BRAUER AND NOHEL: PROBLEM 1, PG. 8

I will disregard the hint and understand the system in a straightforward way without an a priori guess at good coordinates.

For $i = 1, 2$, let x_i be the distance down from the support of the first mass (particle). Let F_i denote the force acting on the ith particle. We then have

$$F_1 = m_1 g - k_1(x_1 - L_1) + k_2(x_2 - x_1 - L_2)$$
$$F_2 = m_2 g - k_2(x_2 - x_1 - L_2) \ .$$

Note that the forces acting on a particle comes only from direct action: a spring connected to the particle, and gravity.

Next we find the values of x_1 and x_2 at equilbrium by setting the forces equal to zero and solving. From the equation for F_2, we find at equilbirium that

$$\frac{m_2 g}{k_2} = x_2 - x_1 - L_2$$

Substituting this expression for $x_2 - x_1 - L_2$ into the equation $0 = F_1$, we get

$$0 = m_1 g - k_1(x_1 - L_1) + k_2(\frac{m_2 g}{k_2}) \ , \quad \text{so}$$
$$k_1(x_1 - L_1) = m_1 g + m_2 g \ .$$

Next we solve the last equation and our earlier equation to get the values $x_i = b_i$ at equilibrium:

$$b_1 = L_1 + \frac{(m_1 + m_2)g}{k_1}$$
$$b_2 = L_1 + \frac{(m_1 + m_2)g}{k_1} + L_2 + \frac{m_2 g}{k_2} \ .$$

Set $y_i = x_i - b_i$ (so, $y_1 = 0 = y_2$ at equilibrium). Now for $i = 1, 2$, substitute $y_i + b_i$ for x_i in the equation for F_i, and simplify. This shows that, indeed,

$$F_1 = -k_1 y_1 + k_2(y_2 - y_1)$$
$$F_2 = -k_2 y_2 \ .$$

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