Overview

• Several important phenomena – the bootstrap current and collisional transport in stellarators, and neoclassical tearing modes (NTM) in tokamaks – must be computed by numerical solution of the drift kinetic equation (DKE) in toroidal geometry.
• Need time-accurate (steady state) solution, like as a poloidal field.
• High resolution is required at least 3 dimensions (poloidal angle, toroidal angle, pitch angle) due to internal boundary layers.
• Existing continuum solvers (e.g. DKES [1] and STAC [3]) have used a direct solver for at least these 3 dimensions, which scales poorly with resolution: large memory and time required for high resolution.
• Multigrid methods [4,5] are state-of-the-art for solving PDEs and can be optimally scaled. However, multigrid is not straightforward for advection-dominant problems like the DKE.
• Here we develop a multigrid solver for the DKE.

(PDEs, Variants with differing complexity:

- Essential goal (as solved in STAC95 code) [2]:
  \[ \frac{1}{\omega} \frac{\partial}{\partial t} \phi + \mathbf{v} \cdot \nabla \phi + \nabla \cdot (D \nabla \phi) = f + \sum_{m} \int \rho_m \nabla \cdot (D_m \nabla \psi_m) \, d\tau \]

- Simpler versions are 2D or 3D (monokinetic tokamak or stellarator), Essential goal is in 5D.
- Steady (time-independent).

Minimal accurate version:

- Monomeric, 1 species, \( E = 0 \):
  \[ \frac{1}{\omega} \frac{\partial}{\partial t} \phi + \mathbf{v} \cdot \nabla \phi + \nabla \cdot (D \nabla \phi) = f \]
  Specified: \( \mathbf{v} \) (and \( \mathbf{v} \cdot \nabla \phi \) and \( \nabla \cdot (D \nabla \phi) \))

- Asymmetry:
  \[ \frac{1}{\omega} \frac{\partial}{\partial t} \phi + \mathbf{v} \cdot \nabla \phi + \nabla \cdot (D \nabla \phi) = f + \sum_{m} \int \rho_m \nabla \cdot (D_m \nabla \psi_m) \, d\tau \]
  \( \psi_m \) is a Steenbeck solution (electron density)

- Solution has internal boundary layers.
- Small distance problem (radial)