EXERCISE 3
Plant Wide Process Control Design
Steady State Gain Calculation

I. OBJECTIVE

The objective of this exercise is to demonstrate how the steady state gain matrix for a process can be calculated by using numerical differentiation. The Tennessee Eastman process is used for illustration. Perturbations in its manipulated variables are introduced, and the effect of the size of the perturbations and their sign is illustrated.

II. CONTROL TECHNOLOGY

The approach illustrated in this exercise can be employed on a steady state process simulation. One needs to start out at the steady state point around which the plant control system is to be designed. In the case of the Eastman process, steady state is defined by a set of states, y(50)[1] , a set of manipulated variables, u(12), a set of measurements, xmeas(41), and the fact that dy/dt=yp=0. The process gain matrix has elements given by:

$$K_{i,j} = \partial(xmeas_i) / \partial(u_j)\Big|_{u_{k,k\neq j}} \qquad (1)$$

To calculate $K_{i,j}$ one can numerically differentiate the simulation. With the process at steady state, perturbations are introduced into each of the manipulated variables sequentially, and changes in the measurements are calculated. In order for the simulation to reach a new steady state all integrating variables, e.g. liquid levels and for some plants gas pressures, need to be under control. If liquid levels are not controlled then either vessels will overflow, or run dry. In the Tennessee Eastman process there are three liquid levels that need to be controlled. These are the reactor level, separator level, and the stripper bottoms level. Thus, prior to introducing the perturbations in manipulated variables, three manipulated variables are chosen to control these three levels. Then $K_{i,j}$ is approximated as:

$$K_{i,j} \cong \left[\Delta(xmeas_i) / \Delta(u_j)\right]_{\substack{levels \\ controlled}} \qquad (2)$$

where the Δ's are calculated from the difference between the original and perturbed steady states. In using eq. 2 one of

[1] In the original Tennessee Eastman paper, y is used to denote states, and xmeas is used to denote measurements. Normally x is used for states, and y for measurements. The notation in the original paper is adopted here.

the issues that arises is the size of the perturbation that is used, and whether the perturbation is positive, negative, or averaged. Since three manipulated variables are used for level control, the size of the gain matrix calculated numerically for the Eastman process is 41x9.

To converge the steady state simulation the MATLAB routine fsolve in the Optimization Toolbox is used. This routine uses a Gauss Newton approach to solve the nonlinear vector equation:

$$f(x) = 0. \tag{3}$$

To incorporate the fact that the three levels are under control the f(x) that is used is:

$$f(x) = \begin{pmatrix} yp_i \\ (l_j - l_{j,set}) \end{pmatrix} \begin{cases} i = 1:50 \\ j = 1:3 \end{cases} \tag{4}$$

and

$$x = \begin{pmatrix} y \\ u_l \end{pmatrix} \tag{5}$$

where l_j is a liquid level and u_l is the vector of three manipulated variables used for level control. The dimension of f and x is 53 for the Tennessee Eastman process.

III. PROCESS DESCRIPTION

A description of the Tennessee Eastman process has been given earlier. In calculating steady state gains for the Tennessee Eastman process, it is possible to enumerate all of the possible level control pairings that are reasonable to consider. These pairings are given in Table 1. In this exercise pairing number 15 will be studied.

IV. COMPUTER EXERCISE

Equations 1 to 5 have been applied to the simulation of the Tennessee Eastman process. The results of this calculation are stored in the mat-file ssgain.mat, which can be loaded into memory. If the Couries font is selected then the columns of the various matrices in ssgain will line up. Perturbations ranging up to 5% were used, and one sided and averaged results were calculated. Note that if extremely small perturbations are used, then numerical problems result from the fact that fsolve only uses a finite convergence tolerance, and digital computers only have a finite accuracy.

Table 1. - Possible Level Control Pairings

Pairing Number	Reactor Level	Separator Level	Stripper Level
1	E-Feed mv-2	Condens CW mv-11	Product mv-8
2	React CW mv-10	Condens CW mv-11	Product mv-8
3	Condens CW mv-11	Sep Exit mv-7	Product mv-8
4	React CW mv-10	Sep Exit mv-7	Product mv-8
5	E-Feed mv-2	React CW mv-10	Product mv-8
6	E-Feed mv-2	Condens CW mv-11	Steam mv-9
7	React CW mv-10	Condens CW mv-11	Steam mv-9
8	Condens CW mv-11	Sep Exit mv-7	Steam mv-9
9	React CW mv-10	Sep Exit mv-7	Steam mv-9
10	E-Feed mv-2	React CW mv-10	Steam mv-9
11	E-Feed mv-2	Sep Exit mv-7	Steam mv-9
12	E-Feed mv-2	React CW mv-10	Sep Exit mv-7
13	E-Feed mv-2	Condens CW mv-11	Sep Exit mv-7
14	React CW mv-10	Condens CW mv-11	Sep Exit mv-7
15	E-Feed mv-2	Sep Exit mv-7	Product mv-8

For this exercise the effect of step size and direction is investigated. Various perturbation sizes will be studied as well as the perturbation direction (+,-, and averaged).

V. EXERCISES

A. SIZE OF PERTURBATION - This exercise illustrates that the size of the perturbation is important in calculating the gain matrix. Three manipulated variables will be studied. These are the A-Feed (3), the C-Feed (4), and the purge (6). Positive changes in the manipulated variables are used.

Case 1. For the A-Feed the calculated gains of each of the 41 measurements for % changes in A of 5%, 3%, 1%, .1%, .01%, and .0001% are given in the matrix AFpos. The first column of AFpos gives the labels for each measurement. The first row gives the perturbation size. Using this matrix answer the questions below.

Case 2. The same calculations for the C-Feed are stored in CFpos. Using this matrix answer the questions below.

Case 3. The same calculations for the purge flow are stored in prgpos. Using this matrix answer the questions below.

Which measurements show the greatest sensitivity to the size of the perturbations in manipulated variables? What is a reasonable step size to use on the Tennessee Eastman simulation to calculate steady state gains?

B. DIRECTION OF PERTURBATION - This exercise illustrates that the direction of perturbation for manipulated variables affects the calculation of the gain matrix. For comparison, a .01% change that is averaged is taken as the most accurate estimate of the true gain.

Case 1. For the A-Feed the calculated gains of each of the 41 measurements for a 1% change that is positive, negative, and averaged are given in the matrix AFdir. Using this matrix answer the questions below.

Case 2. The calculations for the C-Feed are given in the matrix CFdir. Using this matrix answer the questions below.

Case 3. The calculations for the purge flow are given in the matrix prgdir. Using this matrix answer the questions below.

Which measurements show the greatest sensitivity to the direction of the perturbations in manipulated variables? What can you conclude about the direction of perturbations that should be used for calculating gain matrices from steady state simulations?

VI. RESULTS ANALYSIS

Of the three manipulated variables tested, which has the strongest effect on the process? Which has the weakest effect?