Optimized stellarators without optimization

Direct construction of stellarator shapes with good confinement

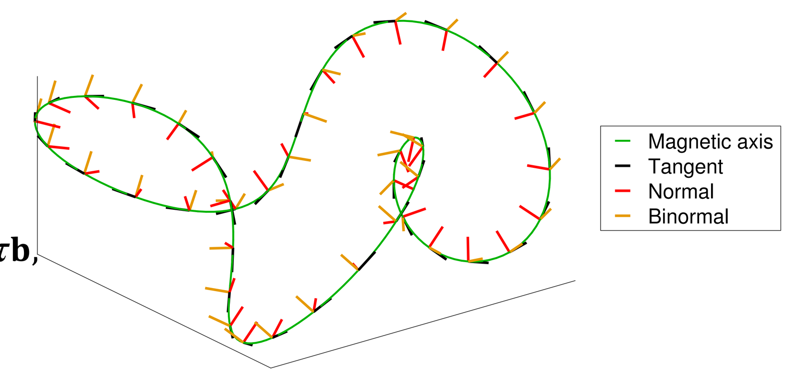
Matt Landreman *(University of Maryland)*, Wrick Sengupta *(NYU)*, Gabriel Plunk *(IPP-Greifswald)*

Using an expansion about the magnetic axis[1,2], we can generate quasisymmetric configurations ~ 10^7x faster than with the conventional optimization approach.

Theory [1, 2]

Setup: Frenet frame $(\mathbf{t},\mathbf{n},\mathbf{b})$ for the magnetic axis:

$$\frac{d\mathbf{x}_0}{d\ell}=\mathbf{t},\quad \frac{d\mathbf{t}}{d\ell}=\kappa\mathbf{n},\quad \frac{d\mathbf{n}}{d\ell}=-\kappa\mathbf{t}+\tau\mathbf{b},\quad \frac{d\mathbf{b}}{d\ell}=-\tau\mathbf{n}$$



$\mathbf{x}_0$ = magnetic axis, κ = curvature, τ = torsion

$\mathbf{t}$ = tangent, $\mathbf{n}$ = normal, $\mathbf{b}$ = binormal, ℓ = arclength

$$\mathbf{x}(r,\theta,\zeta)=\mathbf{x}_0(\zeta)+X(r,\theta,\zeta)\mathbf{n}(\zeta)+Y(r,\theta,\zeta)\mathbf{b}(\zeta)+Z(r,\theta,\zeta)\mathbf{t}(\zeta)$$

$$\mathbf{B}=\beta\nabla\psi+I(r)\nabla\theta+G(r)\nabla\zeta=\nabla\psi\times\nabla\theta+\iota\nabla\zeta\times\nabla\psi$$

Dual relations: $\nabla\psi=(\nabla\psi\cdot\nabla\theta\times\nabla\zeta)\frac{\partial\mathbf{x}}{\partial\theta}\times\frac{\partial\mathbf{x}}{\partial\zeta}$ & permutations.

Expand in $r\propto\sqrt{\psi}$: $\iota(r)=\iota_0+O(r^2)$, $G(r)=G_0+O(r^2)$,

$X(r,\theta,\zeta)=r\left[X_{1s}(\zeta)\sin\theta+X_{1c}(\zeta)\cos\theta\right]+O(r^2)$. Same for Y,Z.

Results, to $O(r^1)$:

$$\frac{d\sigma}{d\zeta}+\iota_0\left[\frac{\bar{\eta}^4}{\kappa^4}+1+\sigma^2\right]-2\frac{\bar{\eta}^2}{\kappa^2}\left[I_2-\tau\right]=0 \qquad (1)$$

I_2 = current density, ι_0 = rotational transform,

$\bar{\eta}$ = some constant: $B=B_0\left[1+r\bar{\eta}\cos\theta+O(r^2)\right]$

Flux surface shape:

$$\mathbf{x}=\mathbf{x}_0(\zeta)+r\frac{\bar{\eta}}{\kappa(\zeta)}\cos\theta\,\mathbf{n}(\zeta)+r\left[\frac{\kappa(\zeta)}{\bar{\eta}}\sin\theta+\frac{\sigma(\zeta)\kappa(\zeta)}{\bar{\eta}}\cos\theta\right]\mathbf{b}(\zeta)+O(r^2)$$

We know *all* solutions,

unlike optimization, where # of local minima is unclear.

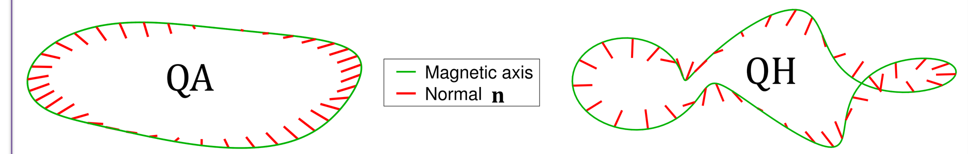
Given $P(\zeta)>0$, $Q(\zeta)$, and $\sigma(0)$, with $P(\zeta)$ and $Q(\zeta)$ 2π-periodic, bounded, and integrable, a solution to

$$\frac{d\sigma}{d\zeta}+\iota\left(P+\sigma^2\right)+Q=0 \qquad (2)$$

is a pair $\{\iota,\sigma(\zeta)\}$ solving (2) where $\sigma(\zeta)$ is 2π-periodic.

Theorem: A solution exists and it is unique [4].

- For every magnetic axis shape with nonvanishing curvature, and 3 real numbers ($\bar{\eta}$, σ(0), and I_2), there is precisely 1 quasisymmetric configuration. Also,
  - For stellarator symmetry, σ(0) = 0.
  - Typically current density on axis I_2 = 0.
- Since $\nabla_\perp B=B\kappa\mathbf{n}$ this solution is quasi-axisymmetric or quasi-helically symmetric depending on whether **n** loops around the axis poloidally when you follow the axis toroidally:



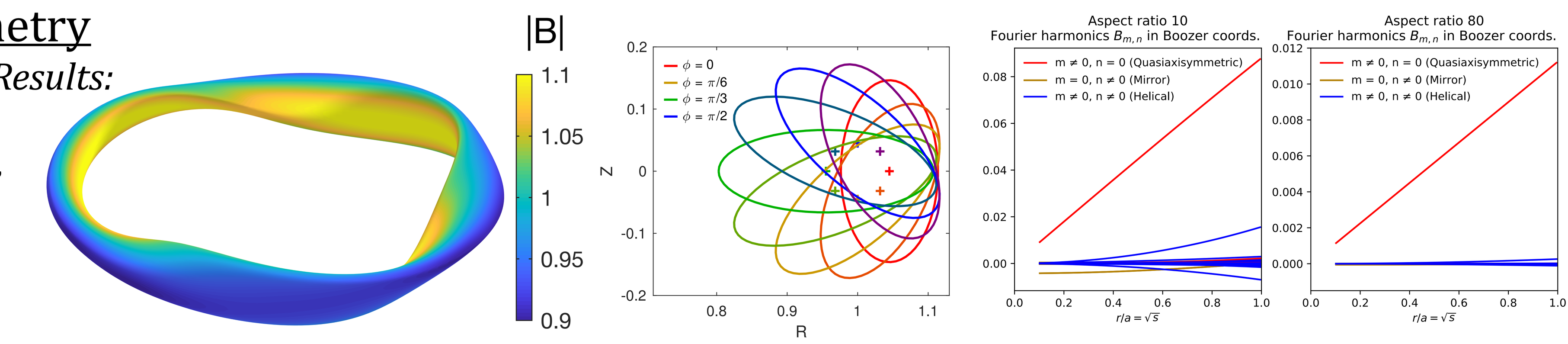
Quasisymmetry can be verified with traditional codes

Quasi-axisymmetry

Inputs: axis shape:

$R_0(\phi)=1+0.045\cos(3\phi)$,
$Z_0(\phi)=-0.045\sin(3\phi)$,
$\bar{\eta}=-0.9$, $\sigma(0)=0$
$R/a=10$

Results:

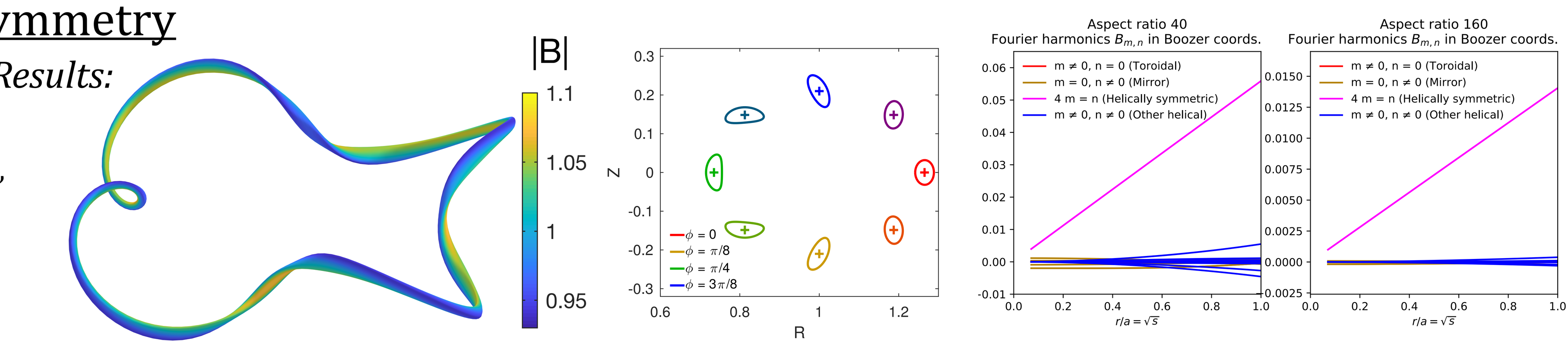


Quasi-helical symmetry

Inputs: axis shape:

$R_0(\phi)=1+0.265\cos(4\phi)$,
$Z_0(\phi)=-0.21\sin(4\phi)$,
$\bar{\eta}=-2.25$, $\sigma(0)=0$
$R/a=40$

Results:



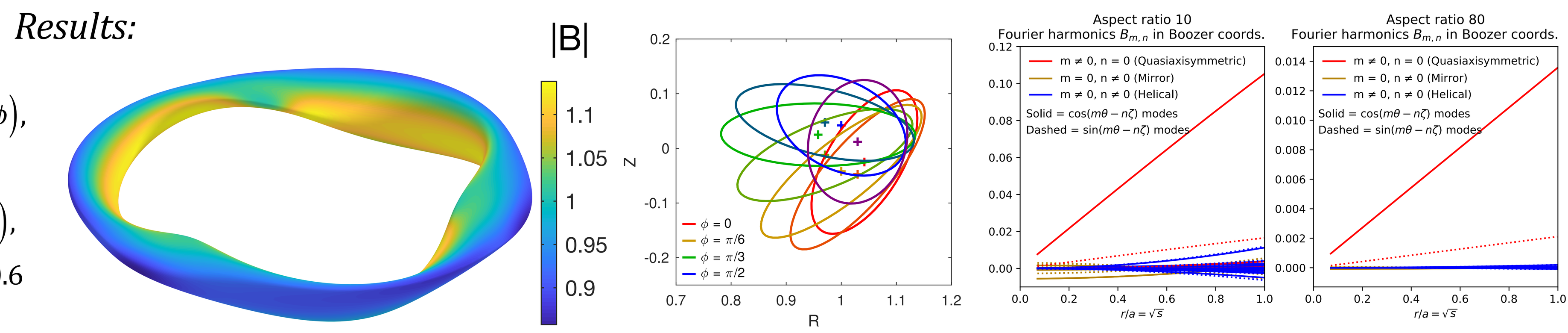
Quasi-axisymmetry without stellarator symmetry

May have large intrinsic rotation?

Inputs: axis shape:

$R_0(\phi)=1+0.042\cos(3\phi)$,
$Z_0(\phi)=-0.042\sin(3\phi)$
$-0.025\cos(3\phi)$,
$\bar{\eta}=-1.1$, $\sigma(0)=-0.6$
$R/a=10$

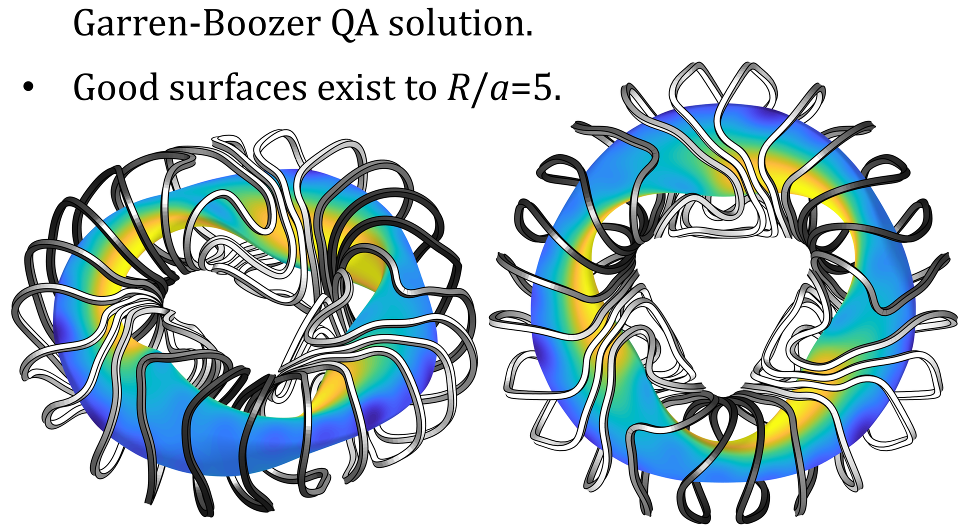
Results:

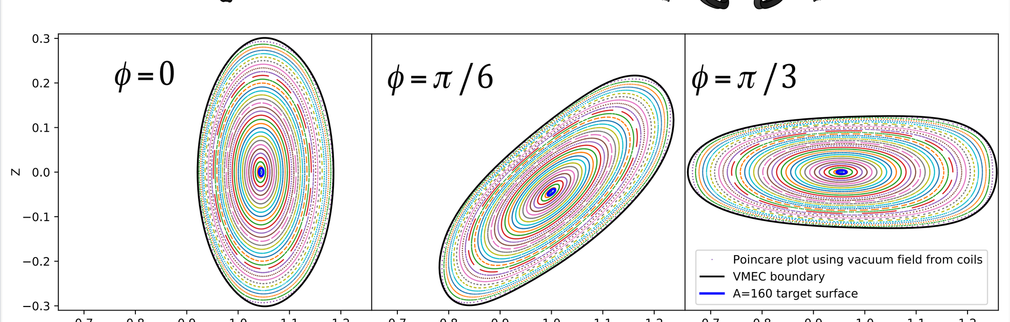


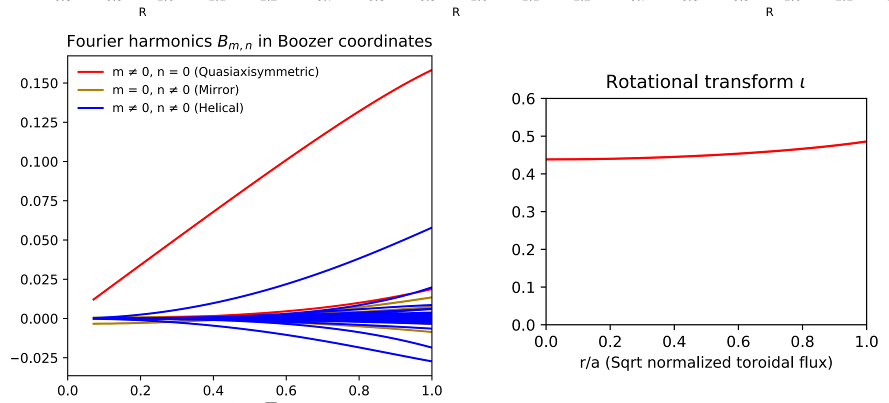
Are there better ways to make aspect ratio finite?

Instead of plugging a finite *r* into expansion,

- Compute coil shapes that produce the *R*/*a*=160 Garren-Boozer QA solution.
- Good surfaces exist to *R*/*a*=5.

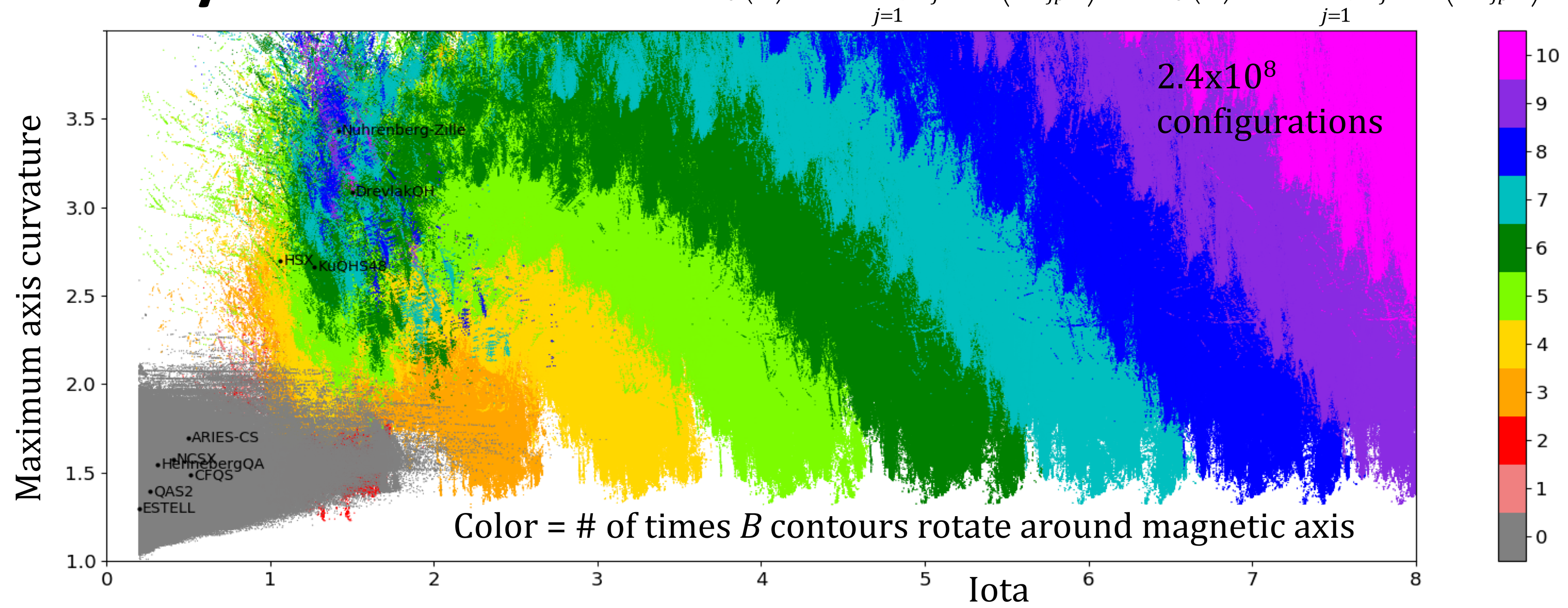


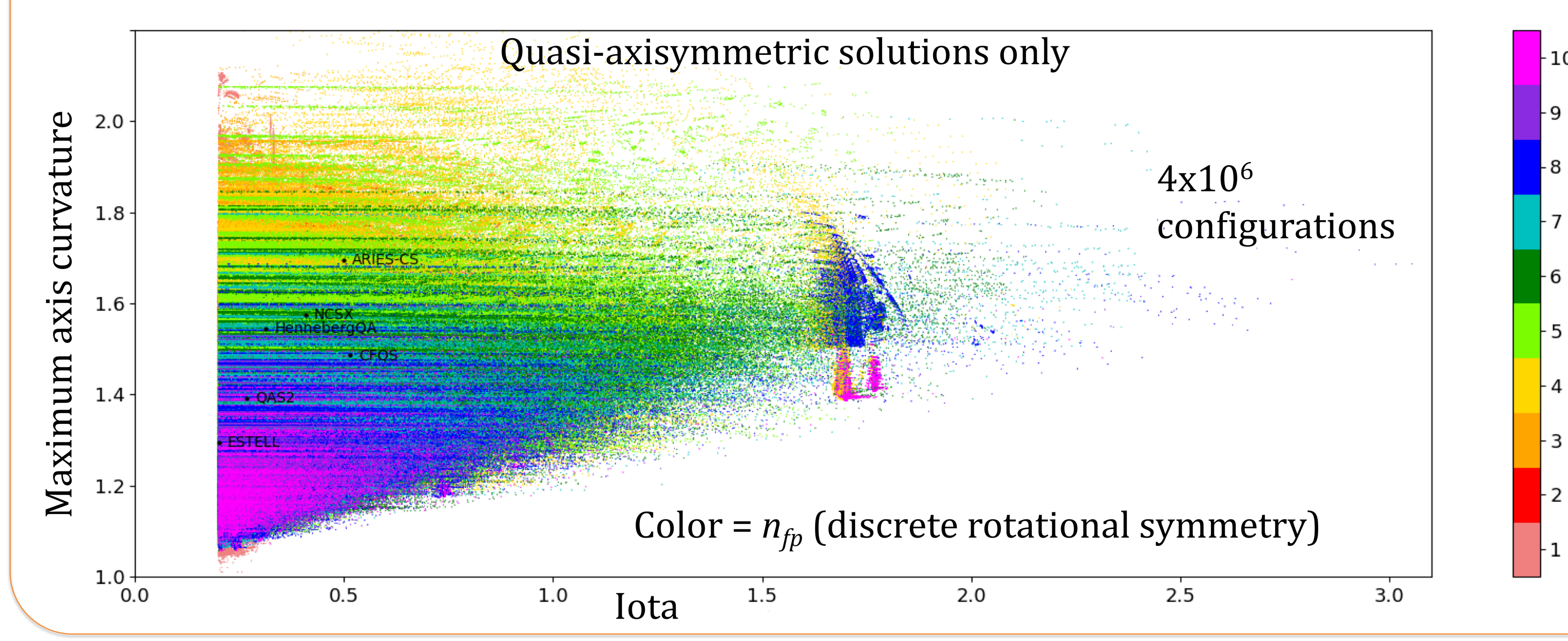




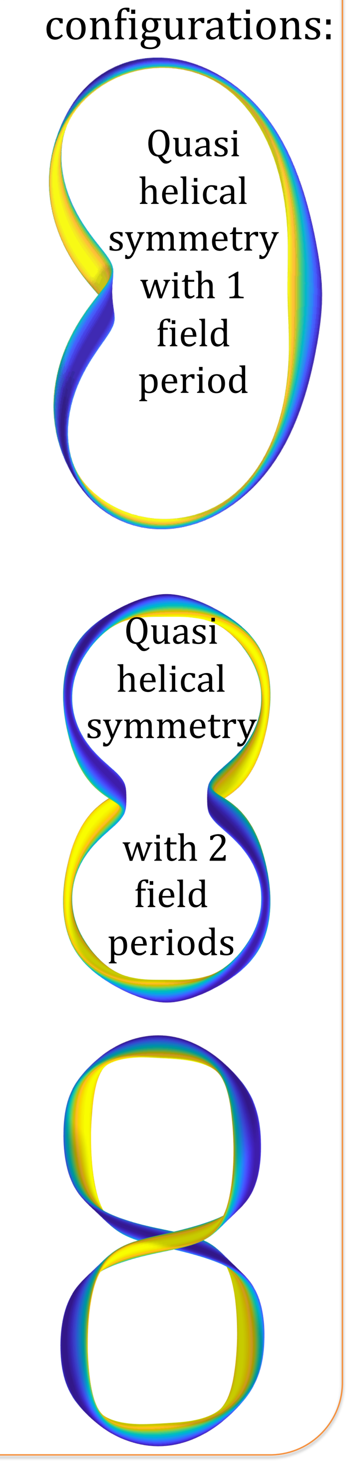
Survey of solutions

Axis: $R_0(\phi)=1+\sum_{j=1}^{3}R_j\cos(jn_{fp}\phi)$, $Z_0(\phi)=1+\sum_{j=1}^{3}Z_j\sin(jn_{fp}\phi)$





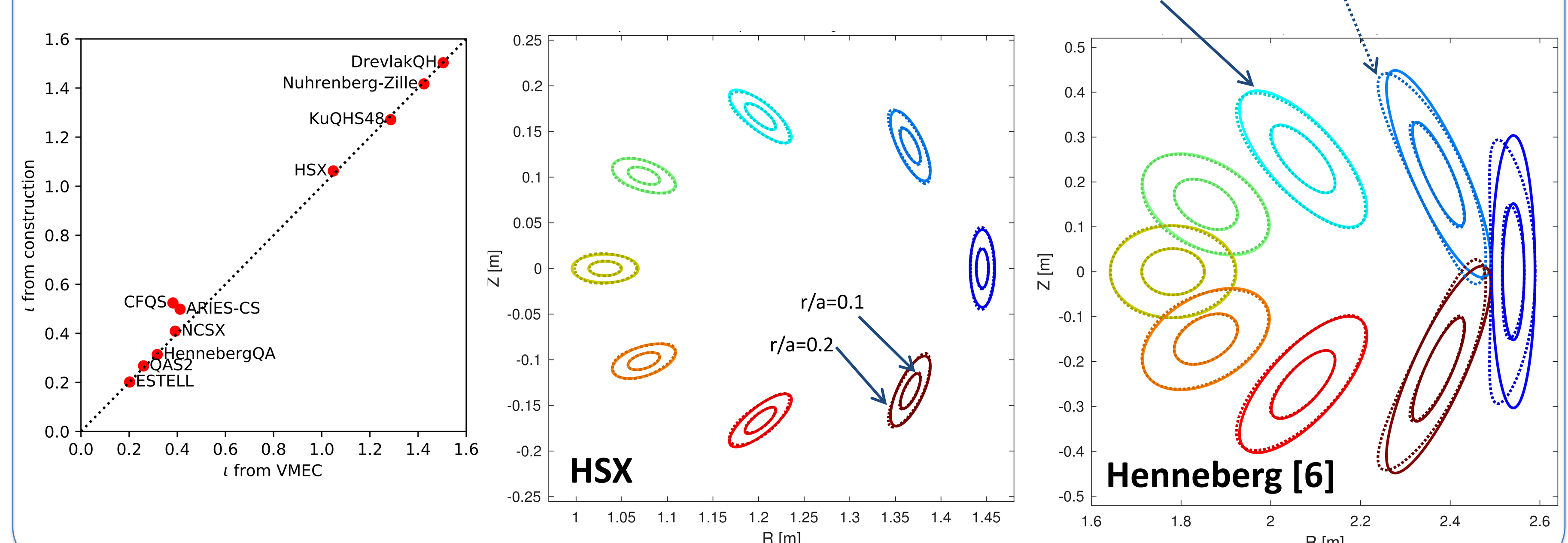
Novel configurations:



The construction is consistent with optimization [5]

Just fit a single number: $\bar{\eta}$

Dashed: Optimized VMEC configuration
Solid: Best-fit construction



Future Directions

- Use construction to generate initial conditions for stellopt.
- Optimize in the space of axis shapes.
- Can we construct shapes with quasisymmetry imposed at a mid-radius surface?
- Construction for omnigenity [7].
- Go to 2nd order in distance from the axis.
- Evaluate the difficulty of producing these plasma shapes with distant coils.

References

[1] D A Garren & A H Boozer, *Phys Fluids B* **3**, 2805 (1991).

[2] D A Garren & A H Boozer, *Phys Fluids B* **3**, 2822 (1991).

[3] M Landreman & W Sengupta, *J Plasma Phys* **84**, 905840616 (2018).

[4] M Landreman, W Sengupta, & G G Plunk, *J Plasma Phys* 85, 905850103 (2019).

[5] M Landreman, arXiv:1902.01672

[6] S Henneberg et al, *Nucl Fusion* **59**, 026014 (2019).

[7] G G Plunk & M Landreman, *in preparation.*

Supported by DE-FG02-93ER54197, DE-FG02-86ER53223, & the Simons Foundation