Optimization of the Magnetic Gradient Scale Length[1] and its Influence on Plasma-Coil Separation in Stellarators

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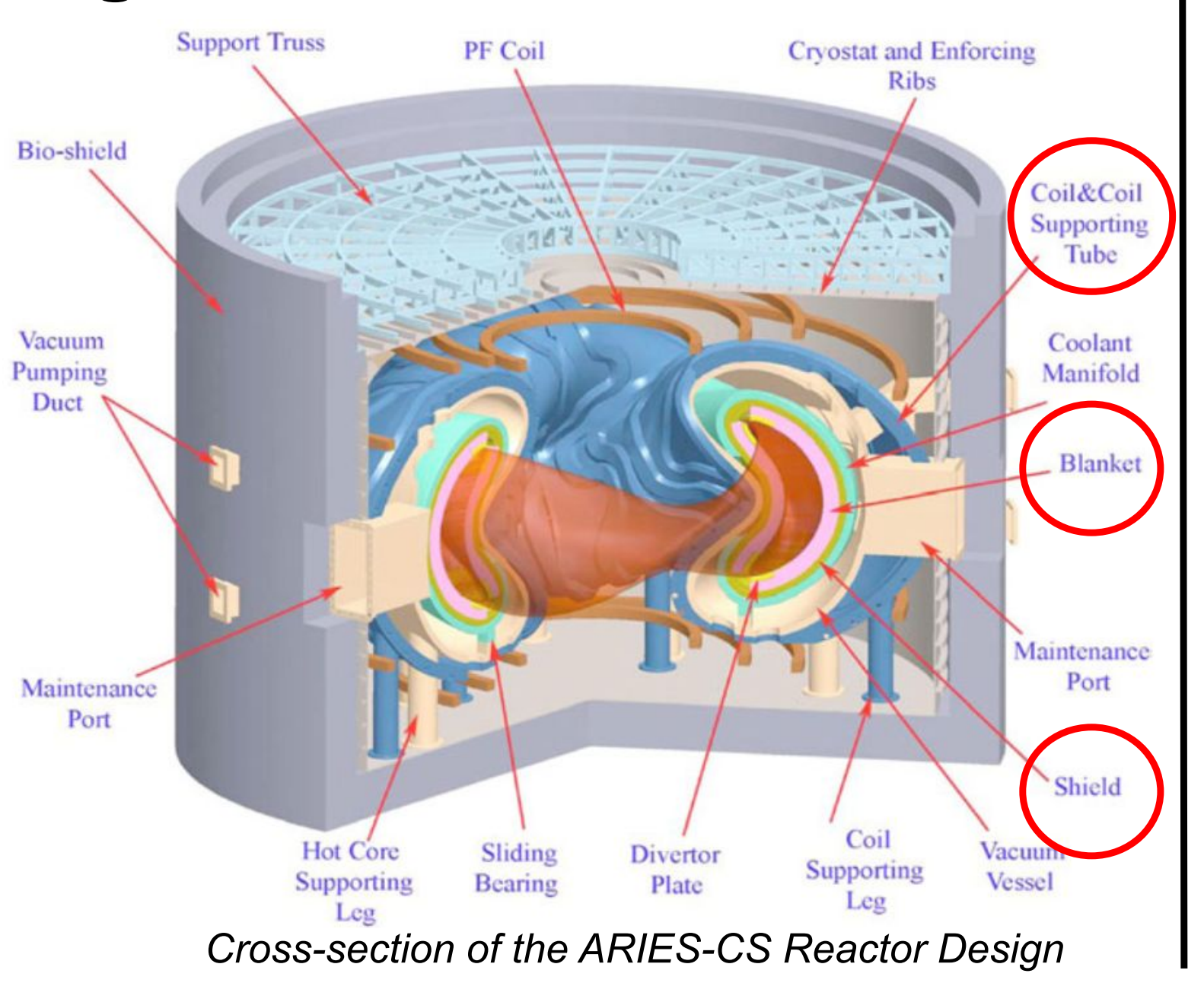
Stellarators Need Space for a Breeding Blanket & Neutron Shielding

During the design of ARIES-CS and W7-X, both configurations experienced engineering issues related to the space between the last closed flux surface and the external coils.[2][3]

This "plasma-coil separation" must be > 1.5m to have enough room for neutron shielding and a blanket.

Larger plasma-coil separation reduces coil ripple, accommodates for shifts during startup and initialization, and can allow larger configurations to be scaled down.



Cross-section of the ARIES-CS Reactor Design

The Magnetic Gradient Scale Length[1] Approximates Plasma - Coil Separation

For a current carrying infinite straight wire, $L_{\nabla \mathbf{B}}$ is equal to the distance between the magnetic field and the wire. Therefore, by measuring the magnetic field and its gradient, we can determine where the nearest wire must be located to create the magnetic field.

$$\nabla \mathbf{B} = \begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_y}{\partial x} & \frac{\partial B_z}{\partial x} \\ \frac{\partial B_x}{\partial y} & \frac{\partial B_y}{\partial y} & \frac{\partial B_z}{\partial y} \\ \frac{\partial B_x}{\partial z} & \frac{\partial B_y}{\partial z} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

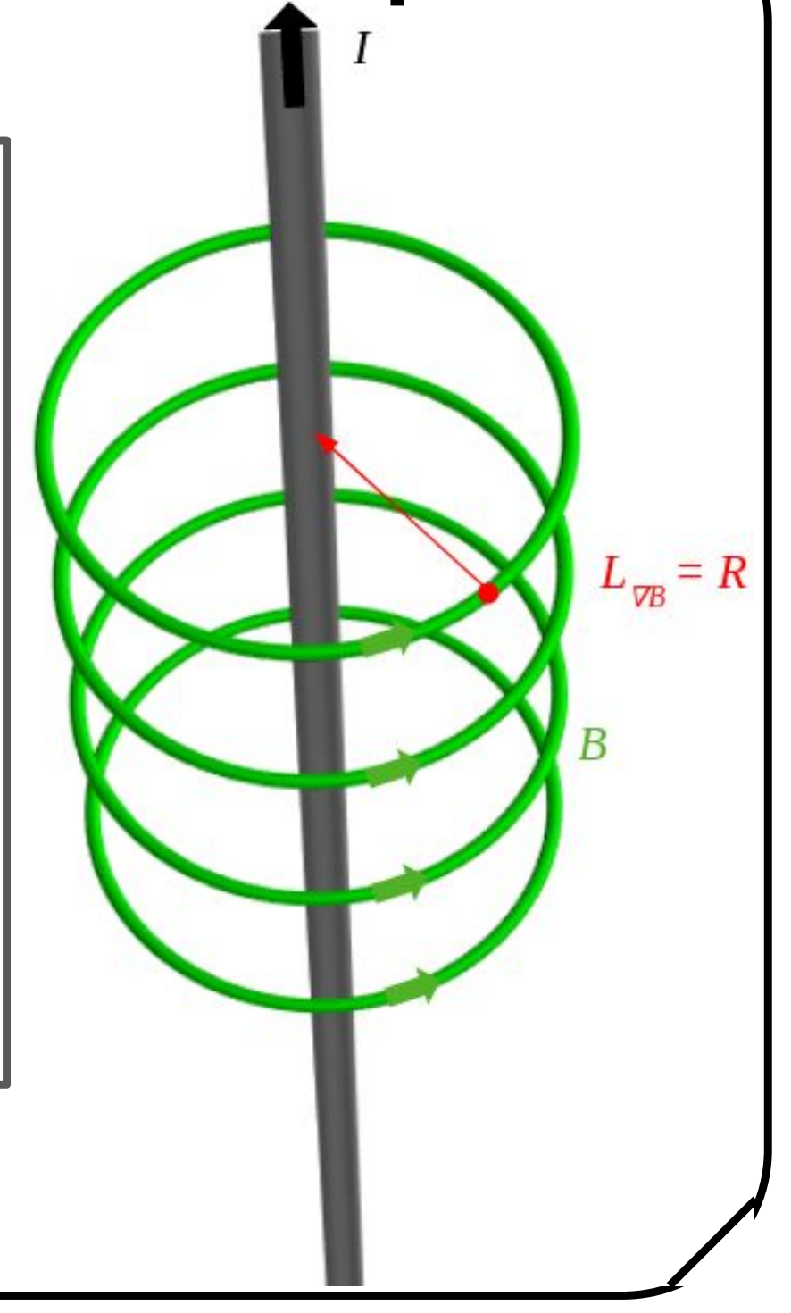
$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} |a_{ij}|^2}$$

$$\mathbf{B}(R) = \frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{\phi}}$$

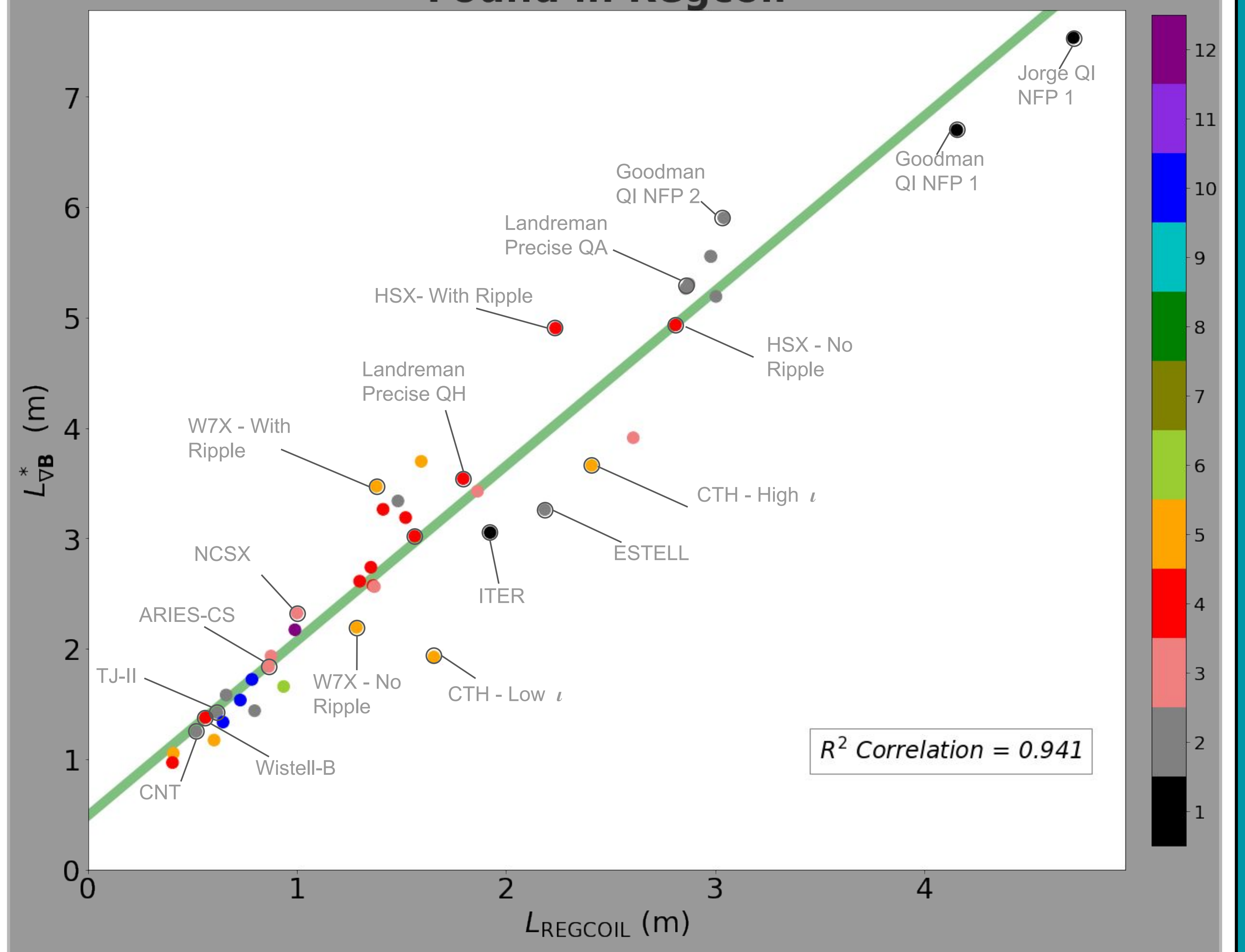
$$\nabla \mathbf{B} = -\frac{\mu_0 \iota}{2\pi R^2}(\hat{\boldsymbol{\phi}}\hat{\mathbf{R}} + \hat{\mathbf{R}}\hat{\boldsymbol{\phi}}),$$

$$\|\nabla \mathbf{B}\|_F = \frac{\sqrt{2}\mu_0 I}{2\pi R^2},$$

$$\boxed{L_{\nabla \mathbf{B}} = \sqrt{2}\frac{B}{\|\nabla \mathbf{B}\|_F} = R.}$$



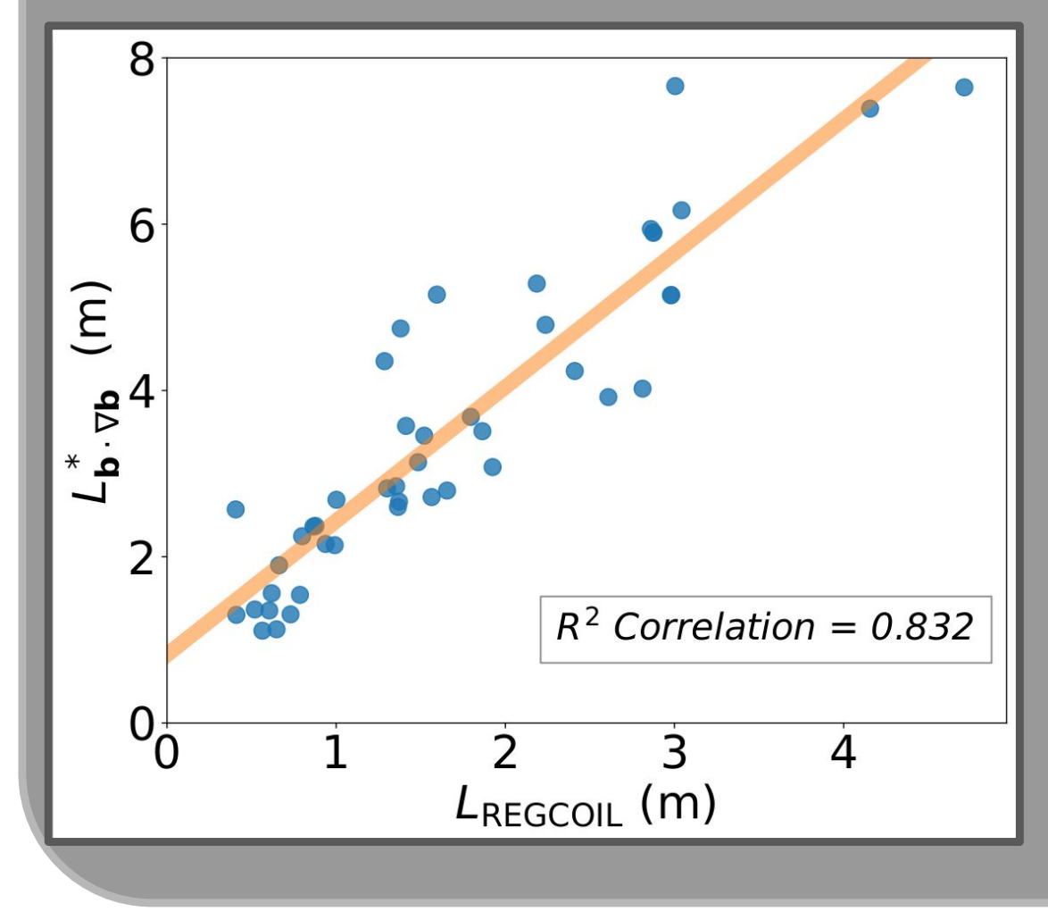
$L^*_{\nabla \mathbf{B}}$ Accurately Predicts Coil-Plasma Separation Found in Regcoil

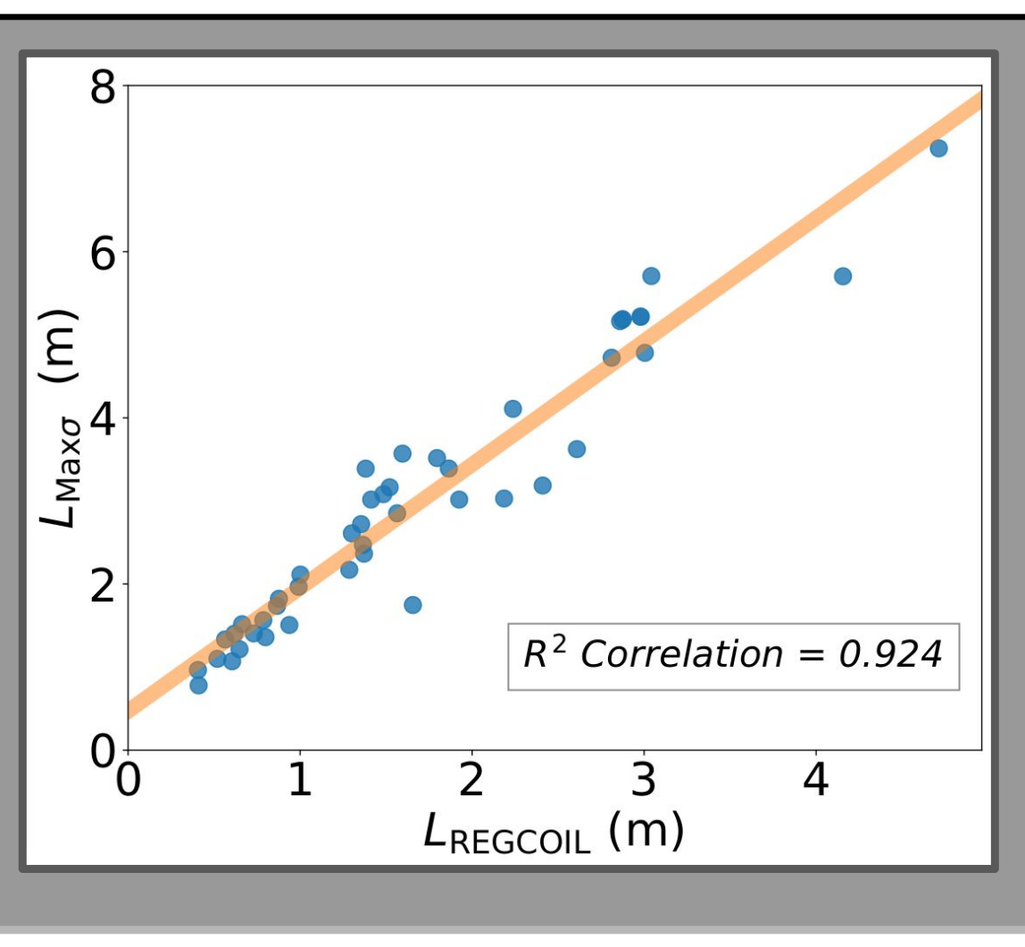


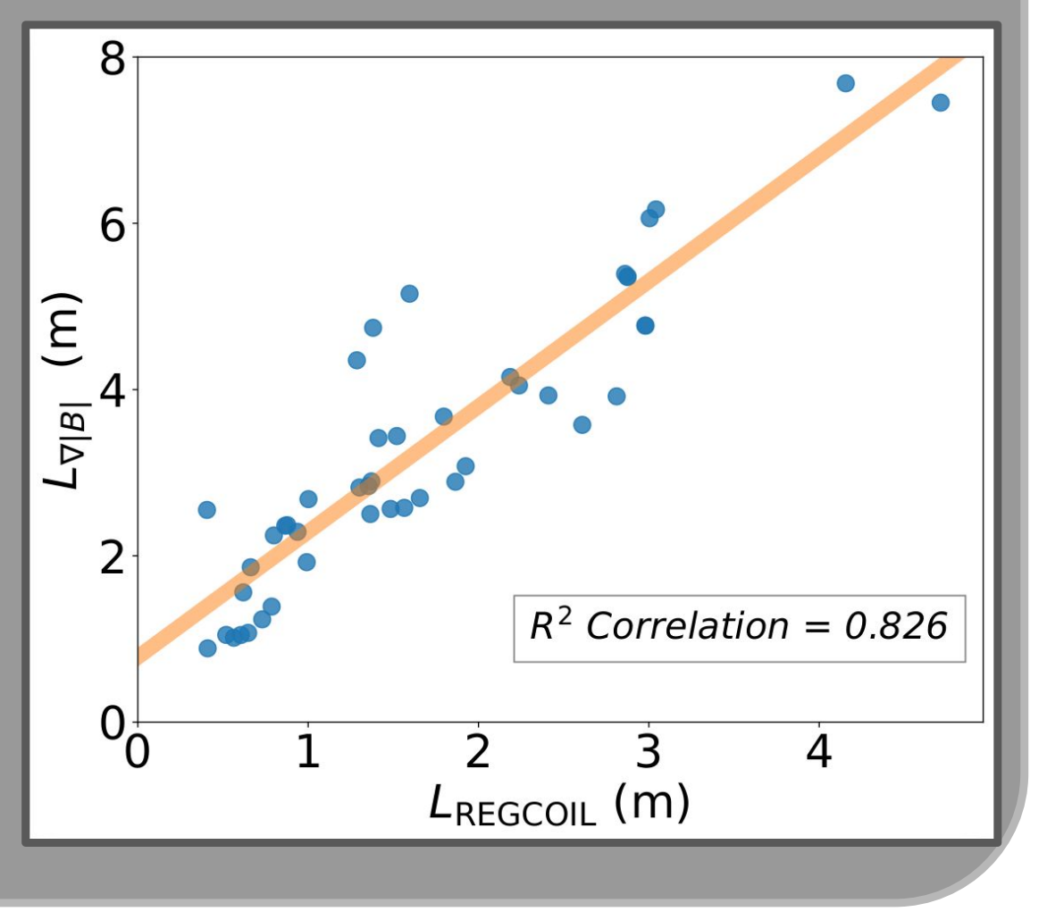
Parameters in RECOIL & VMEC	$\|K\|^*_\infty$ = 17.16 MA/m	B_{Vol} = 5.865 T	m_θ & n_ζ = 96
	a = 1.704 m	B^*_{RMS} = 0.01 T	mpol & ntor = 20

(*Summary of 2023 poster*) We had gathered database of > 40 stellarator and tokamak configurations. Within this database, the coil-to-plasma distance compared to the minor radius varies by over an order of magnitude. **The magnetic scale length** is well correlated to **the coil-to-plasma distance** of actual coil designs generated using the `REGCOIL` method.[4]

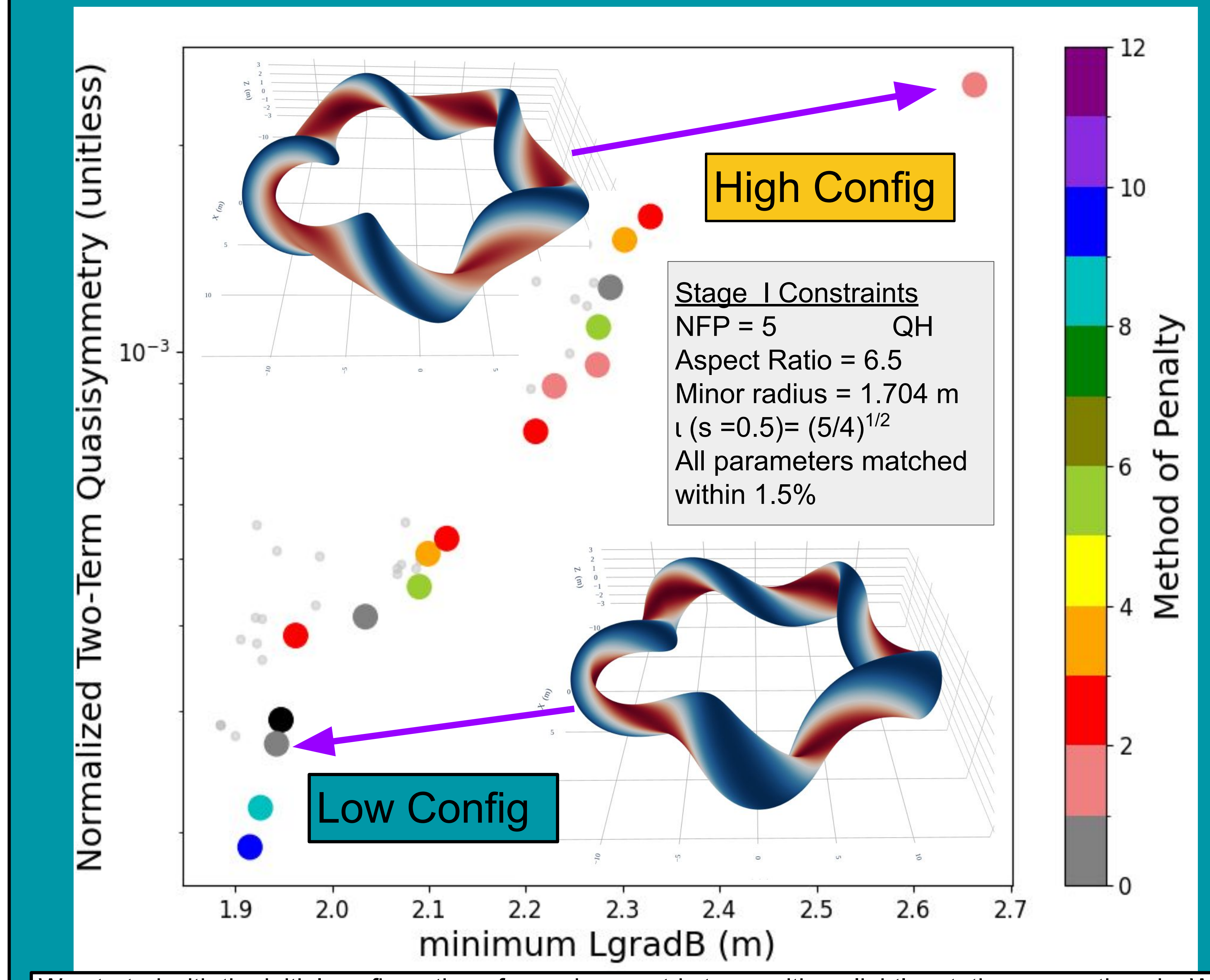
Below, we have plotted alternative scale lengths, which are also correlated with the coil-to-plasma distance.







Tradeoff Between QS and LgradB in 5 Nfp QH



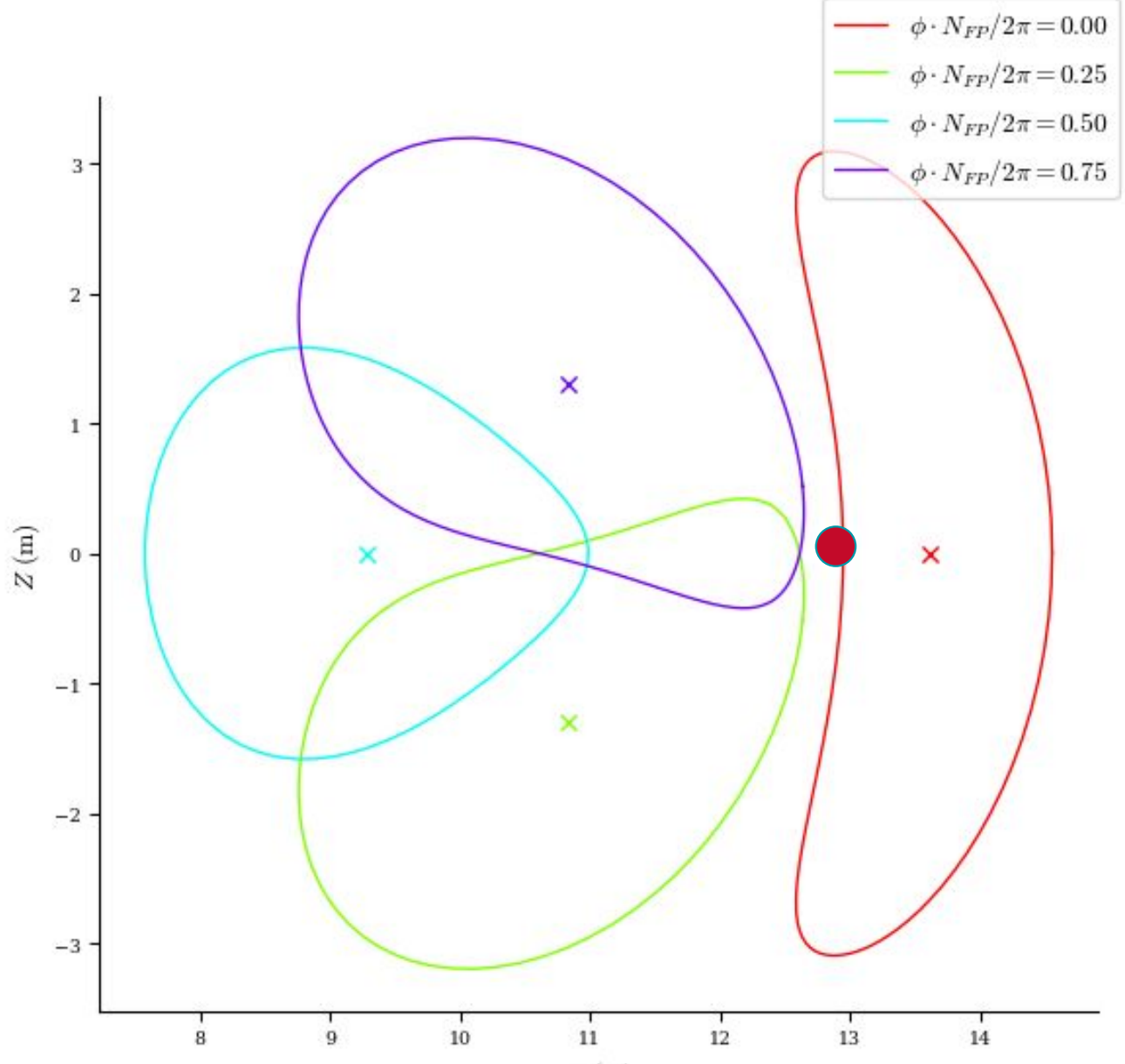
Stage I Constraints
NFP = 5 QH
Aspect Ratio = 6.5
Minor radius = 1.704 m
ι (s =0.5)= $(5/4)^{1/2}$
All parameters matched within 1.5%

We started with the initial configuration of an axisymmetric torus with a slightly rotating magnetic axis. We ran stage I optimization via DESC[5] on the initial configuration with the following objective function:

$$f = f_{QS} + (a - a^*)^2 + (\iota - \iota*)^2 + w_{L\nabla \mathbf{B}}(f_{L\nabla \mathbf{B}})^2$$

See additional slides

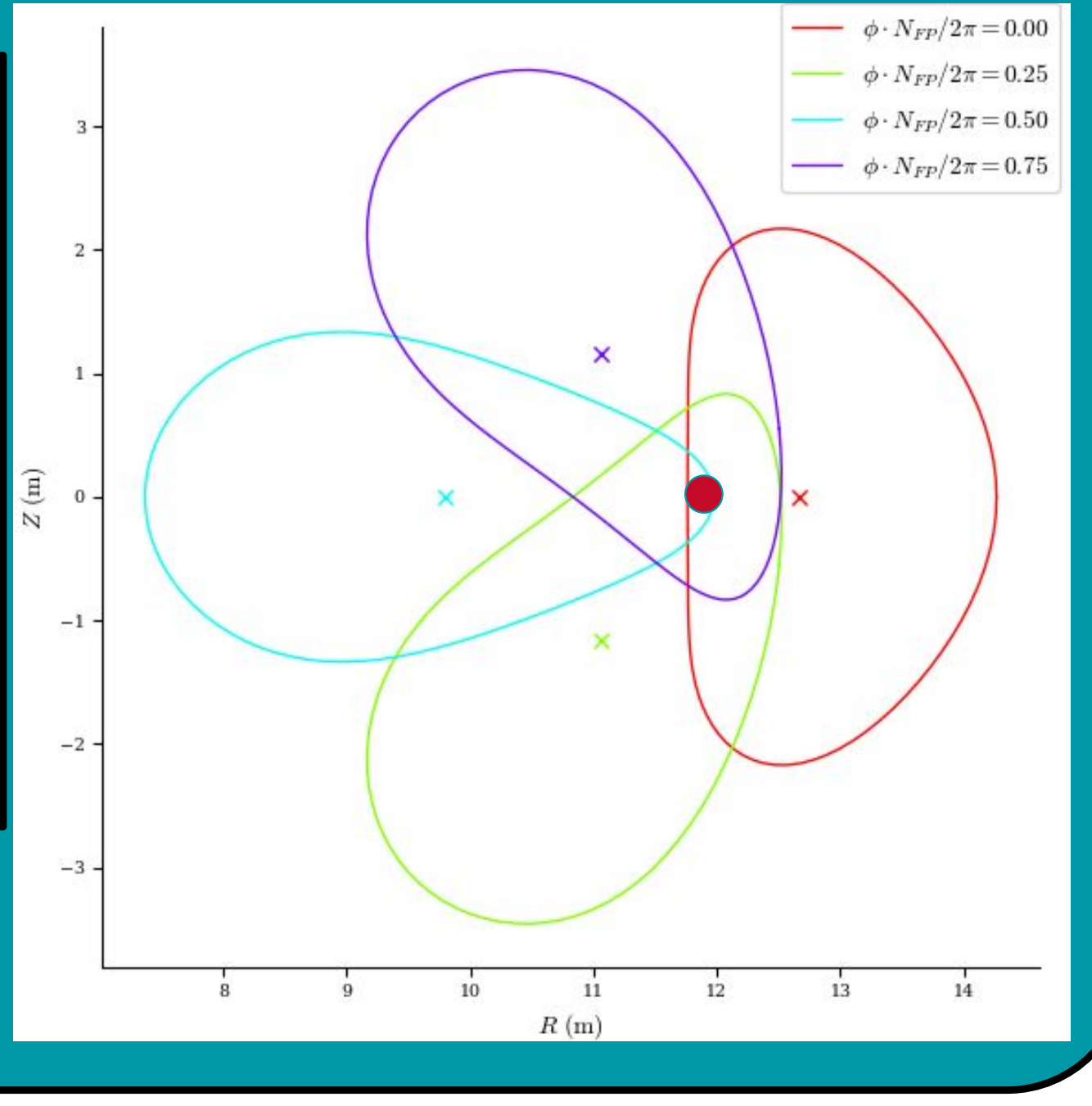
Filtering configurations more than 1.5% away from the target, minor radius, aspect ratio, and rotational transform, we end up with a family of optimized configurations showcase a clear tradeoff between quasisymmetry and the minimum$L_{\nabla \mathbf{B}}$. on last closed flux surface, as shown above.



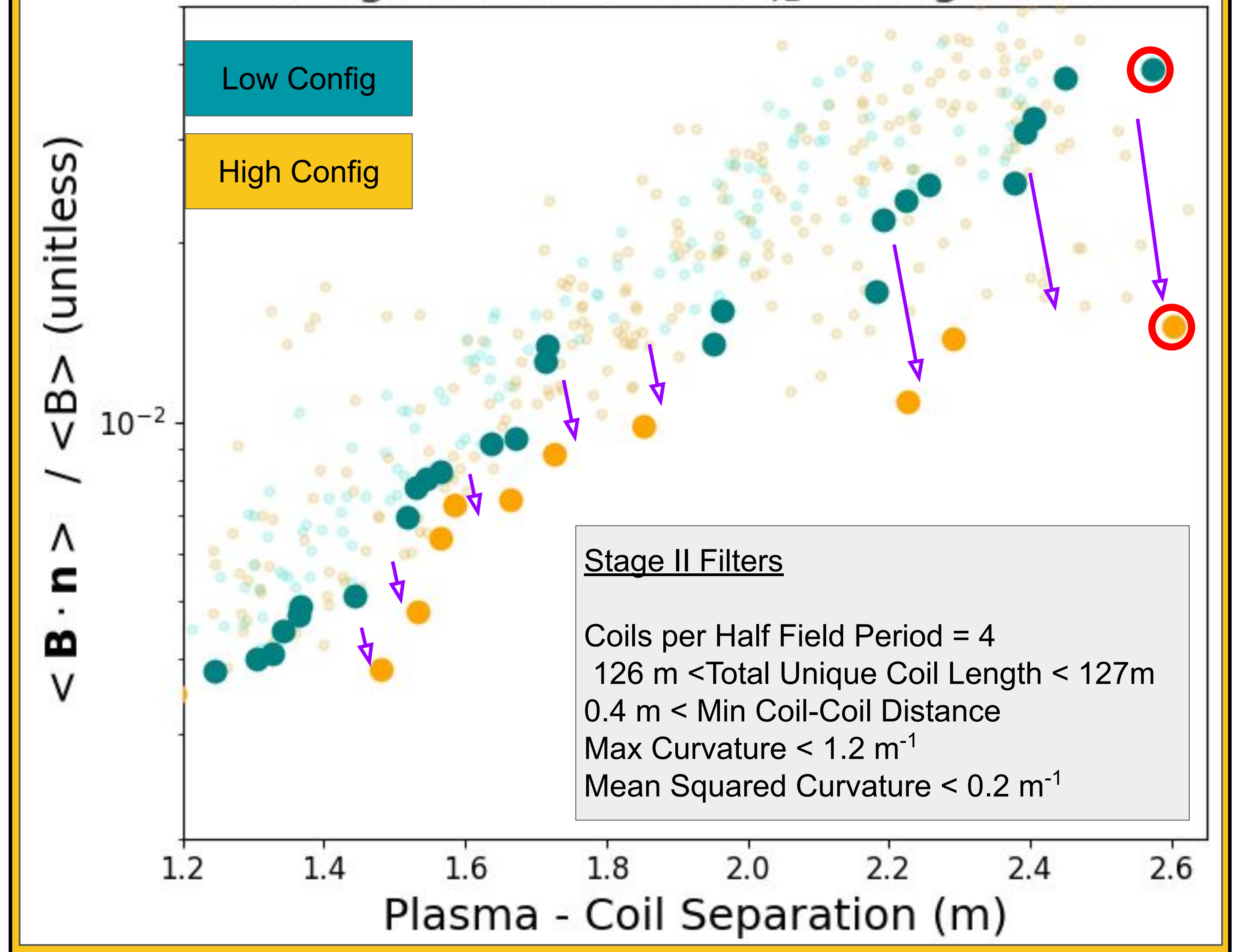
$L_{\nabla \mathbf{B}}$ smallest at the red dot on the inside of the "bean" cross section. The innermost cross section "stretches" to compensate for the reduction in the bean shape. $L_{\nabla \mathbf{B}}$ is **still** in the bean cross section on the High configuration

Low

High



The Pareto Front of Stage II Coils is better in a High Vs. a Low min $L_{\nabla \mathbf{B}}$ Configuration



Stage II Filters

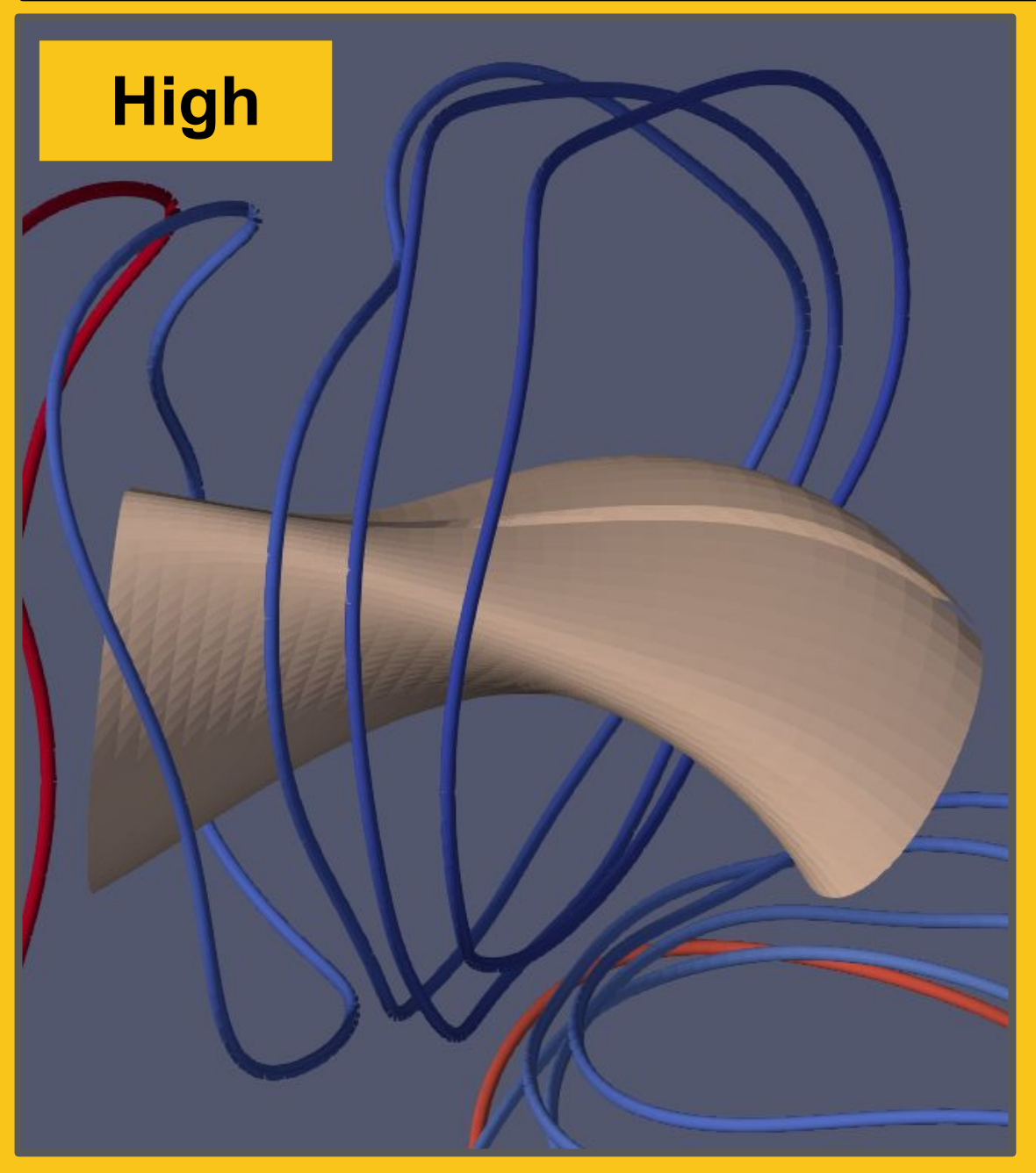
Coils per Half Field Period = 4
126 m <Total Unique Coil Length < 127m
0.4 m < Min Coil-Coil Distance
Max Curvature < 1.2 m^{-1}
Mean Squared Curvature < 0.2 m^{-1}

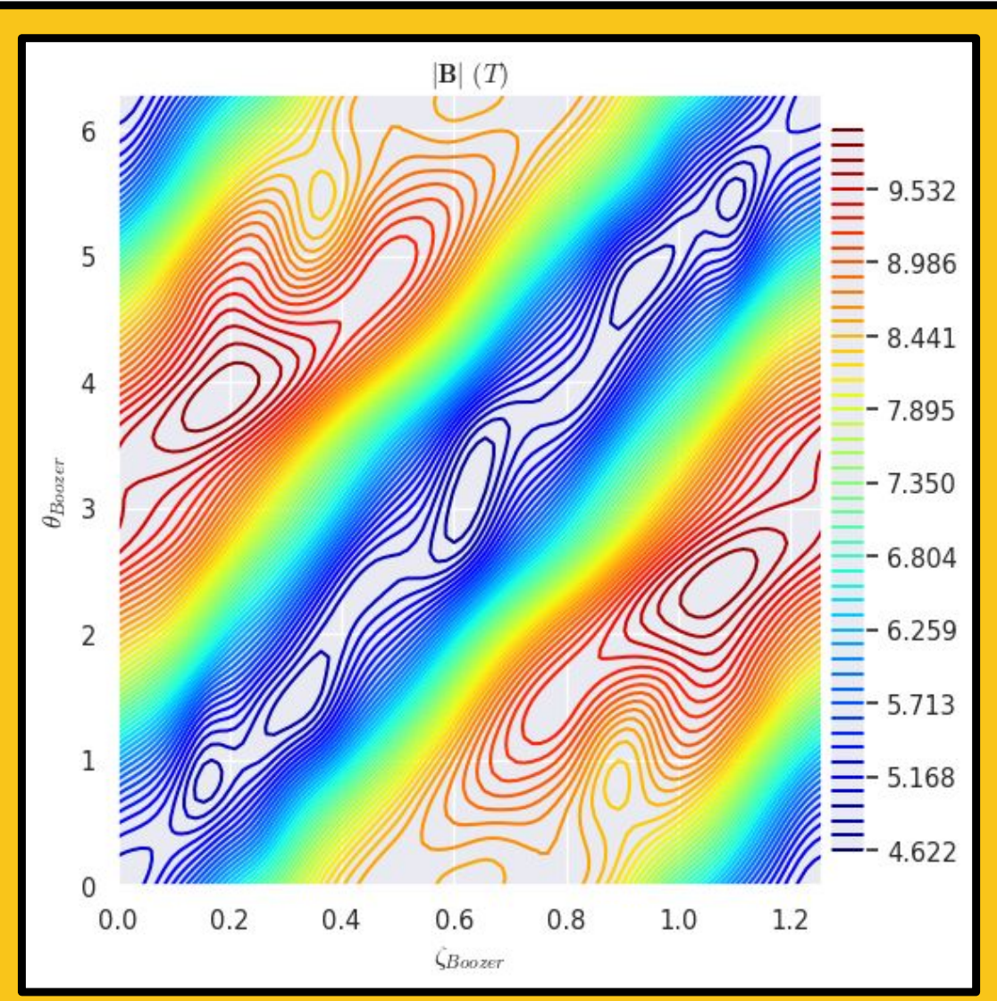
On both the Low and High Configurations, we generated coils using SIMSOPT's[6] Stage II filamentary optimization method using the following objective function:

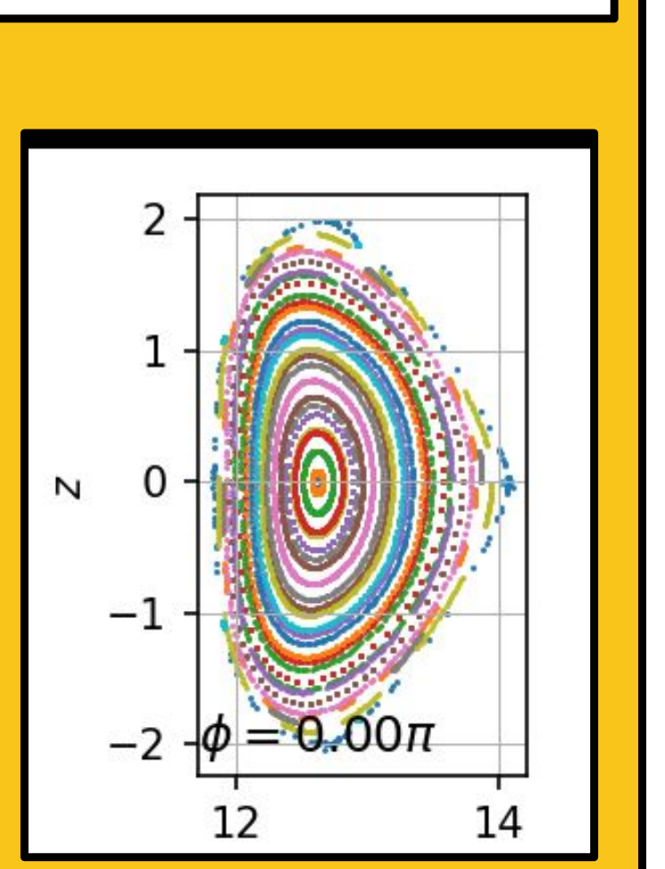
$$f = \left(\int (\mathbf{B}\cdot n)\right)^2 + w_l\left(\min(0, l - l_{max})^2 + \max(0, l - l_{min})^2\right) + w_s\left(\max(0, s_{pc} - s^*_{pc})^2\right)$$
$$+ w_d\left(\max(0, d_{cc} - d^*_{cc})^2\right) + w_{max}\left(\min(0, \kappa_{max} - \kappa^*_{max})^2\right) + w_{msc}\left(\min(0, \kappa_{msc} - \kappa^*_{msc})^2\right)$$
$$+ w_{\text{arc}}\text{Var}(\text{arclength}(\text{coils}))^2 + +w_{\text{link}}\text{LinkingNumber}(\text{coils})^2$$

Above the results are shown. Over 2900 coils were run with randomized weights and thresholds. Of those, only the 451 filtered coils are show, which have good coil complexity as shown on the table in the figure. At large plasma-coil separations, High coil sets can achieve better <**B** ·n>/<B> compared to the Low configuration. Two coils (circled in red above) are showcased below, with respective Boozer and Poincare plots.

High

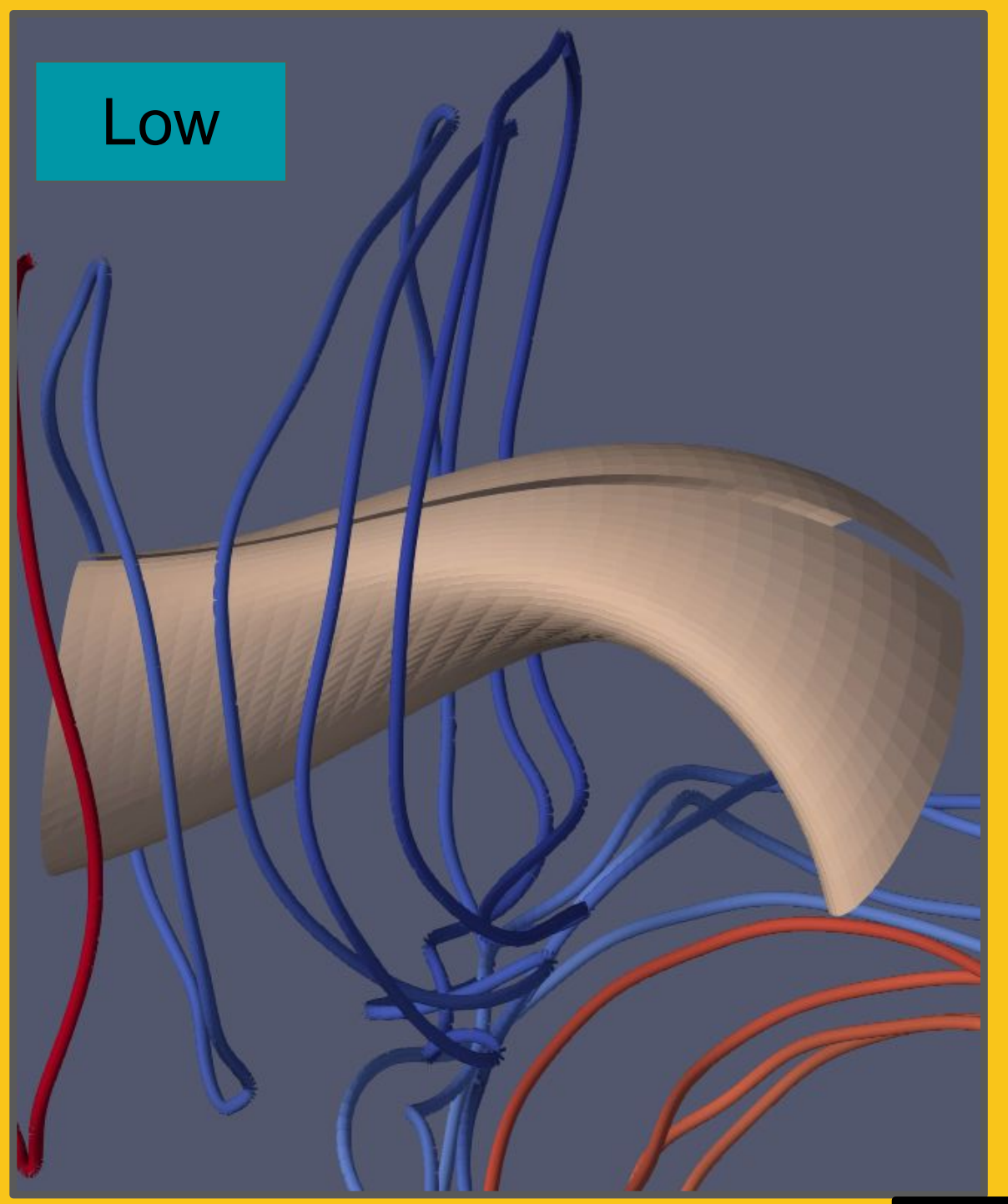


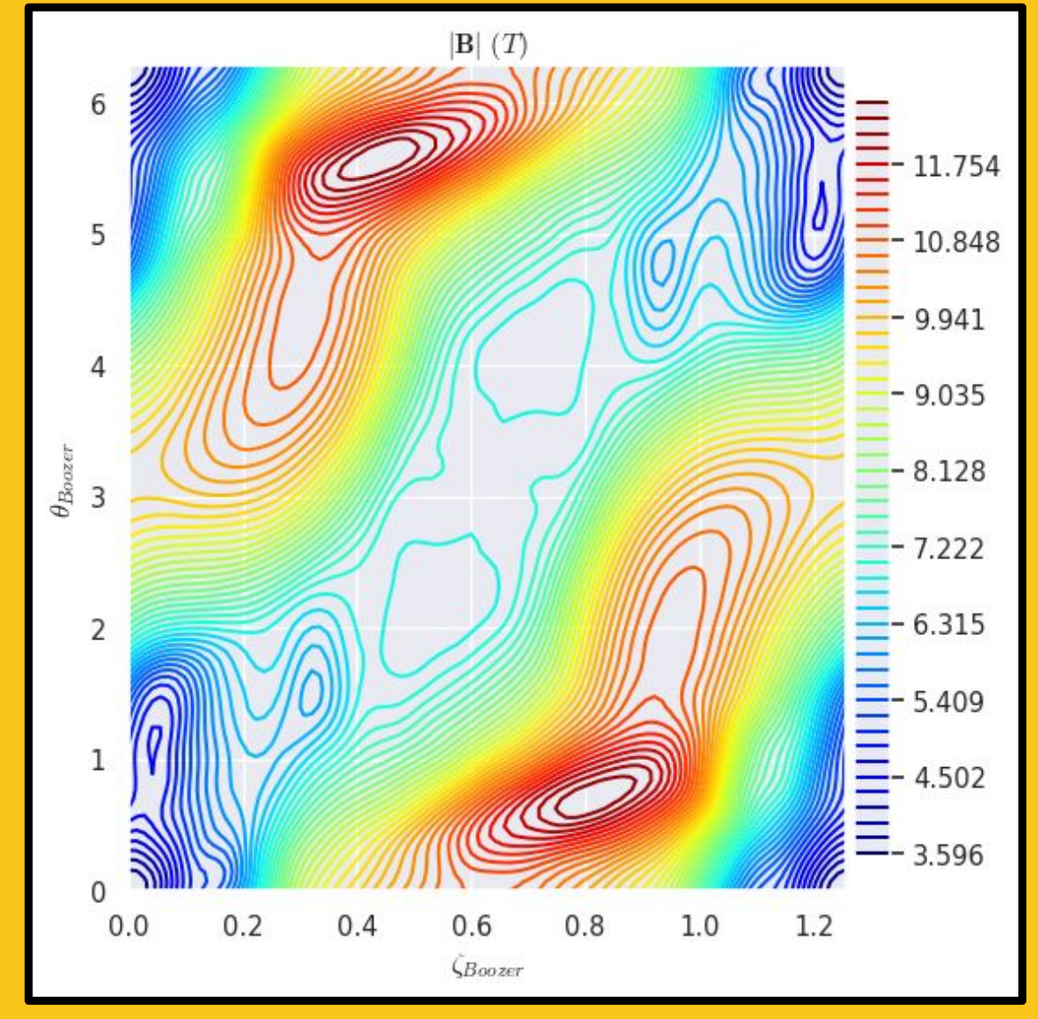


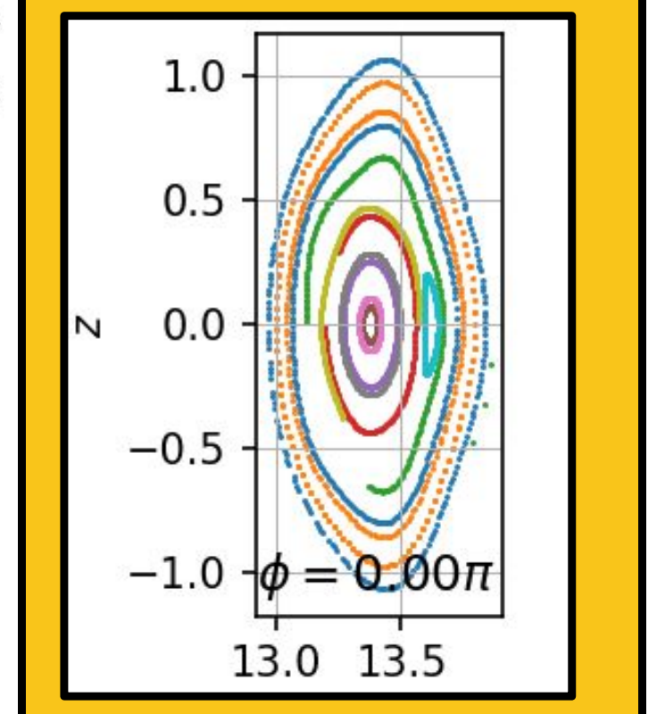


See additional slides for data on coils complexity, QS error, and free boundary equilibria.

Low







Summary and Future Work

We are able to get (a) better quasi-symmetry error and (b) easier to build coils that are (c) slightly farther away from the plasma using $L_{\nabla \mathbf{B}}$. However, there are many ways confinement could be improved. This is because **both** the stage I trade-off and the stage II trade-off (depicted in the graphs above) impact **both** quasisymmetry, coil complexity, and plasma-coil separation. We plan on further comparing the configurations and coils generated by these trade off curves, and directly measuring confinement of the alpha particles using SIMPLE. It is likely the procedures for stage I and for stage II optimization can be improved, allowing us to improve confinement further, such as by adding a magnetic well.