Novel optimized stellarator configurations

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Integrated optimization for good flux surfaces [1]

- Flux surfaces are not guaranteed in 3D fields.
- But, for many stellarator optimization objective functions, it is convenient to assume good surfaces.
- So, at each iteration, compute two **B** representations: one assuming surfaces (VMEC) and one not assuming surfaces (SPEC).
- Penalizing Greene's residues in the objective function [4] makes the two representations consistent by the end of the optimization.

Objective function:

Transform on axis — Aspect ratio — Transform at edge

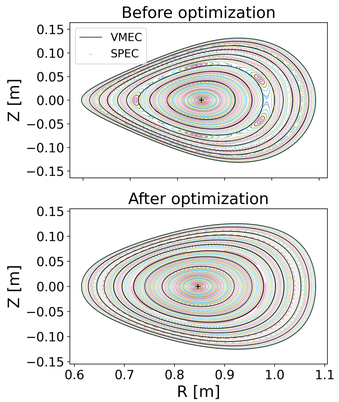
$$f = \left(A-6\right)^2 + \left(\iota_0 - 0.39\right)^2 + \left(\iota_a - 0.42\right)^2$$

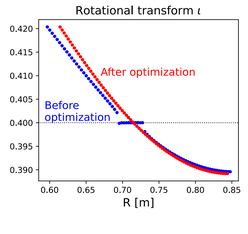
$$+2\sum_{n\neq 0,m}\left(\frac{B_{m,n}\left(s=0.5\right)}{B_{0,0}}\right)^2 + 2R_1^2 + 2R_2^2$$

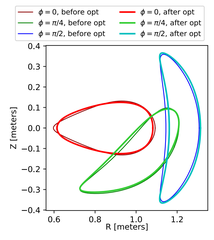
Quasisymmetry error from VMEC + BOOZ_XFORM — Greene's residues for X- and O-points, from SPEC

Corresponding SIMSOPT [3] script:

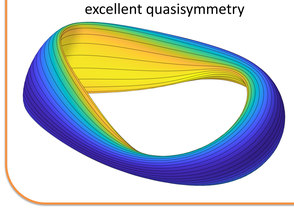
```
# Create objects for the Vmec and Spec equilibrium
mpi = MpiPartition()
vmec = Vmec("input.nfp2_QA_iota0.4", mpi=mpi)
surf = vmec.boundary
spec = Spec("nfp2_QA_iota0.4.sp", mpi=mpi)
spec.boundary = surf  # Identify the Vmec and Spec boundaries

# Configure quasisymmetry objective:
boozer = Boozer(vmec)
qs = Quasisymmetry(boozer,
                   0.5, # Radius s to target
                   1, 0) # (M, N) you want in |B|

# Specify resonant surface by iota = p / q
p = -2
q = 5
residue1 = Residue(spec, p, q)  # X-point
residue2 = Residue(spec, p, q, theta=np.pi)  # O-point

# Define objective function
prob = LeastSquaresProblem([(vmec.aspect, 6, 1.0),
                            (vmec.iota_axis, 0.39, 1),
                            (vmec.iota_edge, 0.42, 1),
                            (qs, 0, 2),
                            (residue1, 0, 2),
                            (residue2, 0, 2)])

for step in range(3):
    # Define parameter space for this step:
    surf.all_fixed()
    max_mode = step + 3
    surf.fixed_range(mmin=0, mmax=max_mode,
                     nmin=-max_mode, nmax=max_mode, fixed=False)
    surf.set_fixed("rc(0,0)")  # Fix the major radius

    # Dynamically increase the resolution parameters each step:
    vmec.indata.mpol = 4 + step
    vmec.indata.ntor = vmec.indata.mpol
    boozer.mpol = 24 + step * 8
    boozer.ntor = boozer.mpol

least_squares_mpi_solve(prob, mpi, grad=True)
```

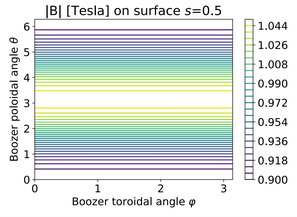


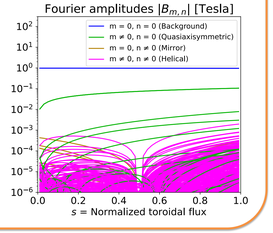




Final configuration has excellent quasisymmetry







References

[1] M Landreman , B Medasani, & C Zhu, *Phys Plasmas* 28, 092505 (2021).

[2] M Landreman & E J Paul, *arXiv*:2108.03711 (2021).

[3] M Landreman, B Medasani, F Wechsung, et al, *J Open Source Software* 6, 3525 (2021).

[4] J D Hanson & J R Cary, *Phys Fluids* 27, 767 (1984).

[5] C G Albert, S V Kasilov, & W Kernbichler, *J Comp Phys* 403, 109065 (2020).

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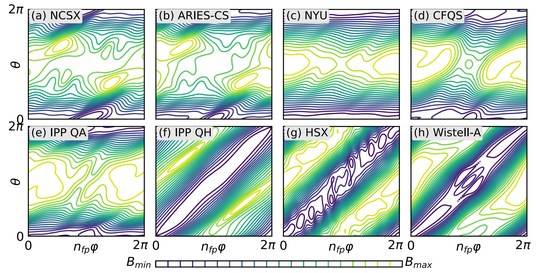
Stellarators with precise quasisymmetry and excellent confinement [2]

Previous quasisymmetric stellarators:



(a) Zarnstorff et al (2001)
(b) Najambadi et al (2008)
(c) Garabedian (2008)
(d) Liu et al (2018)
(e) Henneberg et al (2019)
(f) Nuhrenberg & Zille (1988)
(g) Anderson et al (1995)
(h) Bader et al (2020)

Optimization procedure for new configurations:

- Objective functions: $f_{QH} = \left(A - A_*\right)^2 + f_{QS}$ (Boundary aspect ratio) $\qquad f_{QA} = \left(A - A_*\right)^2 + \left(\iota_* - \int_0^1 \iota\, ds\right)^2 + f_{QS}$

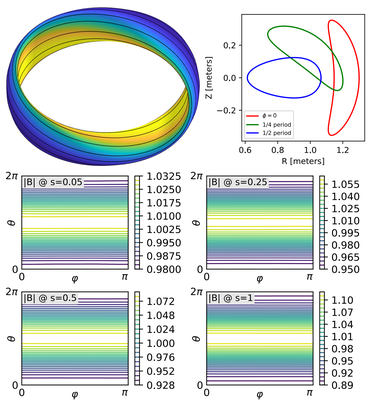
$$f_{QS} = \int d^3x\; w(s)\left(\frac{1}{B^3}\left[\left(N-\iota M\right)\mathbf{B}\times\nabla B\cdot\nabla\psi - \left(MG+NI\right)\mathbf{B}\cdot\nabla B\right]\right)^2$$

- The usual parameter space: $R_{m,n}$ & $Z_{m,n}$ defining a toroidal boundary

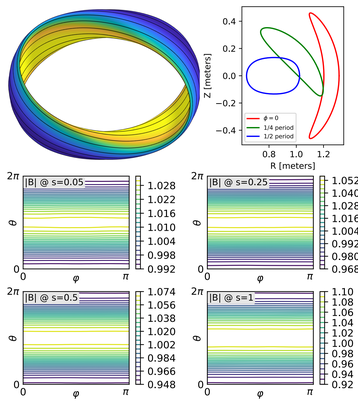
$$R(\theta,\phi) = \sum_{m,n} R_{m,n}\cos\left(m\theta - n\phi\right), \quad Z(\theta,\phi) = \sum_{m,n} Z_{m,n}\sin\left(m\theta - n\phi\right)$$

- 6 stages: increasing # of $R_{m,n}$ & $Z_{m,n}$ modes varied & code resolution
- SIMSOPT [3] with VMEC
- Vacuum fields, to ease confirmation of surface quality
- Algorithm: default for nonlinear least-squares in scipy

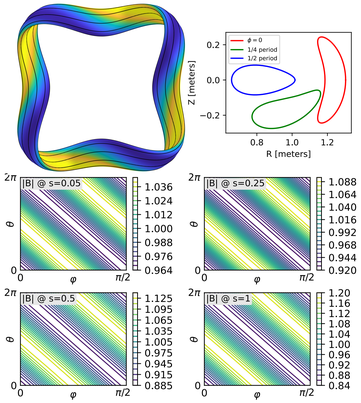
New QA



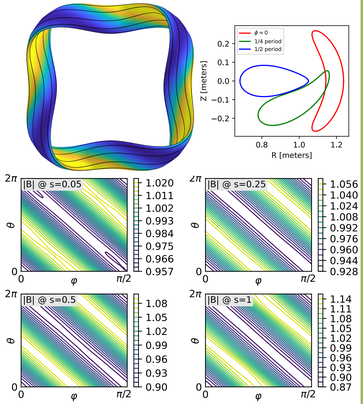
New QA + magnetic well



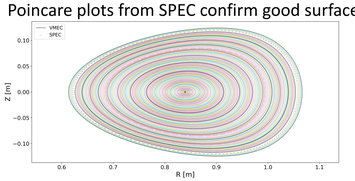
New QH

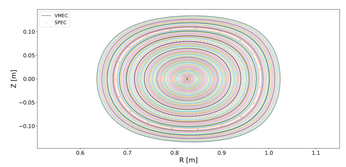


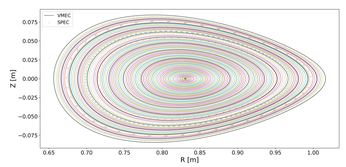
New QH + magnetic well

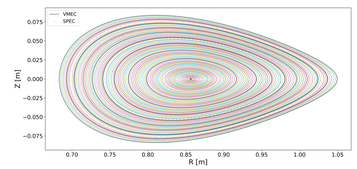


Poincare plots from SPEC confirm good surfaces:

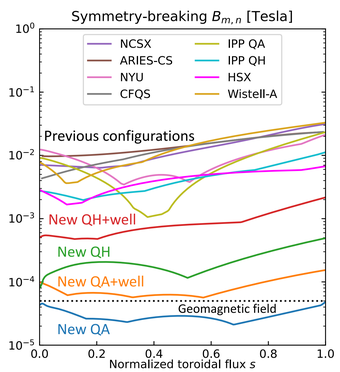




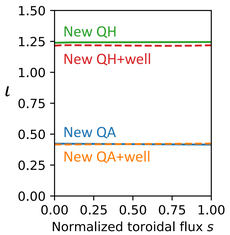




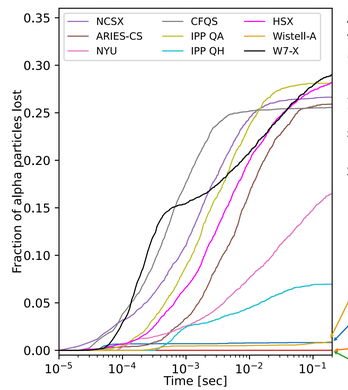
Quasisymmetry is achieved to high precision



The rotational transform turns out strikingly flat



Alpha particle confinement is excellent

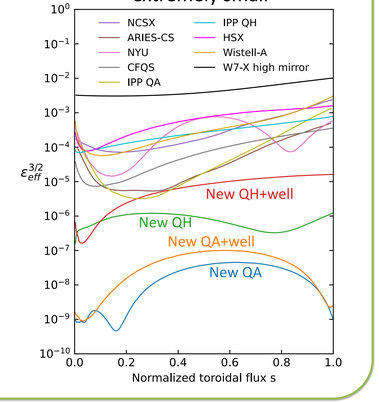


All configurations scaled to ARIES-CS minor radius (1.7 m) and |B| (5.7 T).

5000 alpha particles initialized isotropically at s=0.3.

SIMPLE code [5].

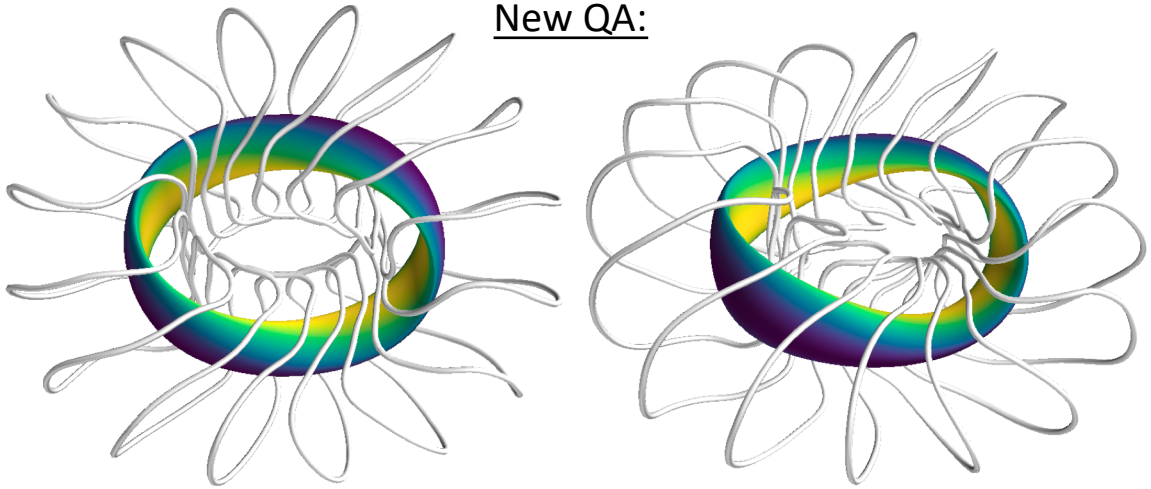
Neoclassial transport is extremely small



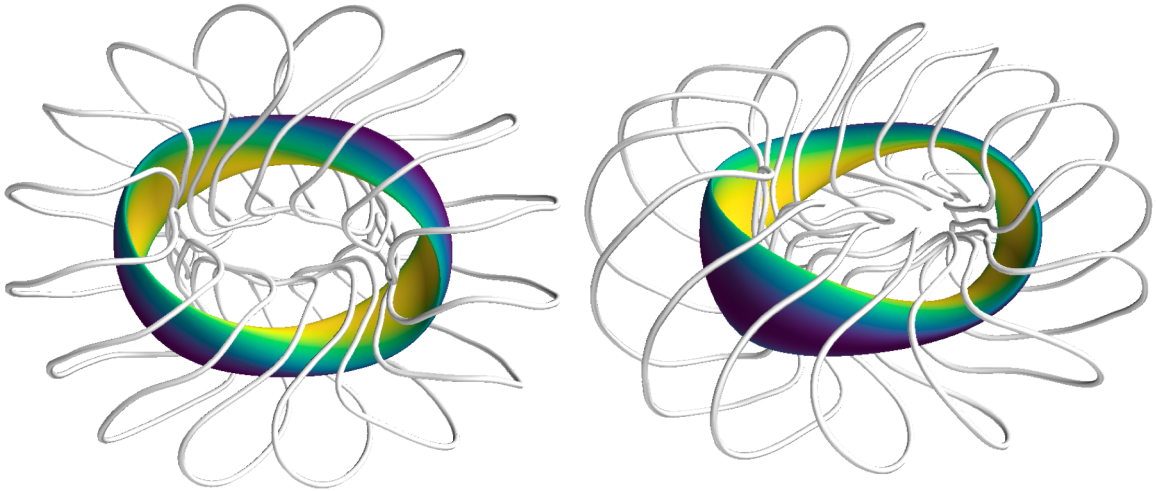
Coils for the new QA configurations

With Florian Wechsung, NYU. Coils optimized using SIMSOPT [3].

New QA:



New QA+well:



<R>/10 between filament centers