Midterm 2B–Stat 100–Spring 1997

You may use calculators, but not books or notes. Each problem is worth 20 points. Different parts of a problem have equal weight unless otherwise indicated. Do not spend too much time on any one problem. Put a box around the final answer to a question.

1. Let Z be a standard normal random variable.
   a. (6 points) Find $P[-0.9 < Z < -0.5]$
   b. (6 points) Find $P[Z < -2.01]$ and $P[Z > -2.01]$
   c. (8 points) Determine the value of a so that $P[-a < Z < a] = 0.668$

2. Suppose we toss a fair coin 100 times. Let X be the number of times we observe heads.
   a. (4 points) What is the exact distribution of X? (Give a brief, complete description.)
   b. (12 points) Use an appropriate approximation to estimate $P[45 < X < 60]$.
   c. (4 points) Justify your use of this approximation.

3. Suppose a random variable X has mean 20 and standard deviation 5. Consider the distribution of $\overline{X}$ for random samples of size 10.
   a. (8 points) What are the mean and standard deviation of $\overline{X}$?
   b. (6 points) Either explain why $\overline{X}$ has approximately normal distribution, or give an assumption about the distribution of X which implies $\overline{X}$ has a normal distribution.
   c. (6 points) On the basis of your explanation or assumption above, give an estimate for P$[\overline{X} > 22]$.

4. The U.S. Transportation Department will randomly sample traffic reports to estimate the proportion of accidents involving people over the age of 70. The Department has no advance estimate of this proportion.

   How many reports should the department select to be at least 97% confident that the estimate is within 0.01 of the true proportion?

5. Cholesterol levels are tested for a random sample of 300 women over the age of 30. The sum of all of the observed levels for the sample group is 52,800. The sample standard deviation of the observations is 15.

   Construct a 90% confidence interval for the actual average cholesterol level of women over the age of 30.