv-8

IV. COMPUTER EXERCISE

The A,B, and C matrices for the Tennessee Eastman process are given in the mat-file exerc8. The following scaling was used to calculate these matrices. The states were scaled as (x/x_S), the manipulated variables as (u/u_S), and the measurements as (y/y_S), where the subscript s stands for steady state. Also given there is the gain matrix, K, that results from combining the appropriate rows of $G_0(0)$ and $G_1(0)$, as well as $G_0(0)$ and $G_1(0)$. The scaling of K is consistent with that of A,B, and C. The m-file *dynamic(K)* can be used to calculate the RGA and Niederlinski index for closing the inner cascade loops, and closing the level plus inner cascade controllers. The following calculations should be carried out.

Case 1. Using the MATLAB command eig(A) to calculate the eigen values of A, determine if the process is open loop stable.

Case 2. From the singular value decomposition of A determine how many integrating variables there are in the model. From $G_1(0)$ determine which measurements are integrating.

Case 3. Using the m-file *dynamic(K)* calculate the RGA and Niederlinski index (NI) for the 15 level pairings given in Table 1. In carrying out these calculations *dynamic(K)* closes ten cascades, and three of the cascaded variables are used to control the three levels.

V. RESULTS ANALYSIS

Is the Tennessee Eastman process open loop stable or not? Which measurements in the Tennessee Eastman process are integrating? Are there some measurement(s) that you would expect to be integrating that are not? If so give an explanation as to why these variables are not integrating. Based on the RGA and NI values, comment on the viability of the 15 level pairings shown in Table 1. Which pairings are appropriate based on your calculations?

REFERENCE

[1] Arkun, Y., and Downs, J., "A General Method To Calculate Input-Output Gains And The Relative Gain Array For Integrating Processes", *Computers and Chemical Engineering*, **14**, 1101-1110 (1990).

[2] Mc Avoy, T., and Miller, R., "Incorporating Integrating Variables Into Steady State Models For Plantwide Control Analysis and Design", I&EC Research, 38, pp. 412-420 (1999).