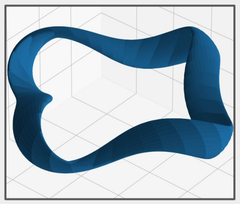
Coil Optimization and Perturbation Analysis for a Quasihelically Symmetric Magnetic Field

Alex Wiedman, Stefan Buller, Matt Landreman
University of Maryland

Overview

- Stellarator fusion reactor designs can have difficulties confining particles, but recently, quasi-symmetric geometries have been found with excellent calculated confinement
- To prove the feasibility of these geometries, coils must be found that both reproduce the target magnetic field and can be easily manufactured
- This project looks at the process of optimizing coils for the precise quasi-helical (QH) symmetric field from (Landreman & Paul, Phys. Rev. Lett. 128 (2022) 035001).
- The resulting coils were highly accurate, resulting in a normalized flux surface average of the magnetic field from the coils normal to the target surface (hereafter referred to as <|B · n|> / <|B|>) of 6×10^{-4}, and no 3.5 MeV alpha particle losses when launched from the surface with normalized toroidal flux of *s* = 0.3



Objective Function

Objective Function

$$F_{min} = \frac{1}{2}\int_s (\boldsymbol{B}\cdot\hat{\boldsymbol{n}})^2 \mathrm{d}S + \sum_{i=1}^{6} w_i f_i$$

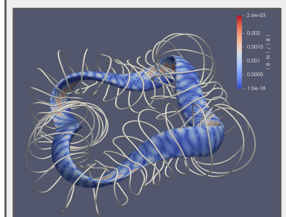
The amplitude of the coil fourier modes and currents are the variables changed in optimization.
The following objectives were included:

- Integral of squared magnetic field normal to target surface
- Coil Length
- Coil to Coil Distance
- Coil to Surface Distance
- Maximum Curvature of the Coil
- Mean Squared Curvature of the Coil
- Linking Number between coils (returning a nonzero integer if the coils are linked, and 0 if they are not)

This optimization was performed using a L-BFGS optimization algorithm from scipy, using objectives from the stellarator optimization framework simsopt.

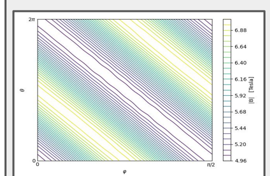
$$\mathrm{link}(\gamma_1, \gamma_2) = \frac{1}{4\pi}\oint_{\gamma_1}\oint_{\gamma_2}\frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}\cdot(d\mathbf{r}_1 \times d\mathbf{r}_2)$$

Best Coil Set

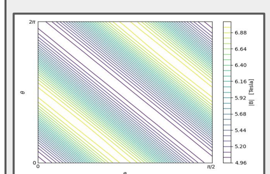


Best set of coils found for the Landreman Paul QH configuration

The best set of coils found for the target surface, with 5 coils per half period and with coil metrics described in this poster all comparable to NCSX and HSX. This baseline set of coils had no 3.5 MeV alpha particle losses when launched from the surface with normalized toroidal flux of *s* = 0.3. The flux surface average <|B · n|> / <|B|> for these coils is 6×10^{-4}



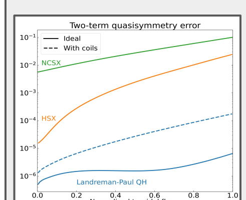
B_{coils} plotted in Boozer coordinates



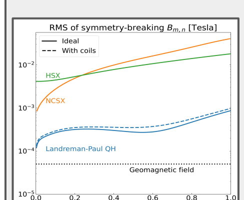
B_{target} plotted in Boozer coordinates

Coils (scaled)	Length (m)	CC Dist (m)	CS Dist (m)	κ (m^{-1})	MSC (m^{-1})
Our Coils	35.56	1.09	1.62	0.67	0.07
NCSX	36.60	0.79	1.00	1.68	0.40
HSX	31.39	1.30	2.01	0.82	0.18

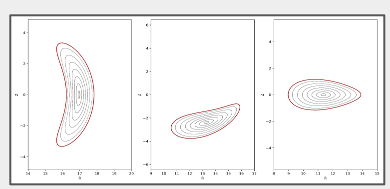
Coil metrics for different configuration scaled to ARIES-CS minor radius and volume average B. Length and MSC are averaged over each coil, while CC Dist and CS Dist are the maximum value, and κ is the maximum value across the coils



Quasisymmetry error against radius



RMS of symmetry-breaking $B_{m,n}$ against radius



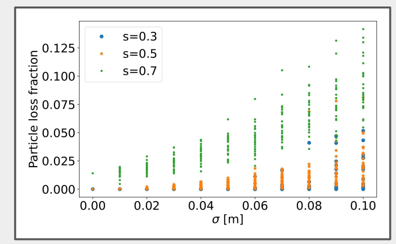
Poincaré plot of the flux surface created by the coils

Coil Perturbation

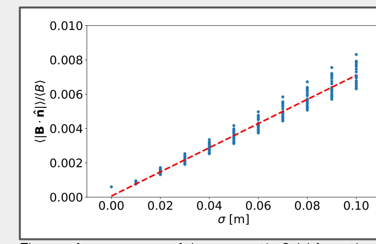
- A test on coil robustness is performed through random perturbations to the shape of the coils
- The perturbations are performed by adding small random changes to the Cartesian coordinates of points along the coils
- The covariance of the random perturbation at points t_1 and t_2 is given by the formula

$$\mathrm{Cov}(X(t_1), X(t_2)) = \sum_{j=-\infty}^{\infty}\sigma^2 \exp\left(\frac{-(t_1 - t_2 + j)^2}{2l^2}\right),$$

- These new coils are analyzed and particle tracing is performed using the guiding center code SIMPLE
- Alpha particle confinement does not scale linearly with either <B · n> / <|B|> (the flux surface average of the B field generated by the coils, normal to the target surface) or quasisymmetry error



Particle loss fraction against amplitude of covariance of perturbation



Flux surface average of the magnetic field from the coils normal to the target surface against amplitude of covariance of perturbation

Planar Coils

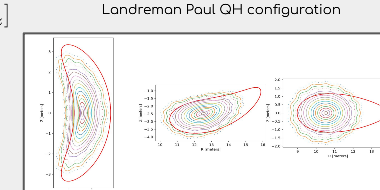
- Additional constraints were placed on coils to make them planar
- These coils are represented as Fourier series over a radius, along with a quaternion based rotation and a center point
- Quaternion rotation is used to avoid gimbal locking during the optimization process
- These coils are shown to have significantly worse <|B · n|> / <|B|> (0.0382) than non planar coils for the Landreman-Paul QH configuration
- The quaternion rotates a point about an axis u_x, u_y, u_z by an angle θ. The quaternion is stored in the form

$$[q, i, j, k] = [\cos\frac{\theta}{2}, \sin\frac{\theta}{2}u_x, \sin\frac{\theta}{2}u_y, \sin\frac{\theta}{2}u_z]$$

Best set of Planar coils found for the Landreman Paul QH configuration

References

Landreman & Paul, Phys. Rev. Lett. 128 (2022) 035001
Wechsung et al. Nuclear Fusion 62 (7) (2022), 076034
Albert, Kasilov & Kernbichler, Journal of Plasma Physics 86 (2) (2020), 815860201.



Poincaré plot of the flux surface created by the planar coils