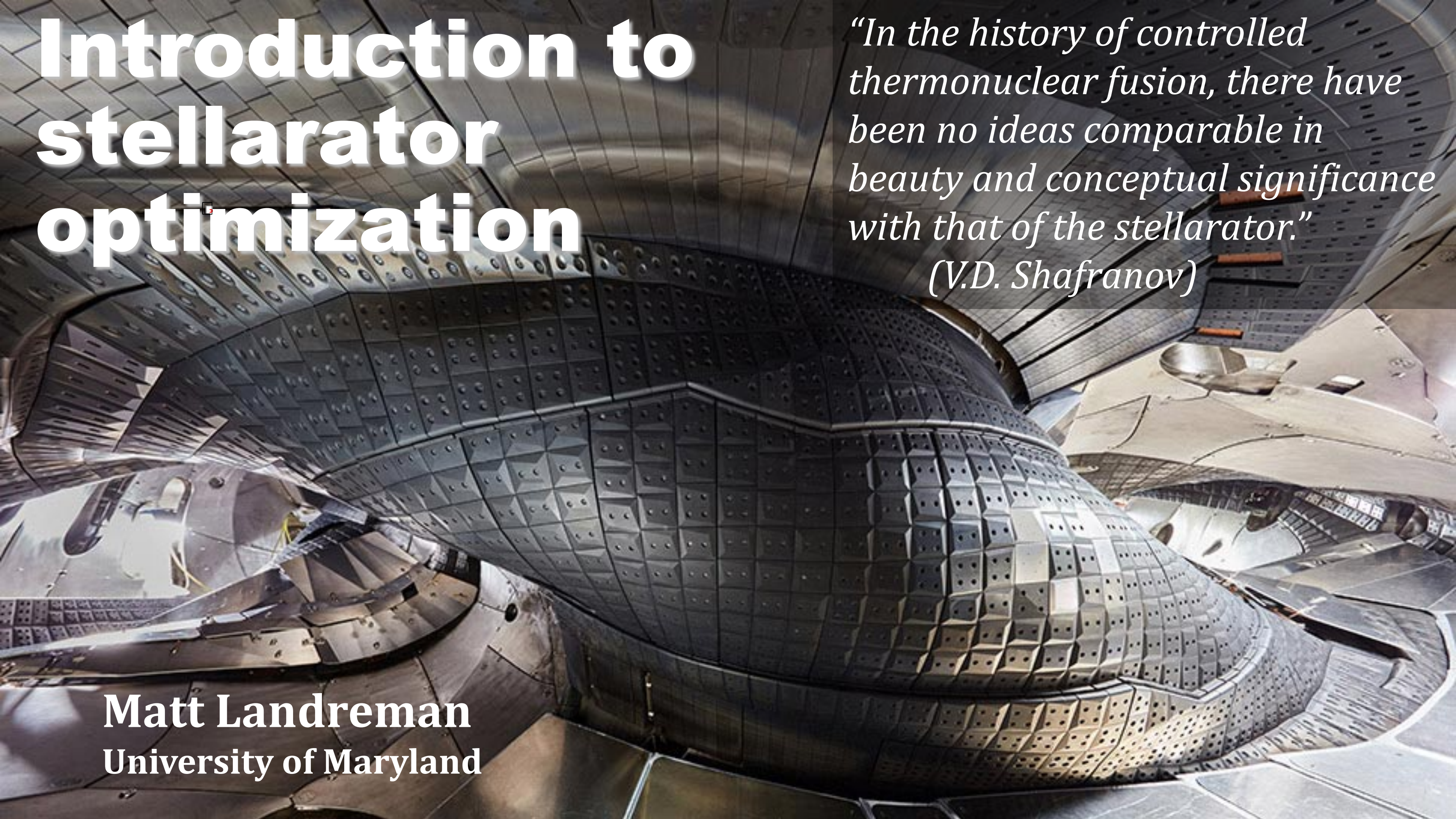


Introduction to stellarator optimization

"In the history of controlled thermonuclear fusion, there have been no ideas comparable in beauty and conceptual significance with that of the stellarator."
(V.D. Shafranov)

Matt Landreman
University of Maryland

Other resources

Review papers:

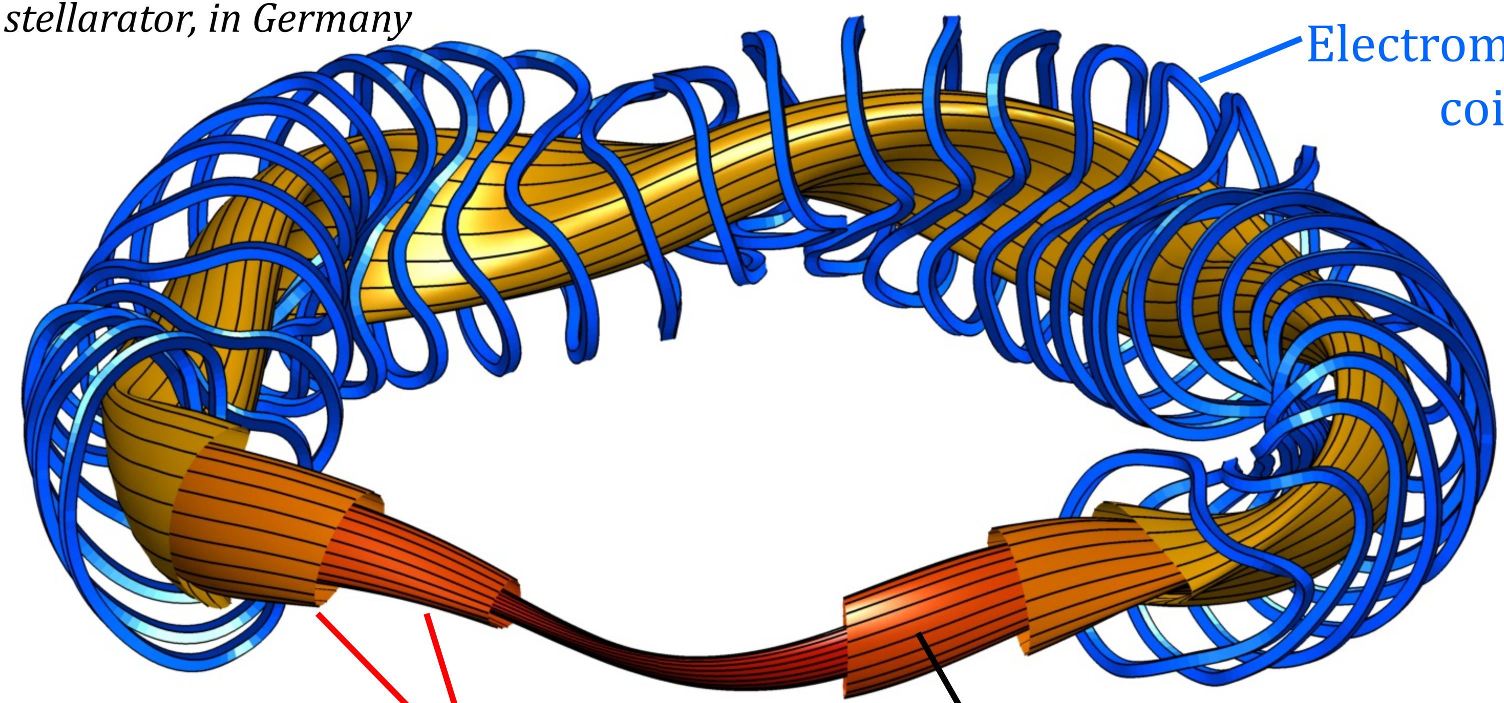
- "*An introduction to Stellarators*", Lise-Marie Imbert-Gerard, Elizabeth Paul, Adelle Wright, arXiv:1908.05360
- "*Theory of plasma confinement in non-axisymmetric magnetic fields*", Per Helander, Rep. Prog. Phys. 77 (2014).

2019 & 2020 Simons Foundation/PPPL Summer Schools: lecture notes & videos
https://hiddensymmetries.princeton.edu/summer-school/summer-school-2020/lecture-notes

PPPL SULI summer course: https://suli.pppl.gov/2021/course/ (or replace 2021 with previous years). Lecture notes & videos on fusion energy, particle motion, stellarators, etc.

A *stellarator* is a configuration of magnets for confining plasma without continuous rotation symmetry

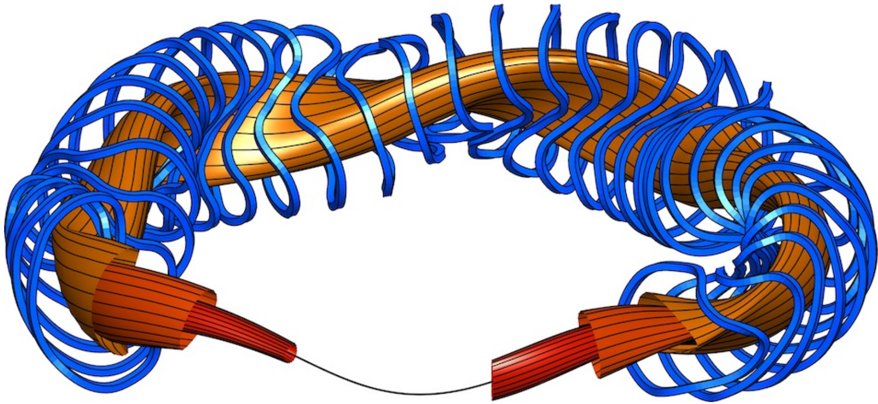
The W7-X stellarator, in Germany

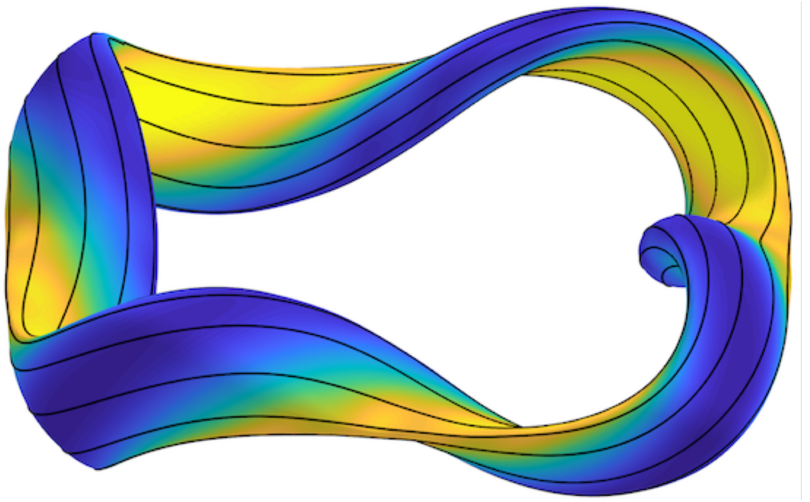


Where did these wiggly shapes come from?   Optimization!

Outline

- Rotational transform & flux surfaces
- Transport & quasisymmetry
- Optimization approaches
- Recent advances in α-particle confinement





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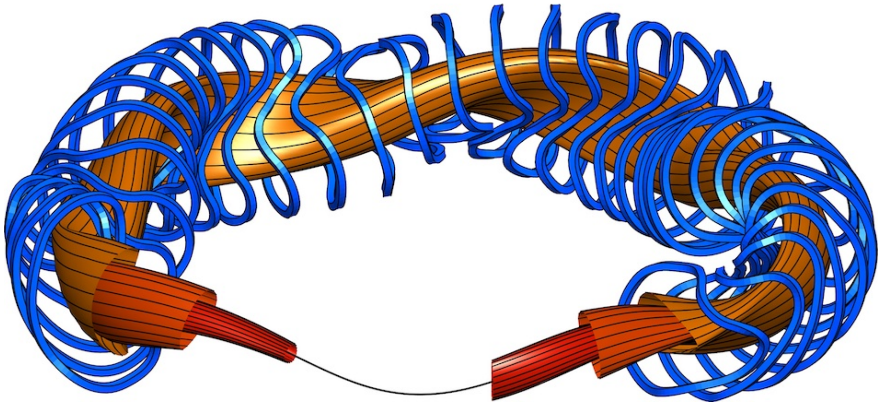
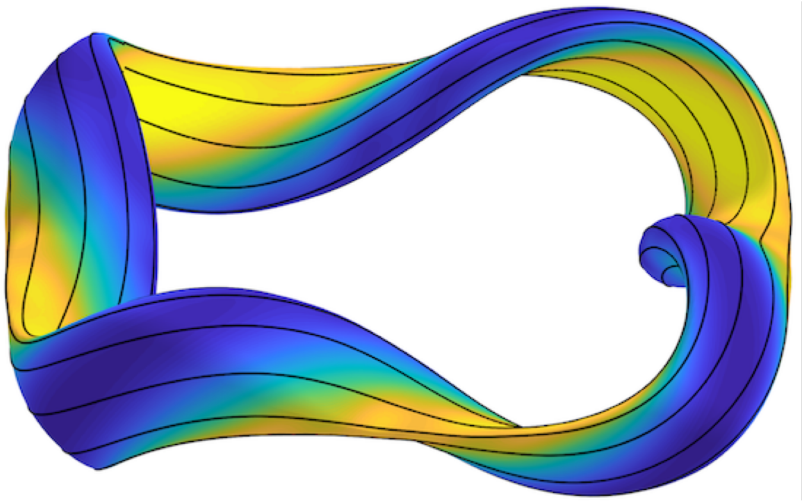
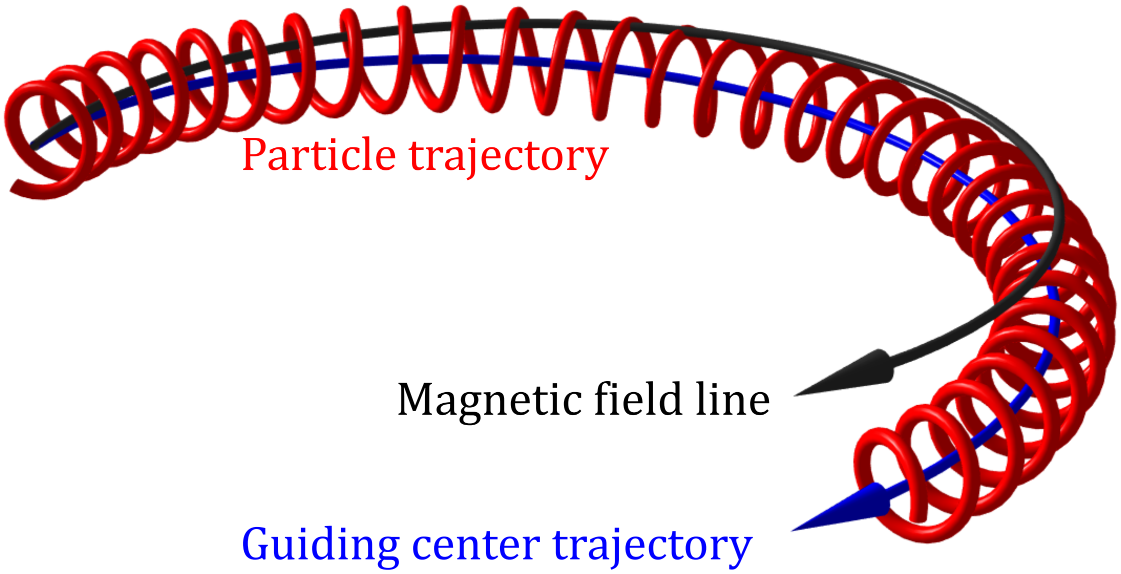


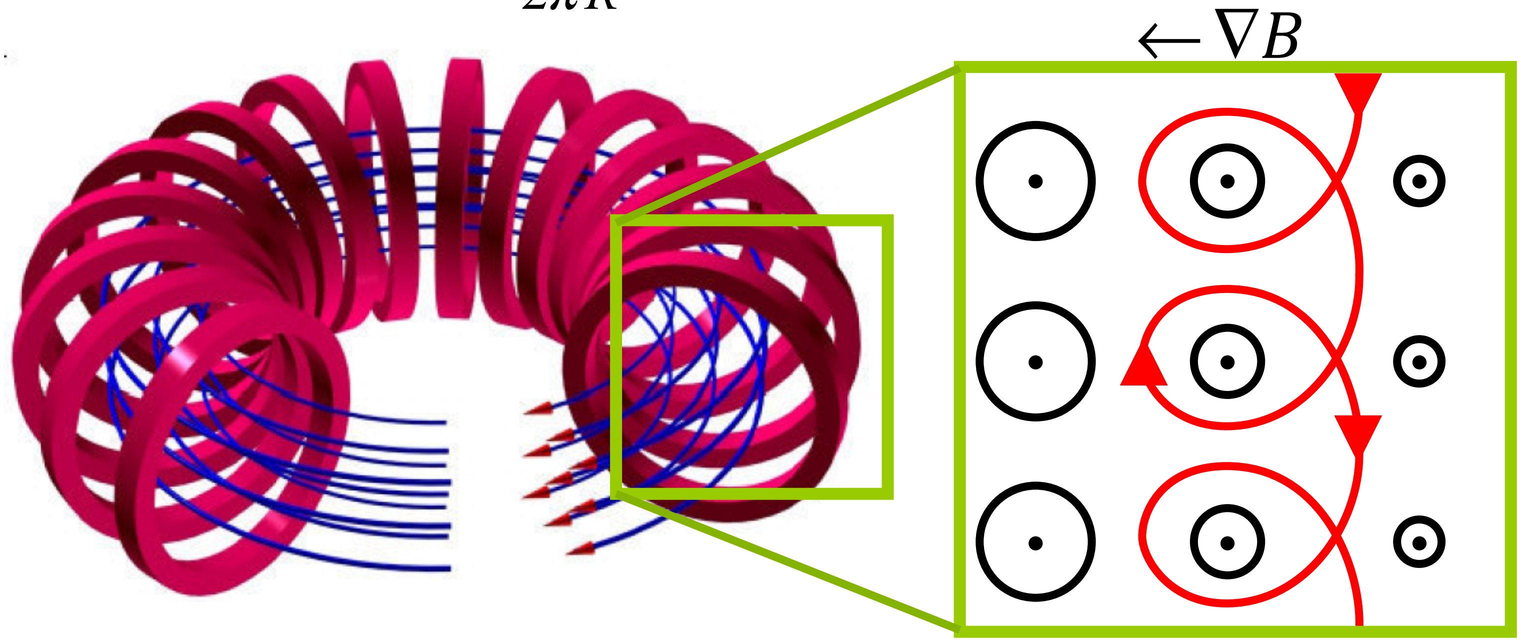


A purely toroidal field does not confine particles.



A purely toroidal field does not confine particles.

Ampere's Law: $B = \dfrac{\mu_0 I}{2\pi R}$ so B is larger on the inside



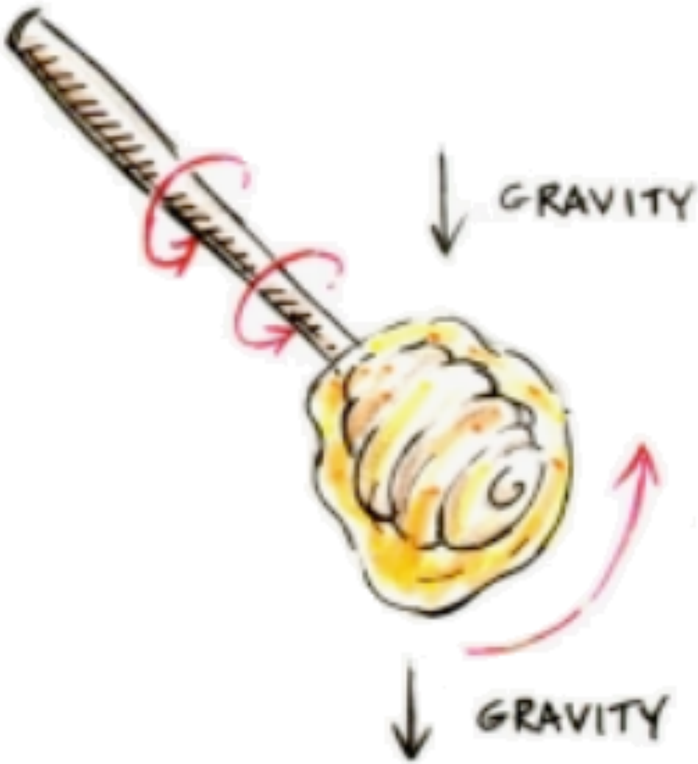
Ions drift down:

they are not confined!

Particles drift in the

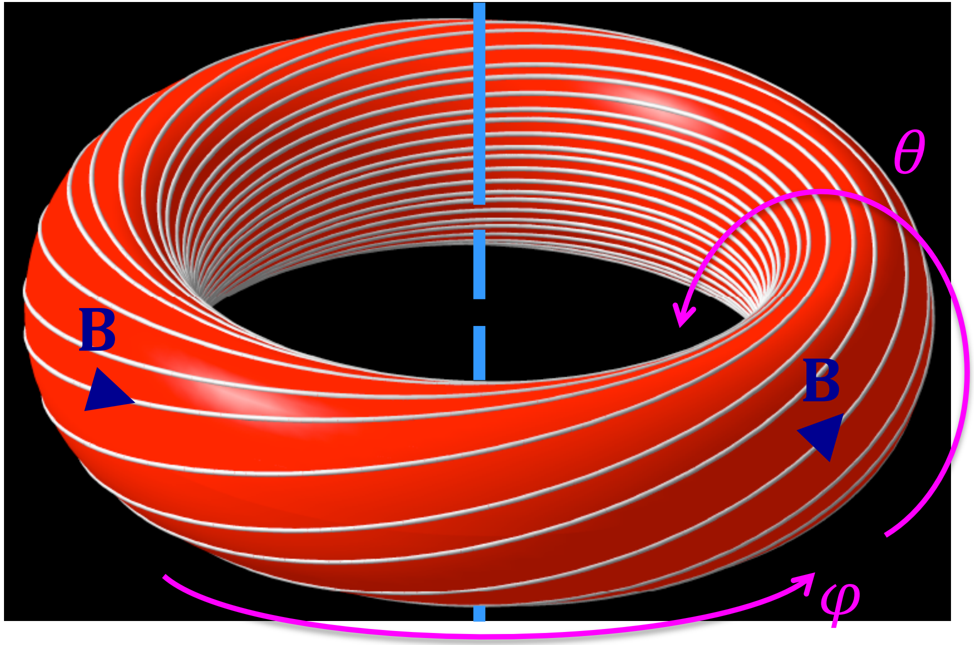
$q\mathbf{B}\times\nabla B$ direction

How can we resolve the problem of the cross-field drifts?



Picture from G Hammett

How can we resolve the problem of the cross-field drifts?

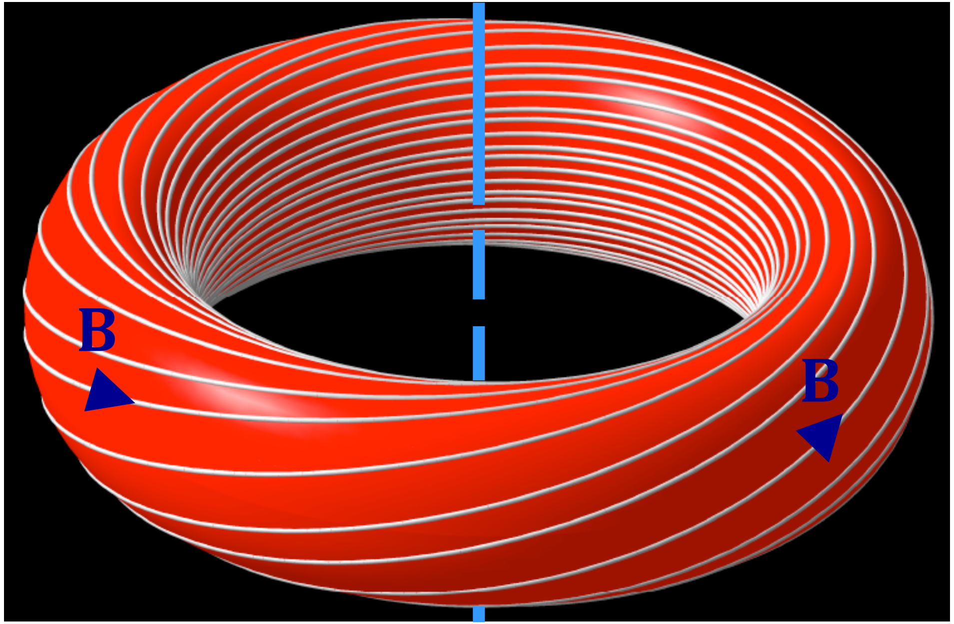


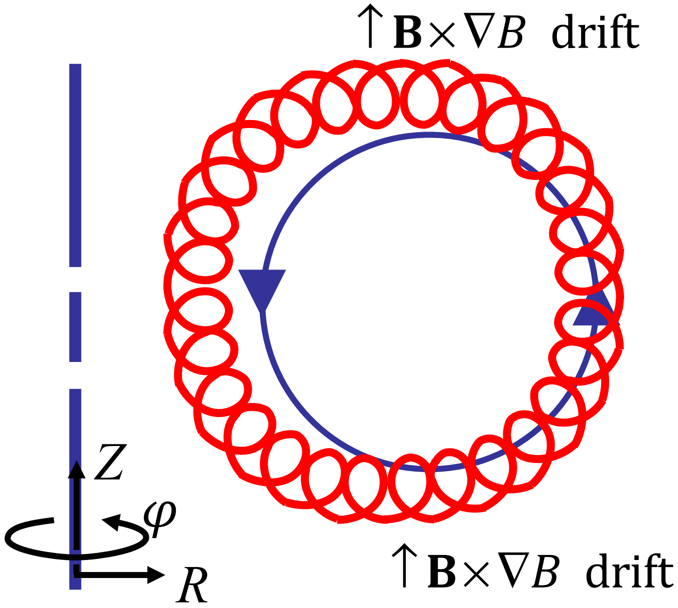
By making the field lines helical rather than toroidal.

$$\iota = \lim \frac{\Delta\theta}{\Delta\varphi}$$

"Rotational transform" ι: If you follow a magnetic field line around the torus once toroidally (i.e. the long way around), you come back to a different place the short way around.

Field lines must be helical: "rotational transform" ι

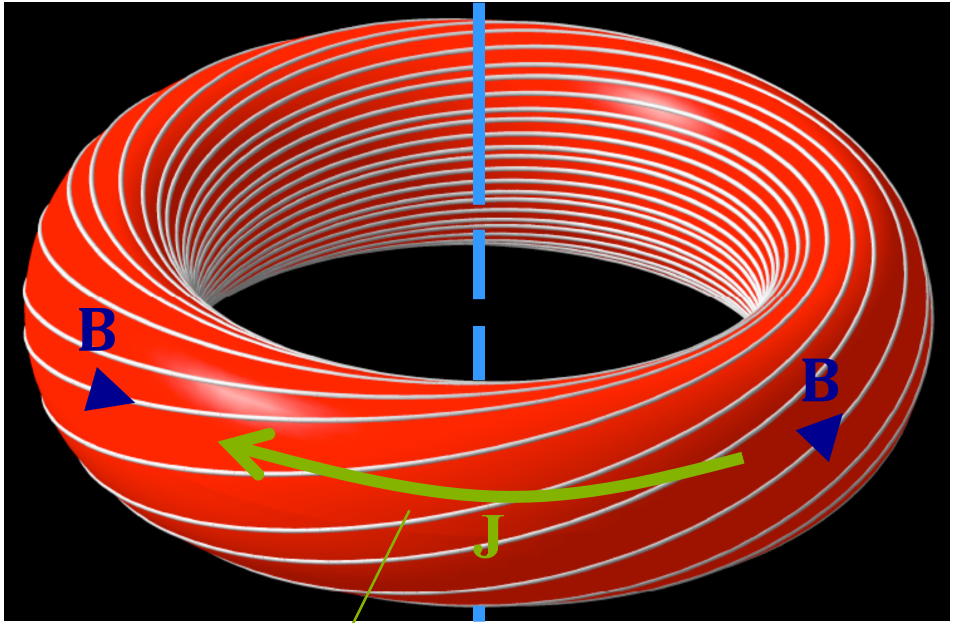


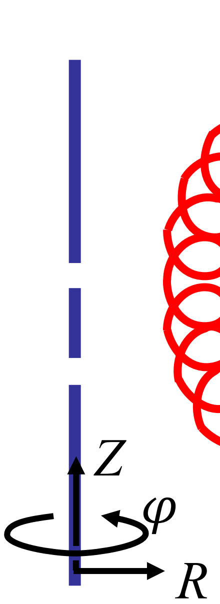


The upward cross-field drift is inward half the time and outward half the time, averaging to 0.

Now particles are confined!

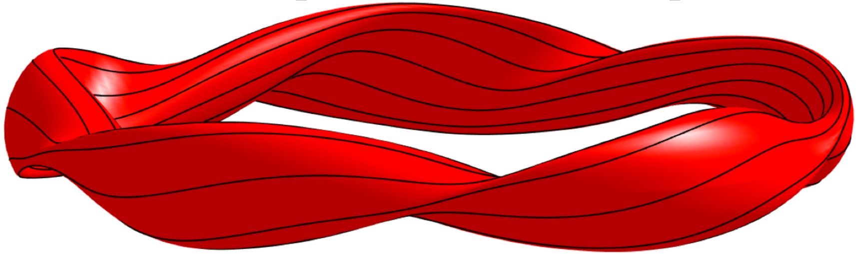
To avoid needing a large current in the confinement region, break continuous rotation symmetry.





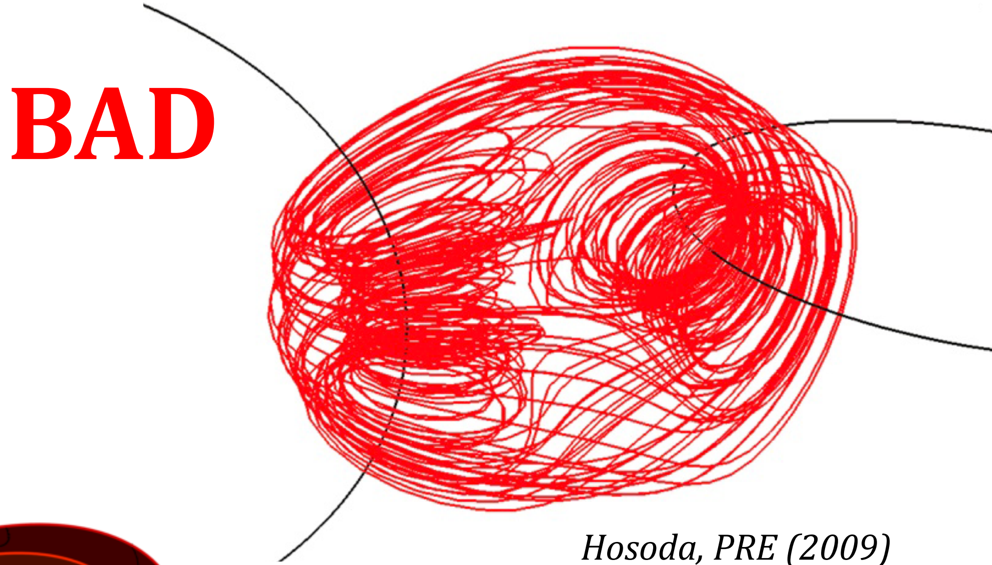
$\nabla\times\mathbf{B}=\mu_0\mathbf{J}$ + axisymmetry $\Rightarrow$ a large current **J** is required *inside* the plasma.

We can still get rotational transform without **J** by breaking axisymmetry:

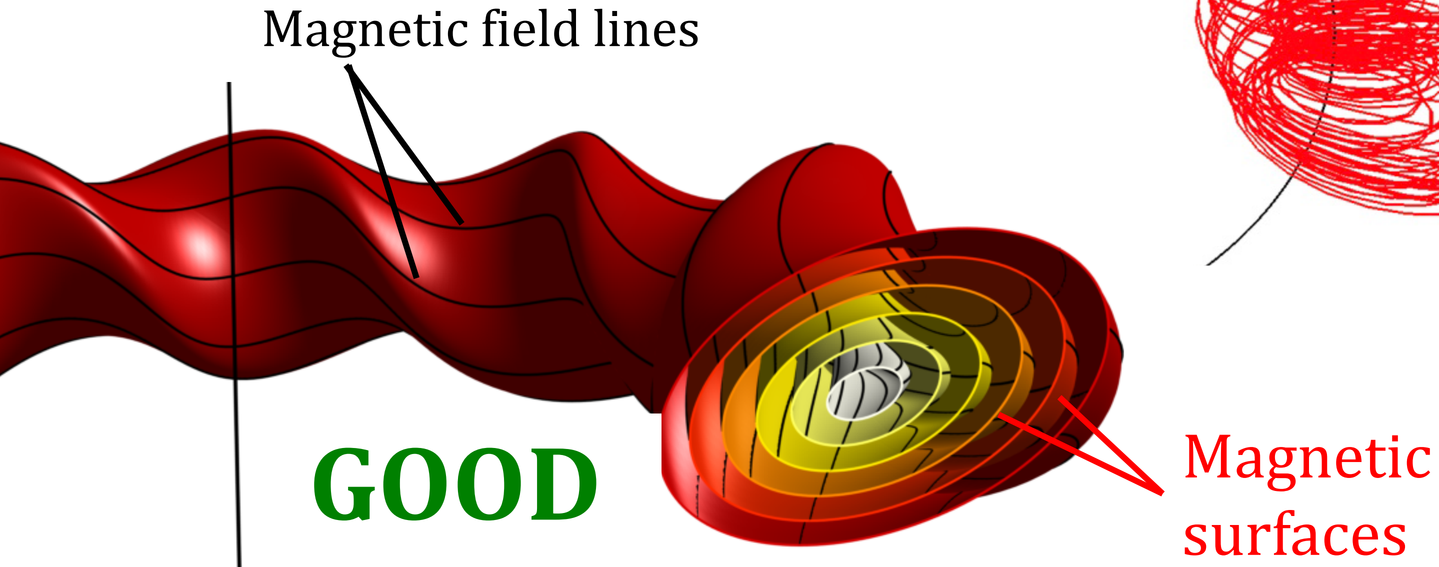


One goal of stellarator optimization is having field lines lie on surfaces.

Chaotic (volume-filling) **B** field lines would allow inside & outside to mix even without cross-**B** drift.

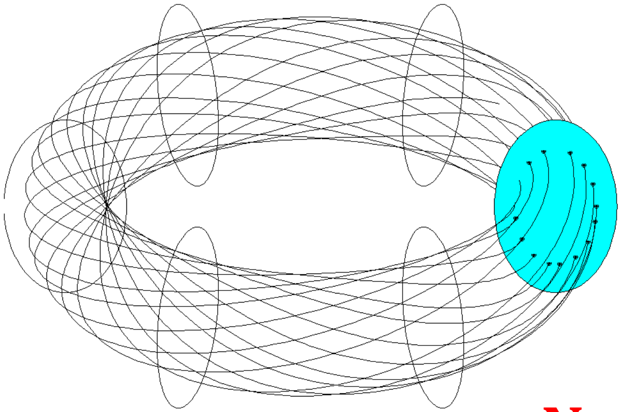


Hosoda, PRE (2009)



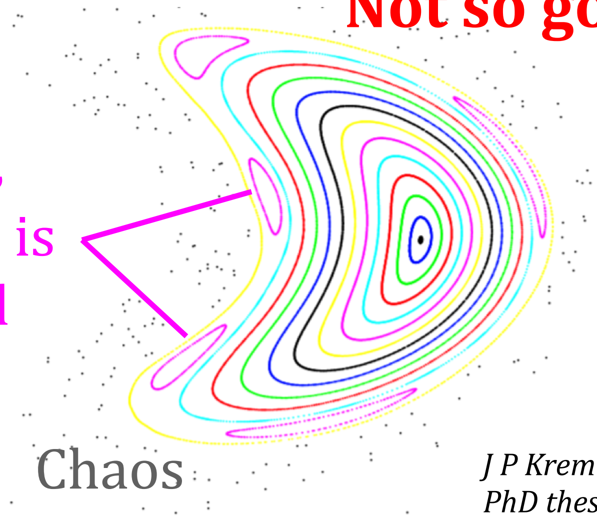
One goal of stellarator optimization is having field lines lie on surfaces.

Magnetic surfaces (a.k.a flux surfaces) can be visualized with a "Poincare plot":



Good

Not so good

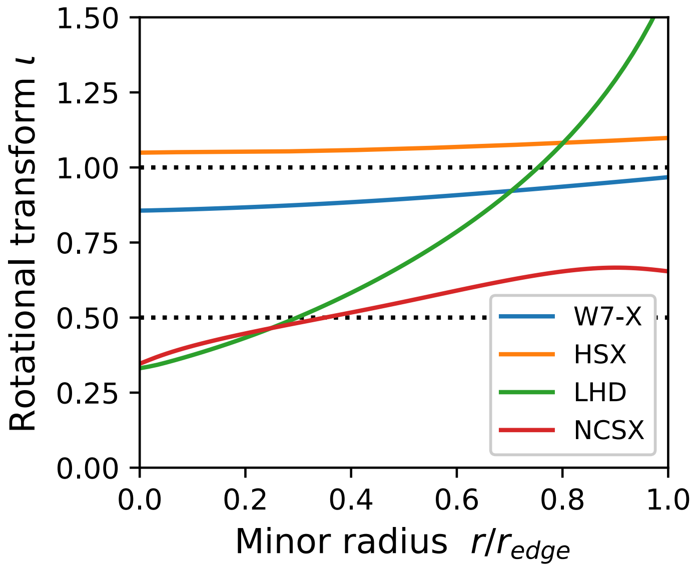


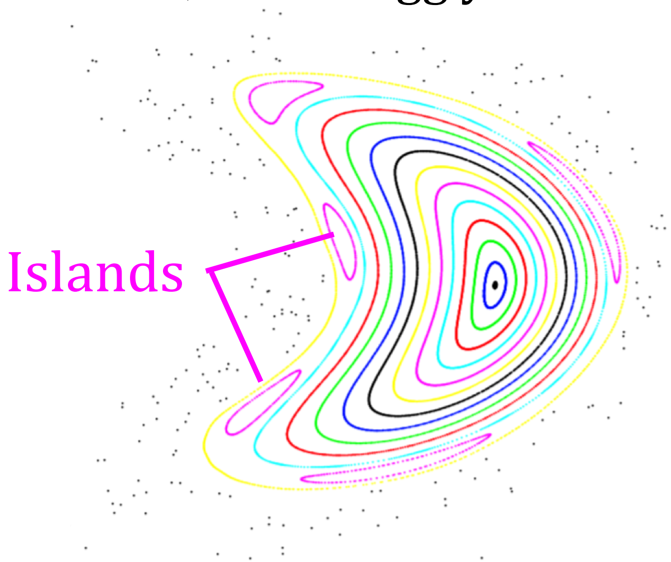
J P Kremer, PhD thesis, Columbia

How much rotational transform do you want?

Larger ι means:

- Thinner orbits, so better confinement.
- **B** changes less due to plasma pressure. (Higher "equilibrium β limit".)
- But, more wiggly coils.





Avoid rationals like ι = 1 or ½: islands form there.

So, maybe want low "magnetic shear" = $|\nabla\iota|$.

Or, maybe want *high* magnetic shear since it makes islands thin. (width $\propto |\nabla\iota|^{-1/2}$)

Outline

- Rotational transform & flux surfaces
- Transport & quasisymmetry
- Optimization approaches
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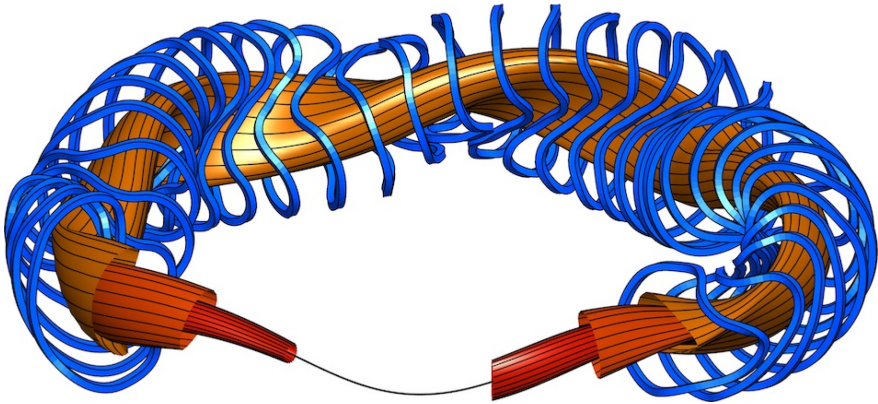
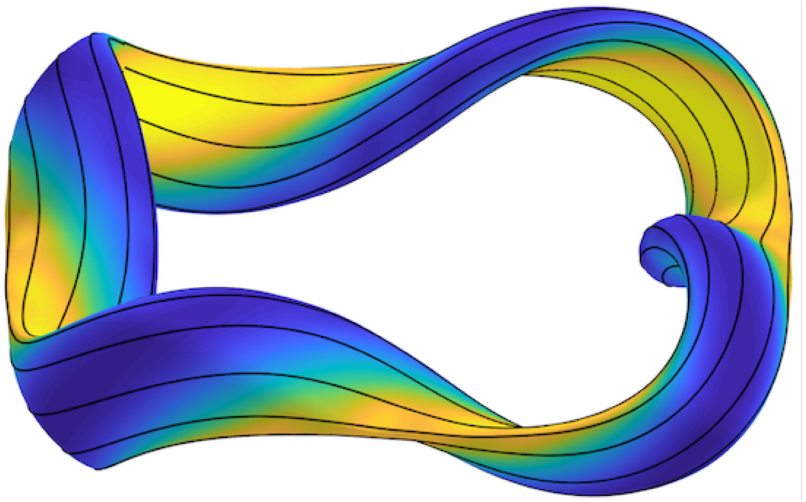




"Transport" = fluxes of heat and particles

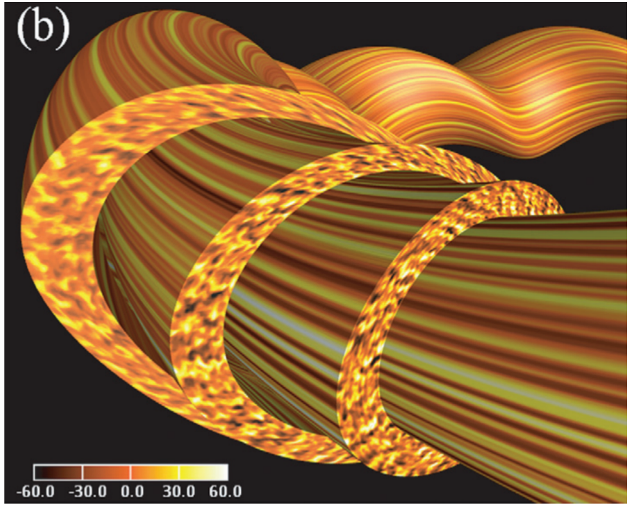
Total flux = Neoclassical flux + turbulent flux.

Turbulent transport: due to instabilities that saturate at low amplitude. Expensive to compute.

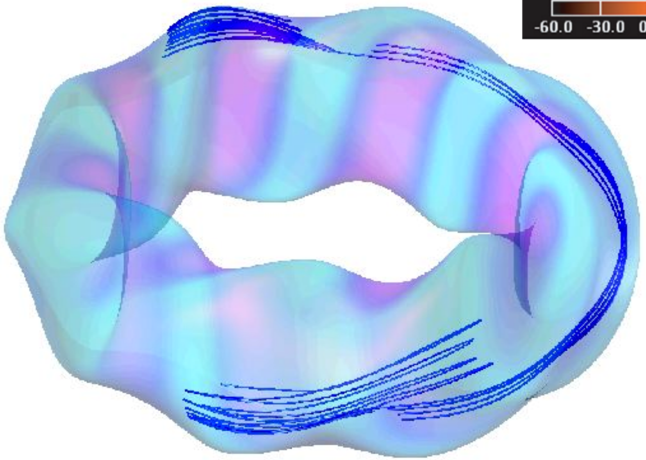
"Neoclassical" transport: due to guiding-center drifts + collisions. The minimum flux possible with no turbulence.

Both depend on geometry.

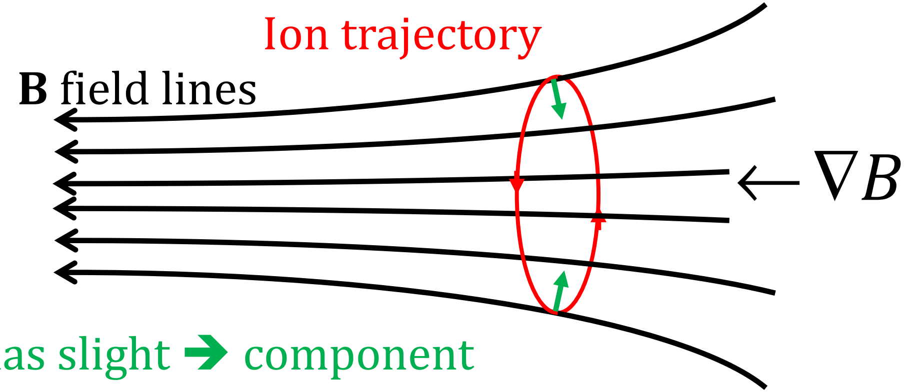
Neoclassical alone would be too large unless it is optimized.



Nunami 2017



Key to neoclassical transport: trapped particles

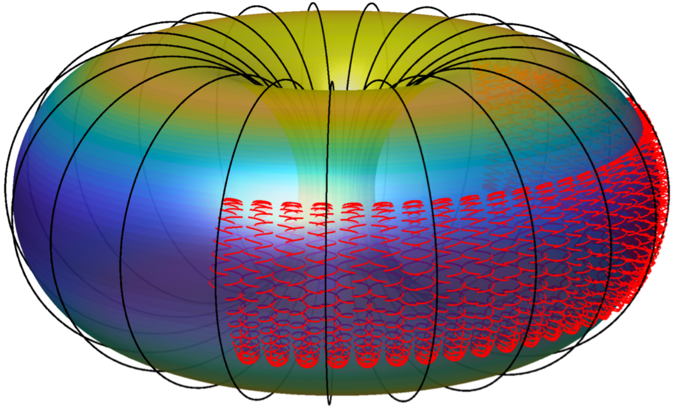


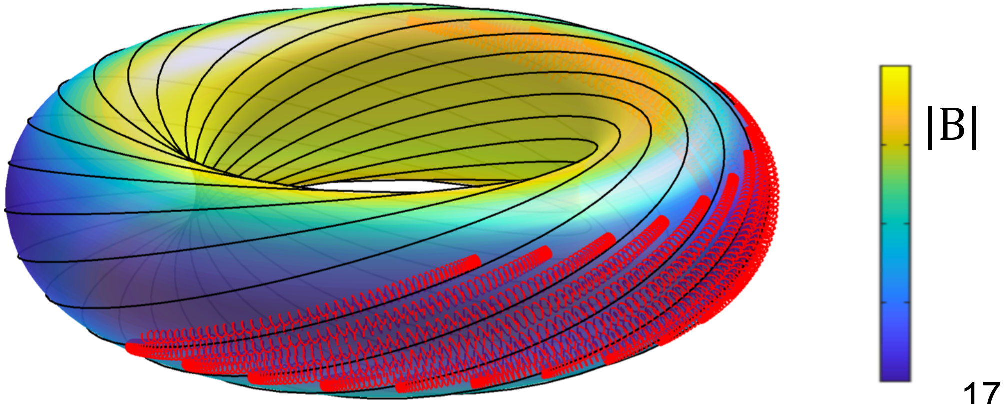
$\mathbf{v}\times\mathbf{B}$ force has slight ➔ component

Mirror force: particles are pushed away from regions of high *B*.

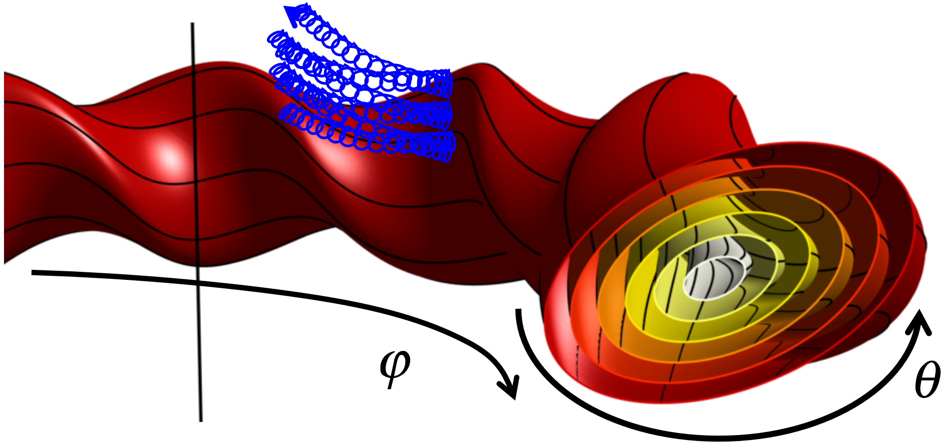
$$\frac{dv_{||}}{dt} = -\frac{v_{\perp}^{2}}{2B}\mathbf{b}\cdot\nabla B$$

A few particles with very small $v_{||} = \mathbf{v}\cdot\mathbf{B}$ "bounce" and are "trapped" in low-|B| regions.





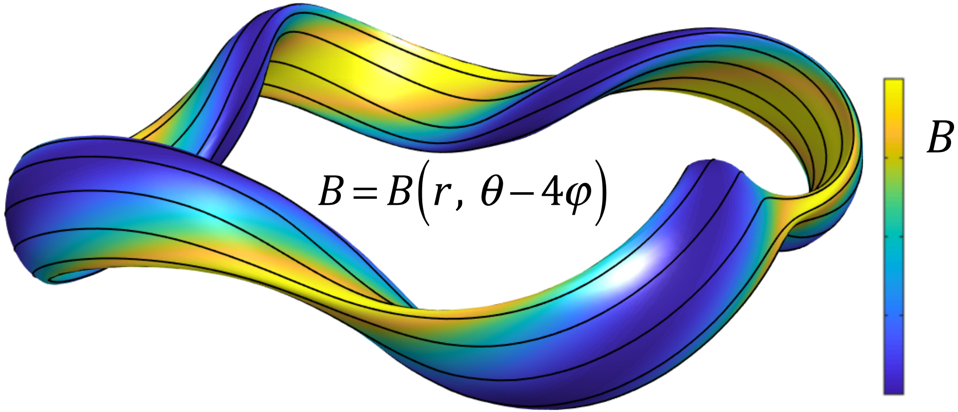
We're not done with confinement: *Trapped* particles are not confined without a further condition like "quasisymmetry"



In general: trapped particles do not sample the whole surface, so cross-field drift does not average to 0.

$\Longrightarrow$ Large neoclassical transport.

A solution is quasisymmetry: make $B(r, \theta, \varphi) = B(r, M\theta - N\varphi)$ for special angles θ, φ.



Symmetry direction

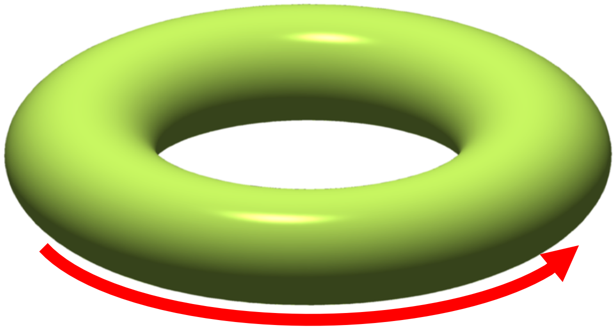
$\Longrightarrow$ Conserved quantity.

$\Longrightarrow$ Drift averages to 0.

For low neoclassical transport, recent stellarators have come in 3 flavors

- Trapped particles should drift toroidally, helically, or poloidally on a surface.
- B contours on a surface have the same topology as these drifts.

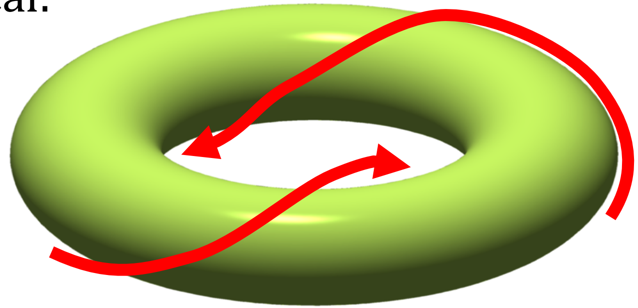
Toroidal:



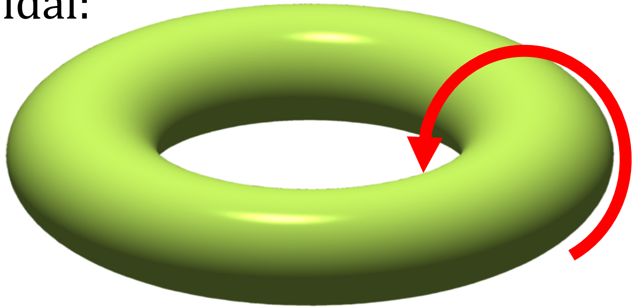
E.g., particles with $v_{||}$=0 move along a constant-B contour:

$$\left(\nabla B \text{ drift}\right)\cdot\nabla B \;\propto\; \mathbf{B}\times\nabla B\cdot\nabla B \;=\; 0$$

Helical:



Poloidal:



For low neoclassical transport, recent stellarators have come in 3 flavors

- Trapped particles should drift toroidally, helically, or poloidally on a surface.
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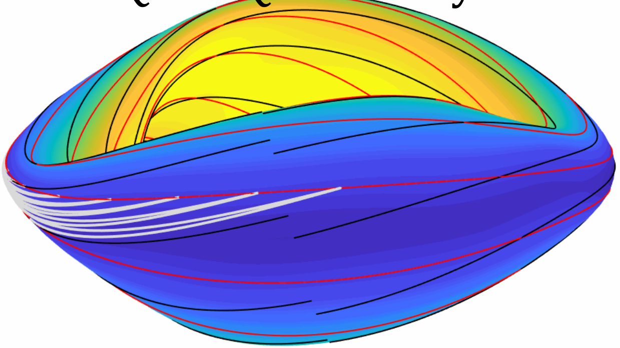
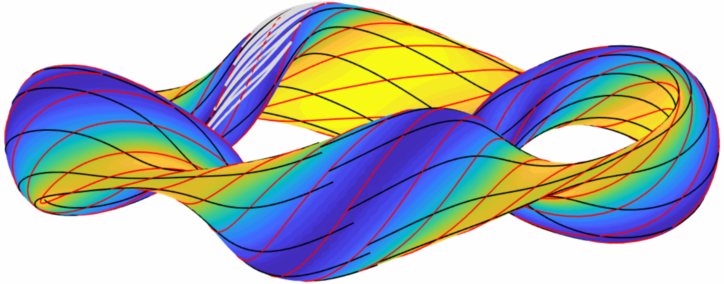
Toroidal: “QA” = Quasi-axisymmetric

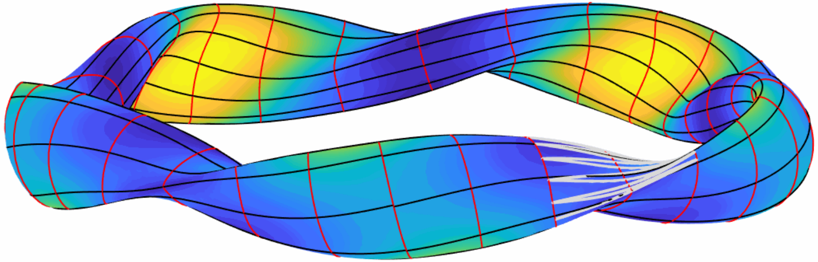


Field lines
|B| contours (slightly idealized)
Trapped particle

Helical: “QH”= Quasi-helically symmetric



Poloidal: “QI”= Quasi-isodynamic



Outline

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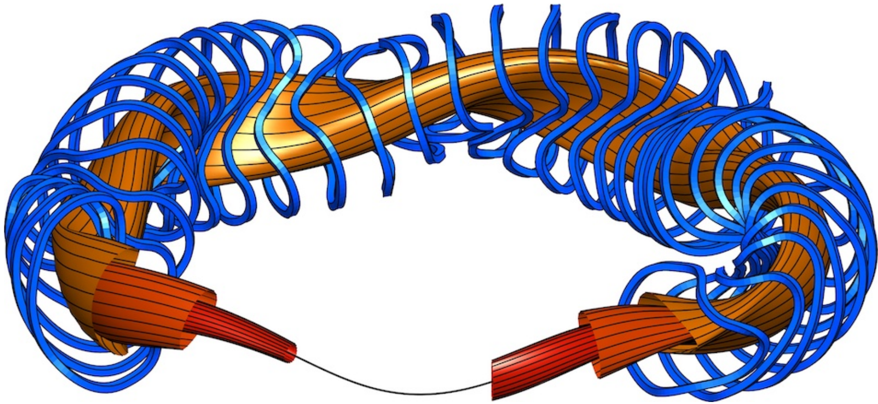
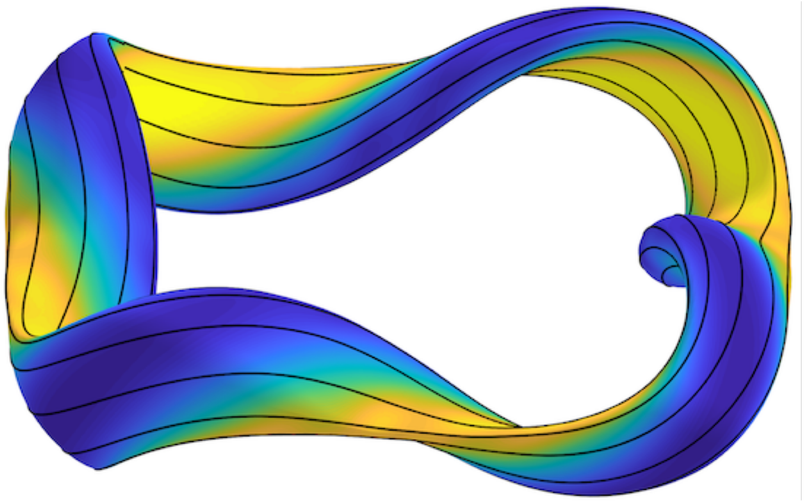
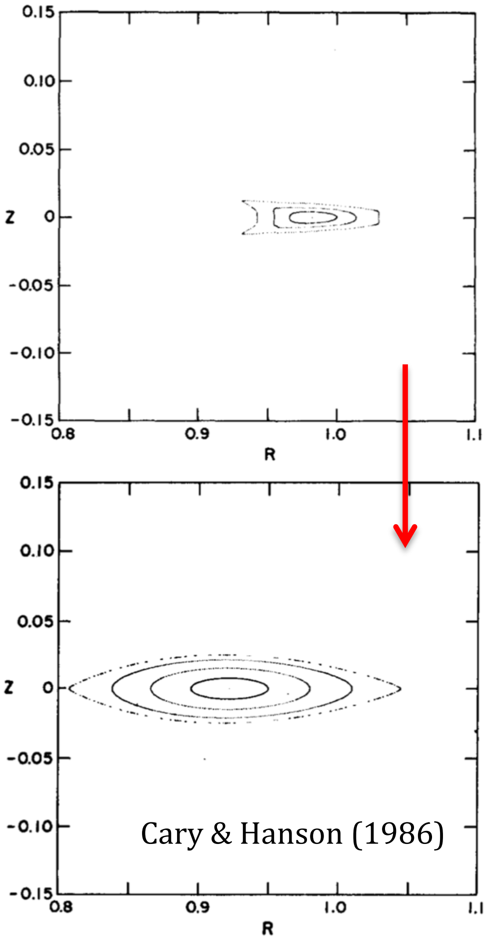
- Transport & quasisymmetry
- Optimization approaches
- Recent advances in α-particle confinement



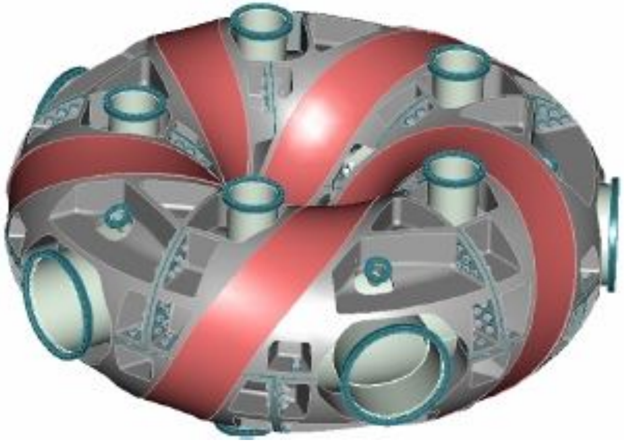


In some stellarators, coil shapes are optimized to maximize the volume of good surfaces

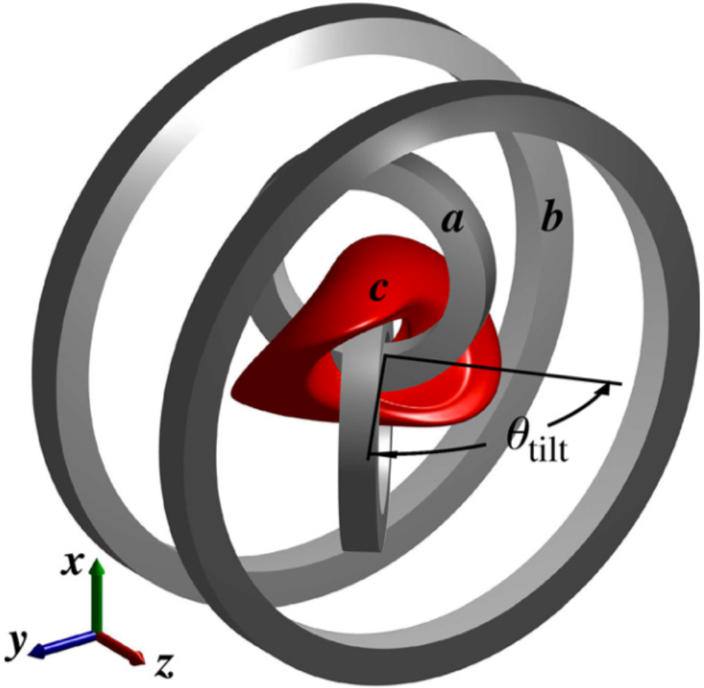


Cary & Hanson (1986)

CTH (Auburn)

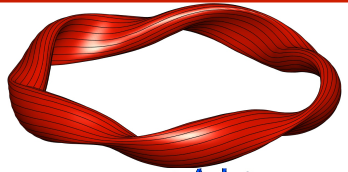


CNT (Columbia):
Optimize *expected* volume over possible coil position errors

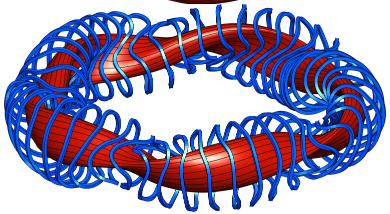


Pedersen (2004), Hammond (2016)

Most transport-optimized stellarators have instead used 2 optimization stages



1. Parameters = shape of boundary toroidal surface.
   Objective = physics (quasisymmetry, stability, etc.)



2. Parameters = coil shapes.
   Objective = error in **B** on boundary shape from stage 1.

Shape of a toroidal boundary surface (+ pressure & current vs *r* inside, & total **B** flux) determines **B** everywhere inside:

Consider a low-pressure plasma so $0 \approx \mathbf{J} = \nabla \times \mathbf{B} \quad \Rightarrow \quad \mathbf{B} = \nabla \Phi.$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \nabla^2 \Phi = 0.$$

$$\mathbf{B} \cdot \mathbf{n} = 0 \text{ on boundary} \quad \Rightarrow \quad \mathbf{n} \cdot \nabla \Phi = 0.$$

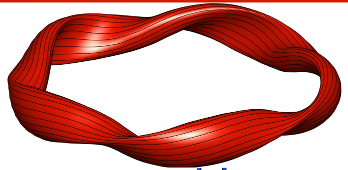
$\Rightarrow$ Laplace's eq with Neuman condition.

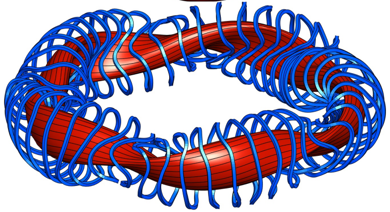
$\Rightarrow$ Unique solution up to scale factor + constant.

Most transport-optimized stellarators have instead used 2 optimization stages

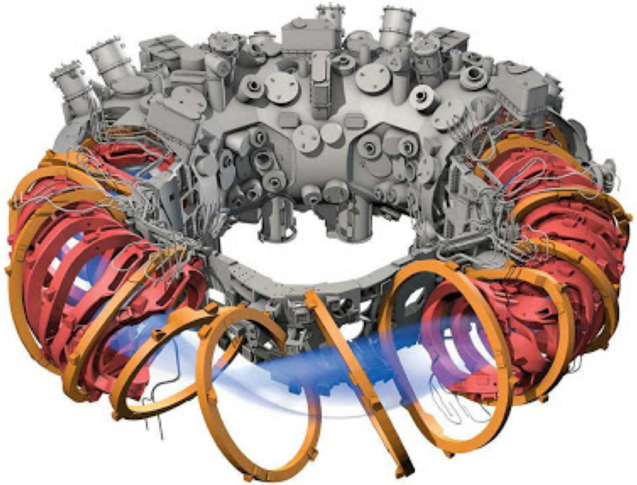
1. Parameters = shape of boundary toroidal surface.
   Objective = physics (quasisymmetry, stability, etc.)

2. Parameters = coil shapes.
   Objective = error in **B** on boundary shape from stage 1.

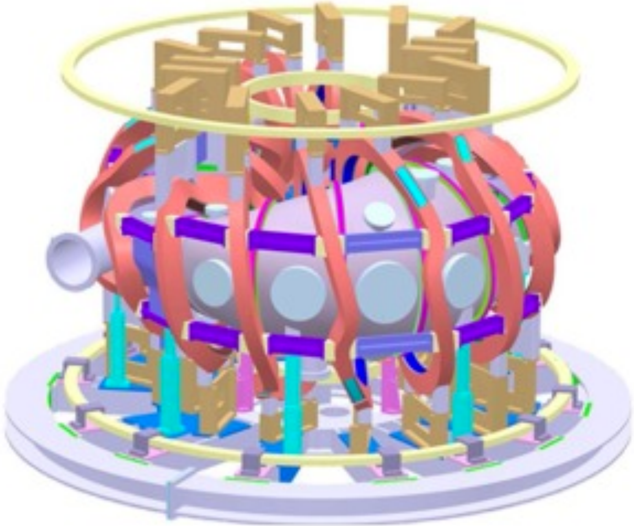




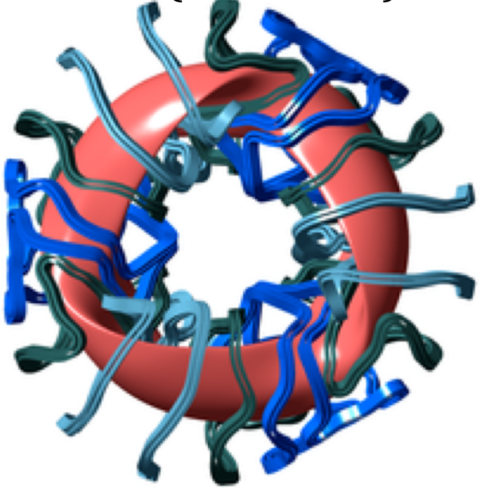
W7-X (Germany)



CFQS (China), under construction

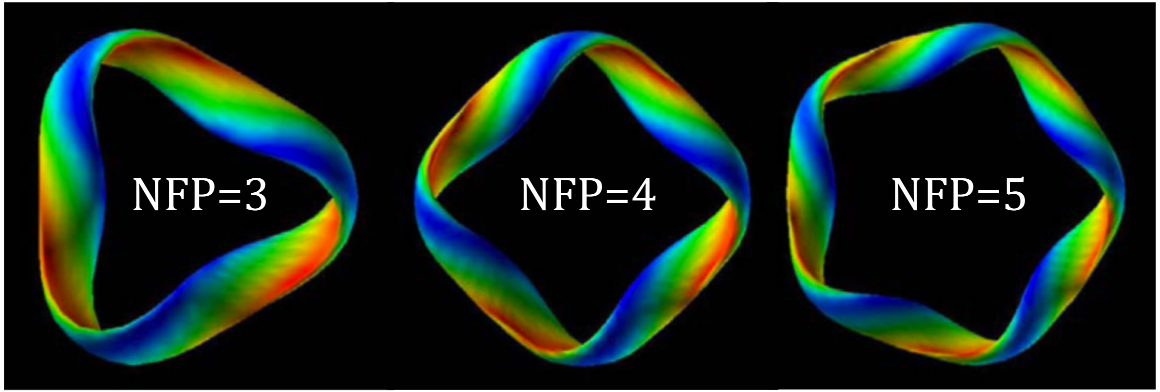


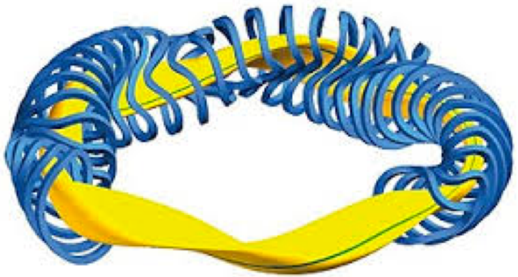
NCSX (Princeton)

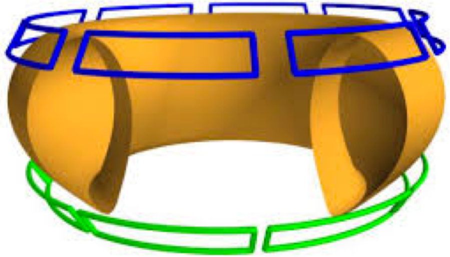


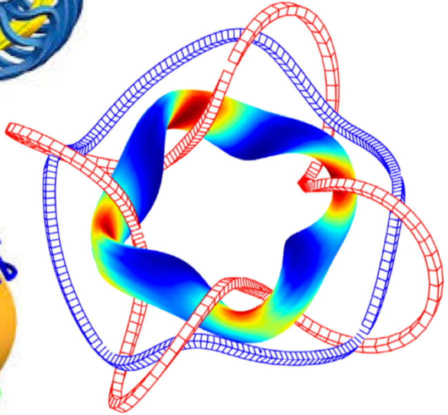
Other design parameters are discrete

- Number of "field periods".
- Number of coils.

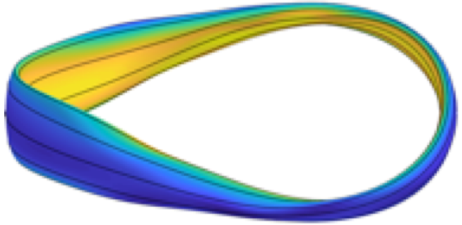


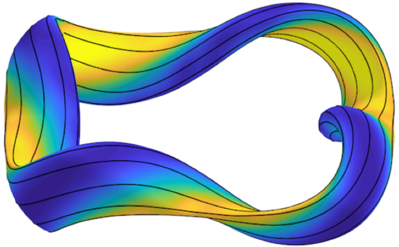






- Do coils link the plasma poloidally, helically, or not at all?
- Do B contours link the torus toroidally (QA), helically (QH), or poloidally (QI)?





Stellarator plasma & coil shapes must be optimized for several objectives

- Large volume of good magnetic surfaces (not islands & chaos)
- Enough rotational transform
- Plasma pressure doesn’t modify **B** too much, i.e. pressure limit is not too low.
- Buildable coil shapes
- Magnetohydrodynamic stability
- Good confinement of particle trajectories
- Low neoclassical transport
- Low turbulent transport

Outline

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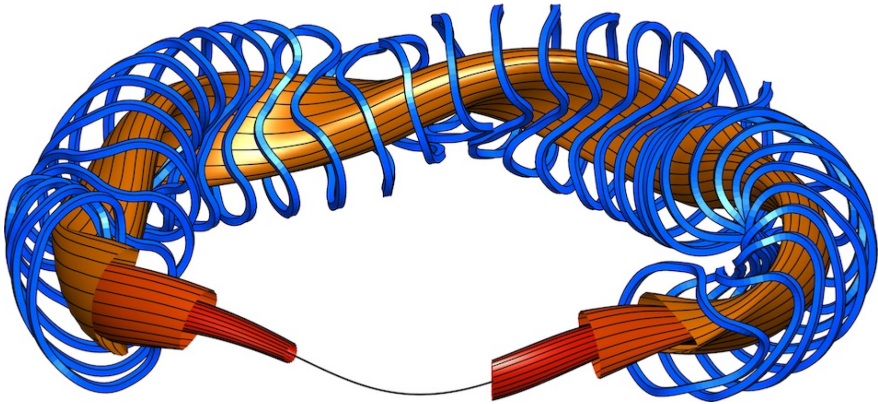
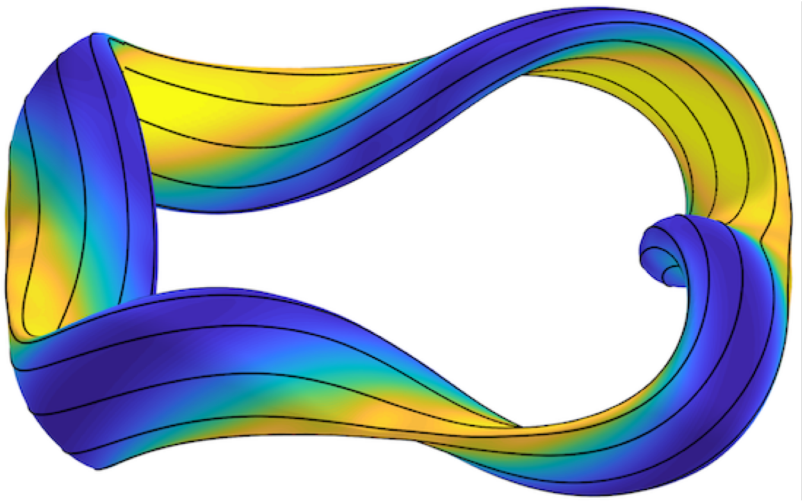
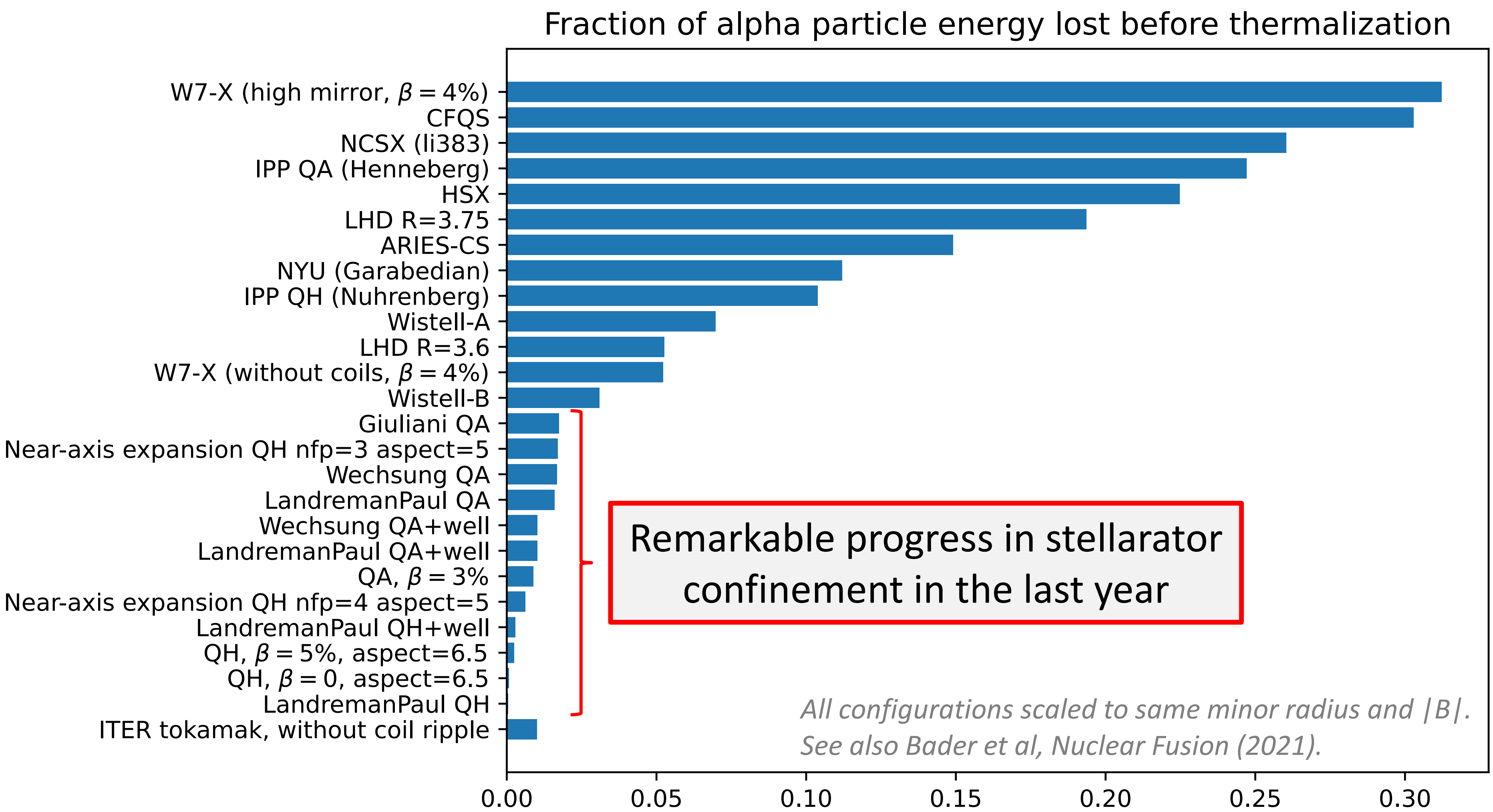
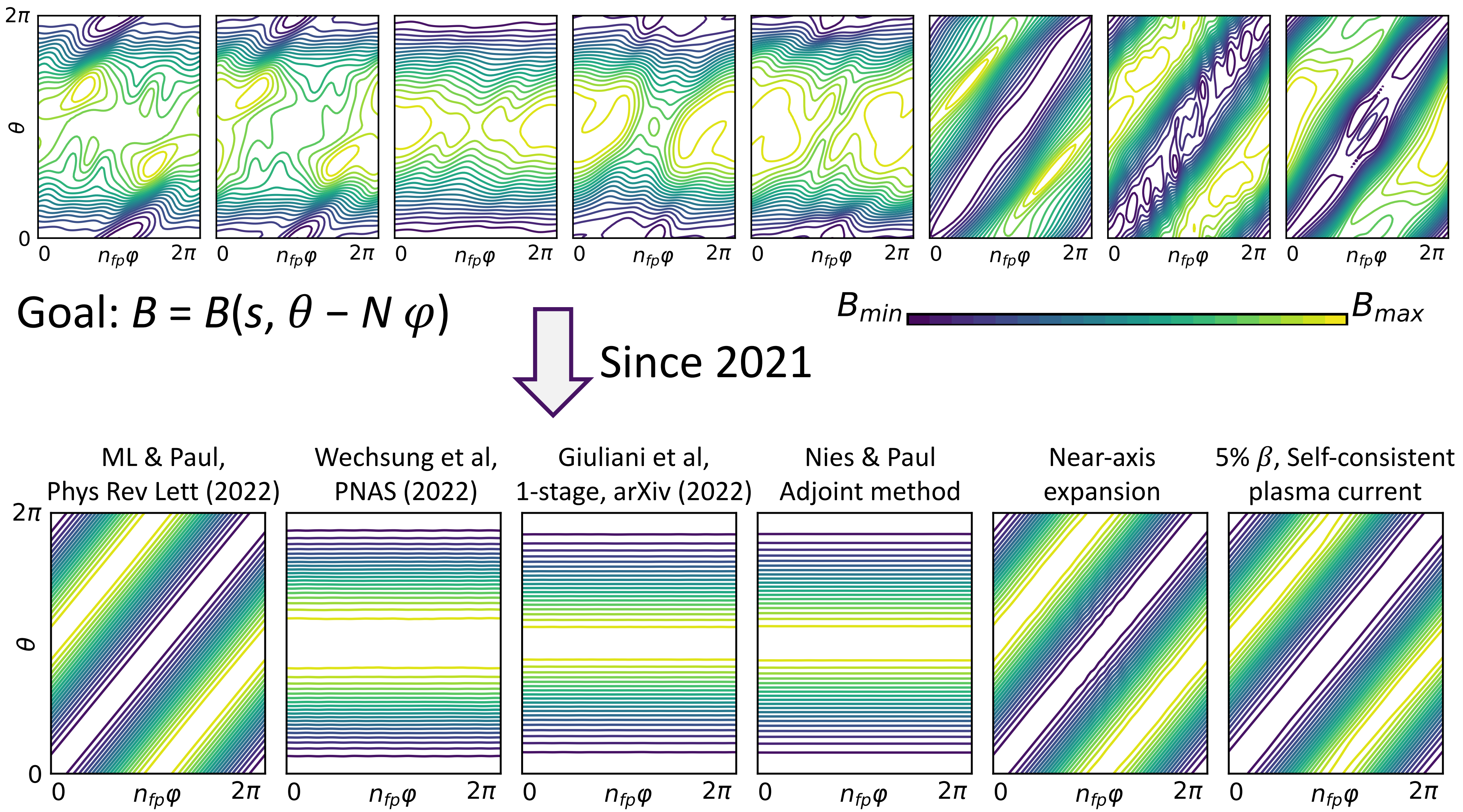
- Transport & quasisymmetry
- Optimization approaches
- Recent advances in α-particle confinement



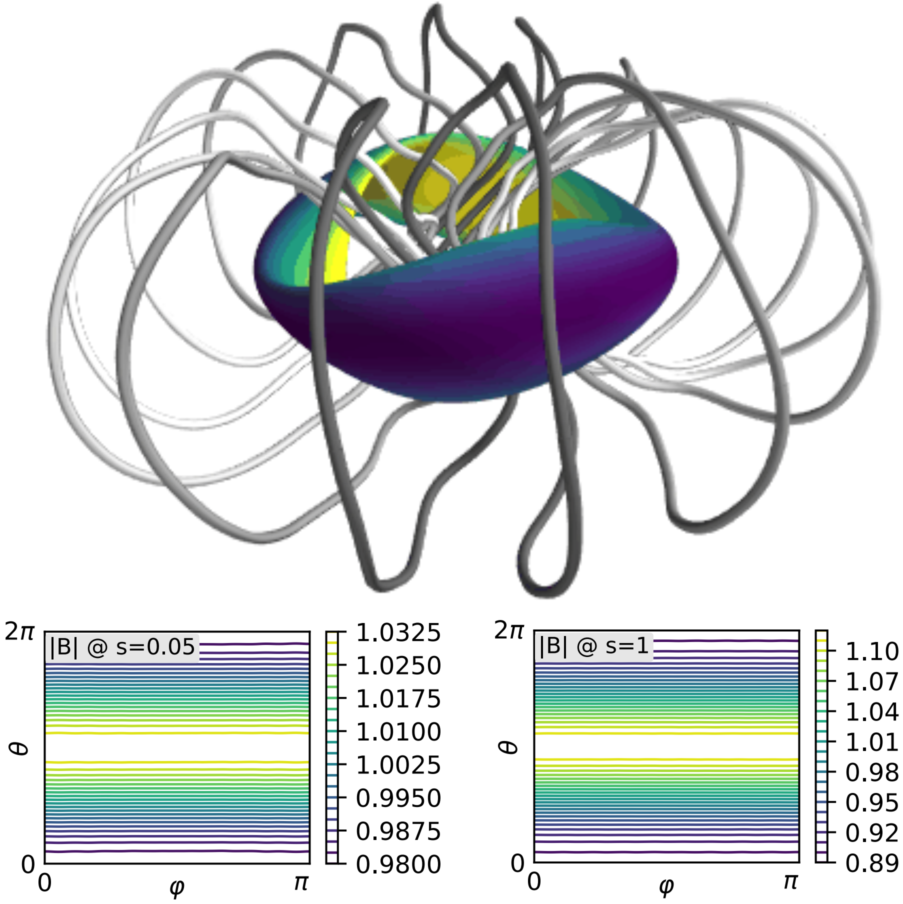


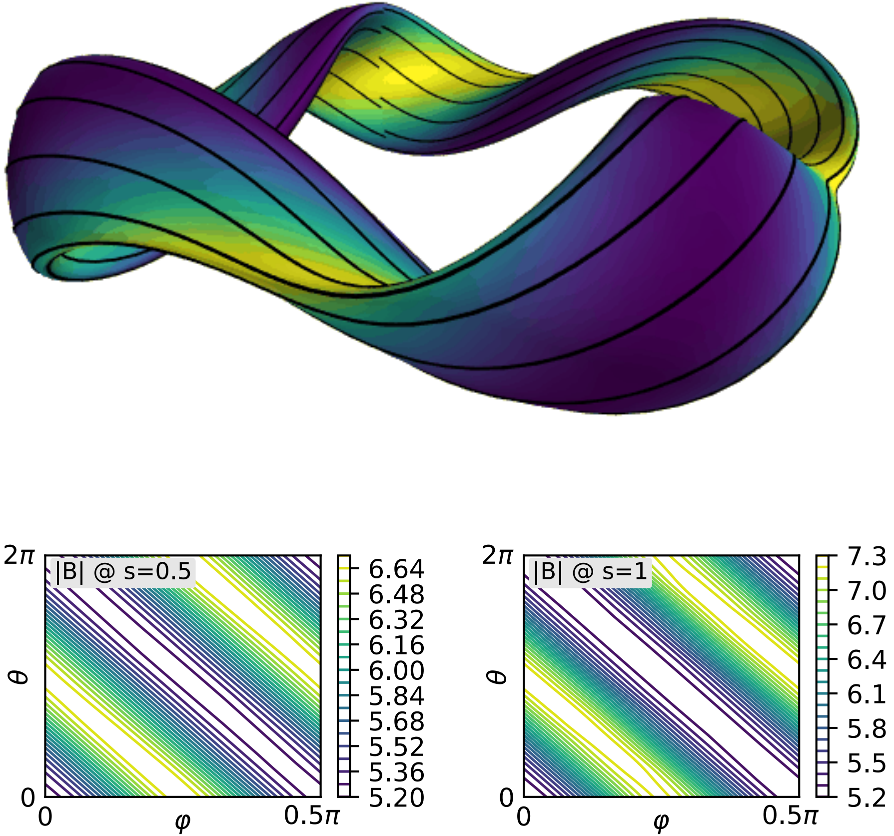

Fraction of alpha particle energy lost before thermalization
W7-X (high mirror, $\beta = 4\%$)
CFQS
NCSX (li383)
IPP QA (Henneberg)
HSX
LHD R=3.75
ARIES-CS
NYU (Garabedian)
IPP QH (Nuhrenberg)
Wistell-A
LHD R=3.6
W7-X (without coils, $\beta = 4\%$)
Wistell-B
Giuliani QA
Near-axis expansion QH nfp=3 aspect=5
Wechsung QA
LandremanPaul QA
Wechsung QA+well
LandremanPaul QA+well
QA, $\beta = 3\%$
Near-axis expansion QH nfp=4 aspect=5
LandremanPaul QH+well
QH, $\beta = 5\%$, aspect=6.5
QH, $\beta = 0$, aspect=6.5
LandremanPaul QH
ITER tokamak, without coil ripple
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Remarkable progress in stellarator confinement in the last year
All configurations scaled to same minor radius and |B|.
See also Bader et al, Nuclear Fusion (2021).



Goal: $B = B(s, \theta - N\varphi)$
Since 2021
B_{min}
B_{max}
ML & Paul, Phys Rev Lett (2022)
Wechsung et al, PNAS (2022)
Giuliani et al, 1-stage, arXiv (2022)
Nies & Paul Adjoint method
Near-axis expansion
5% β, Self-consistent plasma current
θ
$n_{fp}\varphi$
0
2π

Some of the new configurations with excellent alpha-particle confinement





Wechsung et al, PNAS (2022).

Closing thoughts: There are many open questions for stellarator optimization

- How best to combine coil and plasma design?
- How to find designs that tolerate errors in coil shape/position?
- How to avoid getting stuck in little local minima? How to find global optima?
- How to optimize for expensive & noisy objectives (turbulence & fast-particle confinement)?
- How to balance multiple competing objectives?
- How to optimize coil topology?
- How to find configurations that are flexible?
  - Good confinement for different plasma pressures.
  - Ability to tune physics properties by changing coil currents.

Extra slides

Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

$$f_{QS} = \int d^3x \left(\frac{1}{B^3}\left[\left(N - \iota\right)\mathbf{B}\times\nabla B\cdot\nabla\psi - \left(G + NI\right)\mathbf{B}\cdot\nabla B \right] \right)^2$$

$$f_{QH} = \left(A - A_*\right)^2 + f_{QS}$$

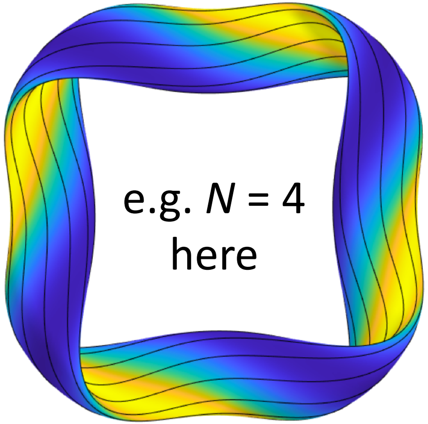
Boundary aspect ratio

$$f_{QA} = \left(A - A_*\right)^2 + \left(\iota_* - \int_0^1 \iota\, ds\right)^2 + f_{QS}$$

Goal: $B = B(s, \theta - N\varphi)$.

For quasi-axisymmetry,
$N = 0$.

For quasi-helical symmetry,
N is the number of field periods,



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$$f_{QH} = \left(A - A_* \right)^2 + f_{QS} \qquad f_{QA} = \left(A - A_* \right)^2 + \left(\iota_* - \int_0^1 \iota \, ds \right)^2 + f_{QS}$$

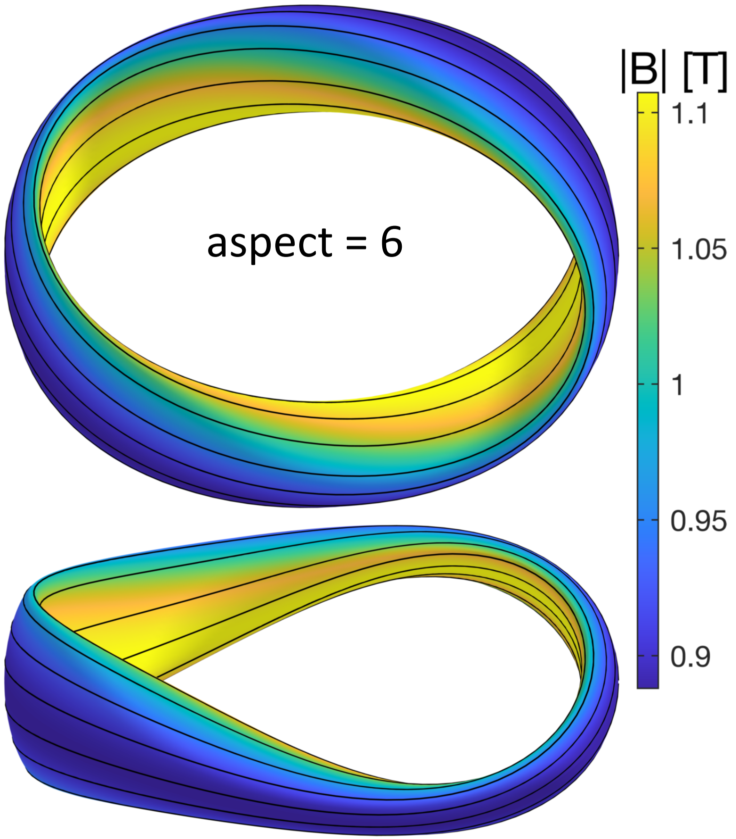
Boundary aspect ratio

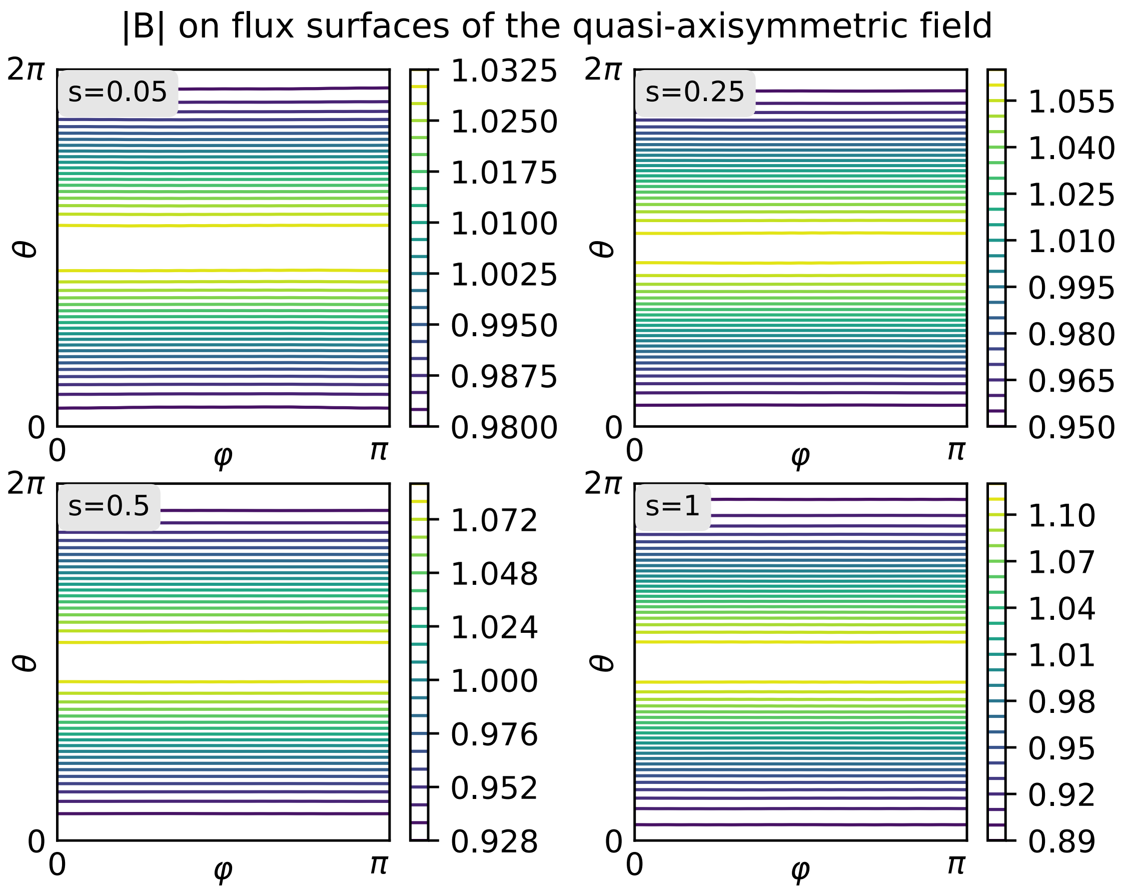
- Parameter space: $R_{m,n}$ & $Z_{m,n}$ defining a toroidal boundary

$$R(\theta,\phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \quad Z(\theta,\phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)$$

- Codes used: SIMSOPT with VMEC
- Cold start: circular cross-section torus
- Vacuum fields at first, allowing precise checks
- Algorithm: default for least-squares in scipy (trust region reflective)
- 6 steps: increasing # of modes varied & VMEC resolution
- Run many optimizations, pick the best

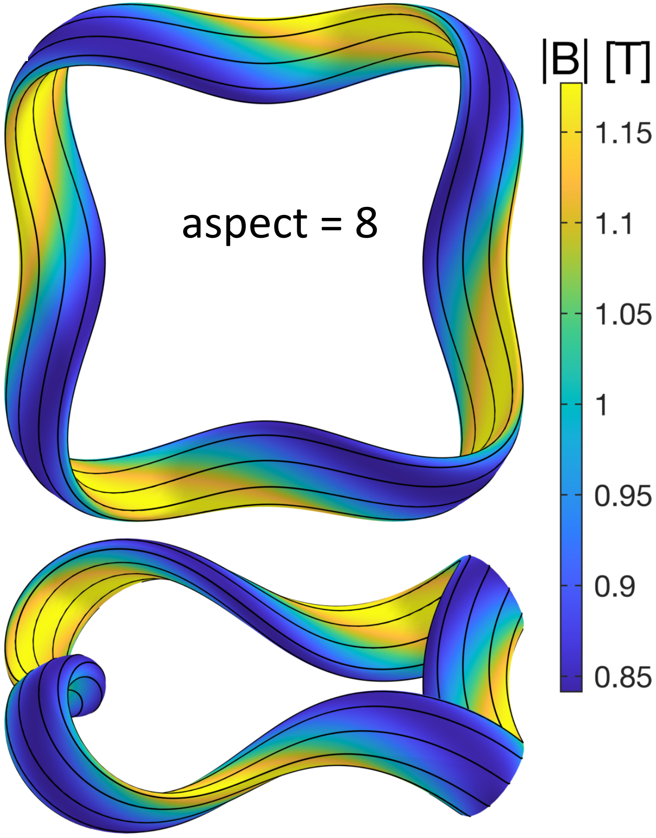
Straight |B| contours are possible for quasi-axisymmetry



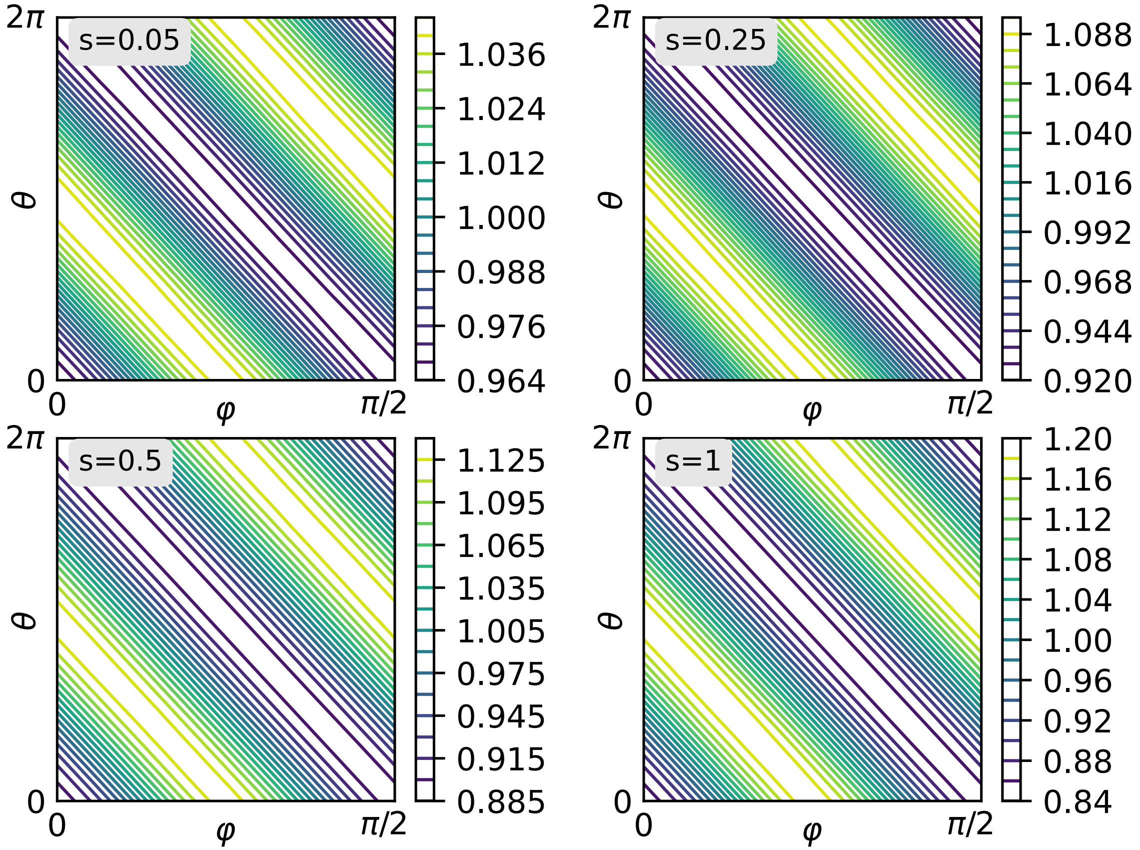


ML & Paul, PRL (2022). All input/output files and optimization scripts online at doi.org/10.5281/zenodo.5645412

Straight |B| contours are possible for quasi-helical symmetry



|B| on flux surfaces of the quasi-helically symmetric field



ML & Paul, PRL (2022).

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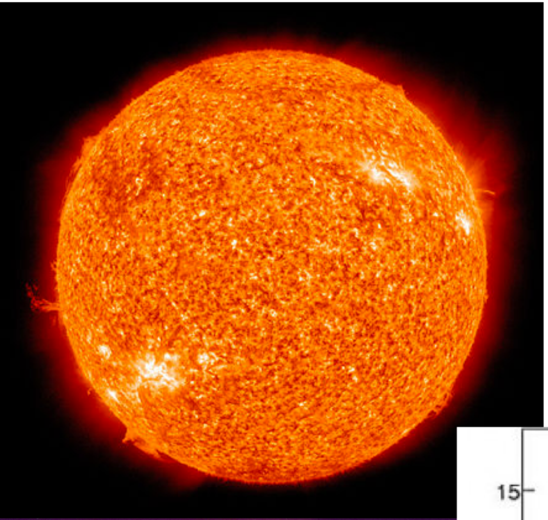
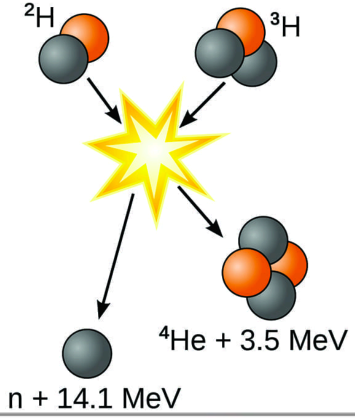
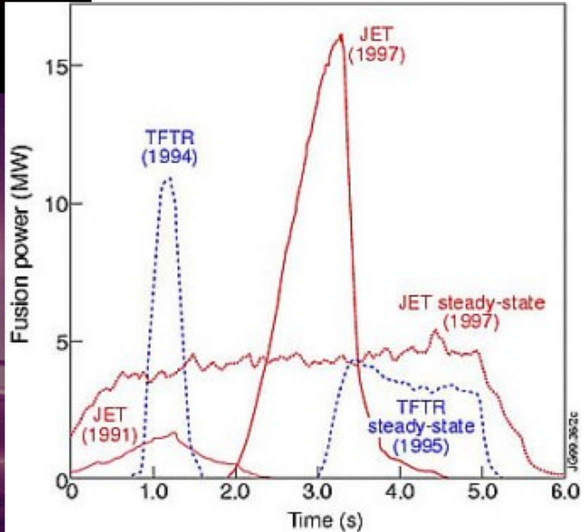
Controlled fusion would be a transformational energy source







- No CO_2 from operations.
- Generation could be near cities.
- Negligible land use.
- Not intermittent – no storage needed.
- No risk of criticality accident.
- Product is inert helium.
- No weapons proliferation risks.
- Plentiful fuel: water + lithium.
- Isotopes produced are short-lived.

But, capital cost may be prohibitive, etc.

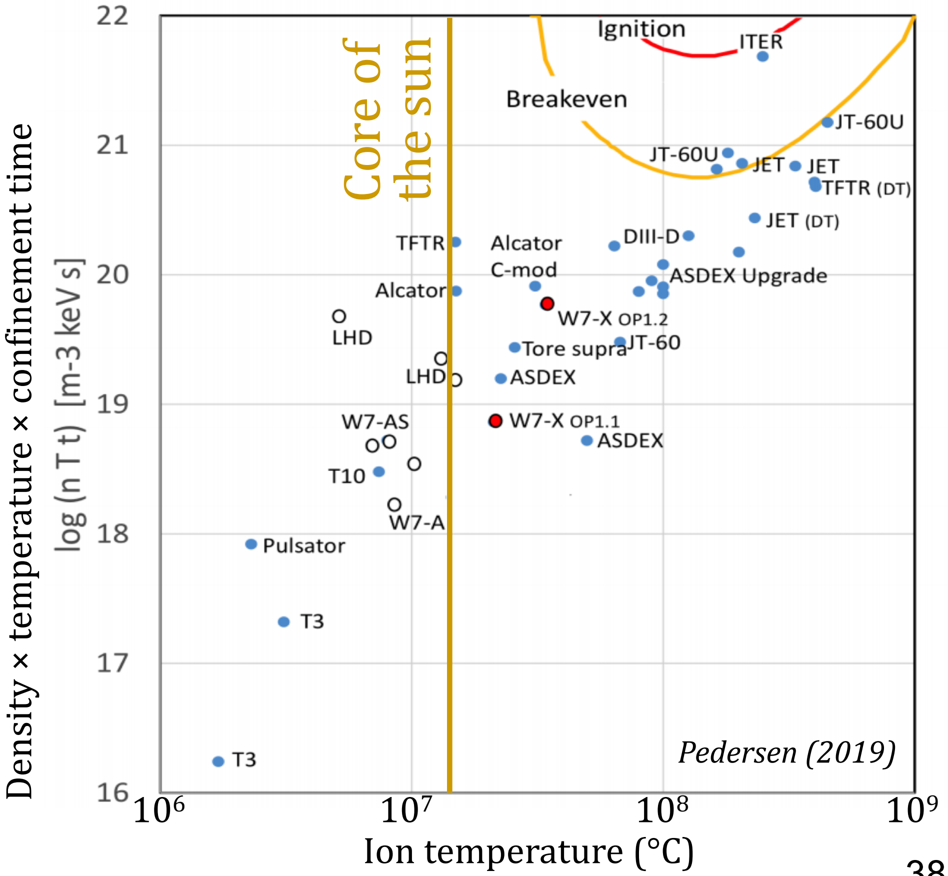
Fusion requires high temperatures and confinement

Must overcome electrostatic repulsion

Need high velocity.
➔ Need high temperature, ~10^8 C.

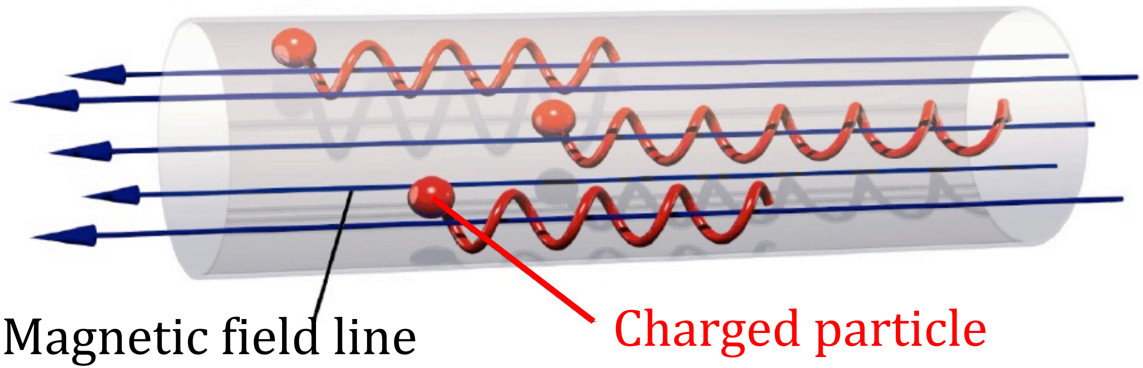
Electrons separate from nuclei: a plasma.
➔ Strong response to **E** and **B** fields.

Necessary temperature has been achieved!



Confining charged particles with a magnetic field is tricky.

Uniform straight **B**: confinement ⊥ to **B**, but end losses.



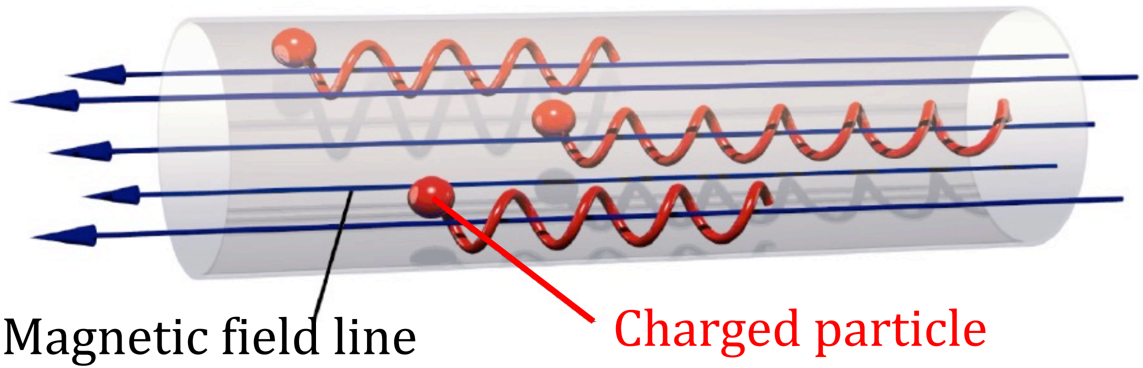
Magnetic fields are the best insulators we know –

Can support 1000x the temperature gradient of spacecraft reentry tiles!

But, there is no confinement *along* the field…

Confining charged particles with a magnetic field is tricky.

Uniform straight **B**: confinement ⊥ to **B**, but end losses.



But if field lines are bent, particles drift off them due to *guiding-center drifts.*