**Final Exam–Stat 100–Spring 2001**

You may use calculators, and ONE page of notes (writing allowed both sides). Each problem is worth 20 points. Put a box around the final answer to a question. EACH PROBLEM SHOULD BE ANSWERED ON A SEPARATE PAGE. Use the back side of a page if necessary. Don't spend too much time on any one problem.

1. Let $A$ and $B$ be two events. The probability that $A$ occurs is .2, the probability that $B$ does not occur is .7, and the probability that $A$ or $B$ occurs is .4.

a. What is the probability that $B$ occurs?

b. What is the probability that both $A$ and $B$ occur?

c. Are the events $A$ and $B$ independent? Briefly justify your answer.

d. What is the probability that neither $A$ nor $B$ occurs?

2. From a group of 12 people, 4 are chosen randomly to work on Project A, and 4 are chosen randomly and independently to work on Project B.
What is the probability that none of the 12 works on both projects?

3. Suppose the number of speeding tickets $X$ issued in a workday by Officer Joe has the probability distribution

| $x$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Prob$[X = n]$ | 0.2 | 0.3 | 0.4 | 0.1 |

a. What is $E(X)$?

b. What is the standard deviation of $X$?

c. What is the expected number of speeding tickets Officer Joe will issue in his next 100 days of work?

d. Suppose the numbers of tickets issued on different days is independent. What is the probability that Officer Joe will issue exactly one ticket over the next four days?

4. Suppose that the probability a voter will believe a rumor about a politician is 1/5.

a. (5 points) If 20 voters are told the rumor, what is the probability at least 3 believe it?

b. (5 points) If 20 voters are told the rumor, what is the expected number of believers?

c. (10 points) If 400 voters are told the rumor, estimate the probability that fewer than 70 believe it.

5. The amount of dirt picked up in one scoop by a diesel shovel has expected value 25 pounds with standard deviation 4 pounds.

In 100 scoops, approximately what is the probability that the shovel will pick up less than 2520 pounds of dirt?

6. A scientist is studying the effect of communication modes on the problem solving capability of teams. For one mode of communication a random sample of 35 teams completed a specific task in an average of 25.7 minutes with a standard deviation of 3.8 minutes.

Construct a 90% confidence interval for the mean time in minutes to complete the task. (Substitute correctly into the appropriate formula, but do not spend time doing the final calculator computations.)

**********THERE ARE MORE PROBLEMS ON THE BACK SIDE**********

7. An educational psychologist is studying differences in intelligence scores produced by two test procedures, A and B. Four subjects were randomly selected and given both tests. The resulting scores are in the following table (each column gives the scores of one subject).

| Test A | 98 | 84 | 91 | 83 |
|---|---|---|---|---|
| Test B | 86 | 86 | 82 | 84 |

Assume the population of test score differences is approximately normal. Construct an appropriate test to determine whether the mean scores are equal, at level of significance $\alpha = .10$ .

(a) Formulate the null and alternative hypothesis
(b) Write the test statistic
(c) State the rejection region
(d) Compute the test statistic for the sample data and estimate the P-value.
(e) State your conclusion. Do you conclude that the mean scores for the two tests differ?

8. Ten rattle snakes in Arizona are found to have a mean length of 1.2 meters with a standard deviation of 0.25. In California a sample of 8 rattle snakes is found to have a mean length of 0.9 meters with a standard deviation of 0.6 meters. Construct a 98% confidence interval for the mean difference in lengths, $\mu_A - \mu_C$. (Substitute correctly into the appropriate formula, but do not spend time doing the final calculator computations.)

What assumptions do you make to justify your procedure?

9. In a study on the effect of diet and life-style on heart disease, 96 patients with severe coronary blockage were randomly assigned, 49 to an experimental group and 47 to a control group. The patients in the experimental group had a low-fat vegetarian diet, regular exercise, and stress-management training, whereas those in the control group had a low-fat diet and moderate exercise. The condition of their coronary blockage was monitored throughout the study period, and the following results were noted.

| | Worse | No Change | Improved | Total |
|---|---|---|---|---|
| Experimental | 4 | 8 | 36 | 48 |
| Control | 8 | 25 | 15 | 48 |
| Total | 12 | 33 | 51 | 96 |

a. (18 pts.) Analyze the data to determine if the changes in coronary blockage were significantly different for the two groups of patients. Test at significance level $\alpha = .05$. State your conclusion clearly. Estimate the P-value and interpret it.
b. (2 pts.) What rule-of-thumb criterion justifies the use of the test you used?

10. Answer each part TRUE or FALSE. No justification required.

a. If $X_1$ and $X_2$ are normally distributed random variables with respective means $\mu_1, \mu_2$ and standard deviations $\sigma_1, \sigma_2$, then $\text{Prob}(|X_1 - \mu_1| < 3\sigma_1) = \text{Prob}(|X_2 - \mu_2| < 3\sigma_2)$.

b. The significance level of a hypothesis test is the probability of incorrectly choosing the alternate hypothesis.

c. If $X$ is a random variable with the Binomial$(n, p)$ distribution, with $n = 1000$ and $p = .999$, then it is appropriate to use the normal approximation for this distribution.

d. If we run a hypothesis test at level of significance $\alpha = .1$, and the $P$-value is .2 , then we should reject the null hypothesis.

e. If $\overline{X}_n$ denotes the average salary of a random sample of $n$ persons from a certain neighborhood, then $E(\overline{X}_{900}) = (1/3)E(\overline{X}_{100})$.