BONUS PROBLEM ON THE LAW OF LARGE NUMBERS

The Law of Large Numbers states a convergence without guaranteeing a rate, as follows.

THEOREM (The Law of Large Numbers)

Suppose $X_1, X_2, \ldots$ are i.i.d. random variables, each with expected value μ.

Then for every $\epsilon > 0$,

$$\lim_{n\to\infty} \text{Prob}\left[\ \left| \frac{X_1 + X_2 + \cdots + X_n}{n} - \mu \right| \ > \epsilon \ \right] \quad = \quad 0 \ .$$

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However we can guarantee a rate of convergence when the underlying distribution has finite variance. Here is an example.

BONUS PROBLEM: Suppose that $X_1, X_2, \ldots$ are i.i.d. random variables, each with expected value 7 and standard deviation 5. Find an positive integer n for which you can PROVE

$$\text{Prob}\left[\ \left| \frac{X_1 + X_2 + \cdots + X_n}{n} - 7 \right| \ > .01 \ \right] \ \leq \ .0001 \ .$$

Hint. You can solve this using the Chebyschev Inequality (proved in a handout on the course page), which states that if a random variable Y has mean μ and standard deviation σ, then for any positive number k,

$$\text{Prob}(|Y - \mu| \geq k\sigma) \ \leq \ \frac{1}{k^2} \ .$$

What should you use for Y?
Then, what should σ be?
Then, what should k be?
Now solve.

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