The Magnetic Gradient Scale Length Explains Why Certain Plasmas Require Close External Magnetic Coils

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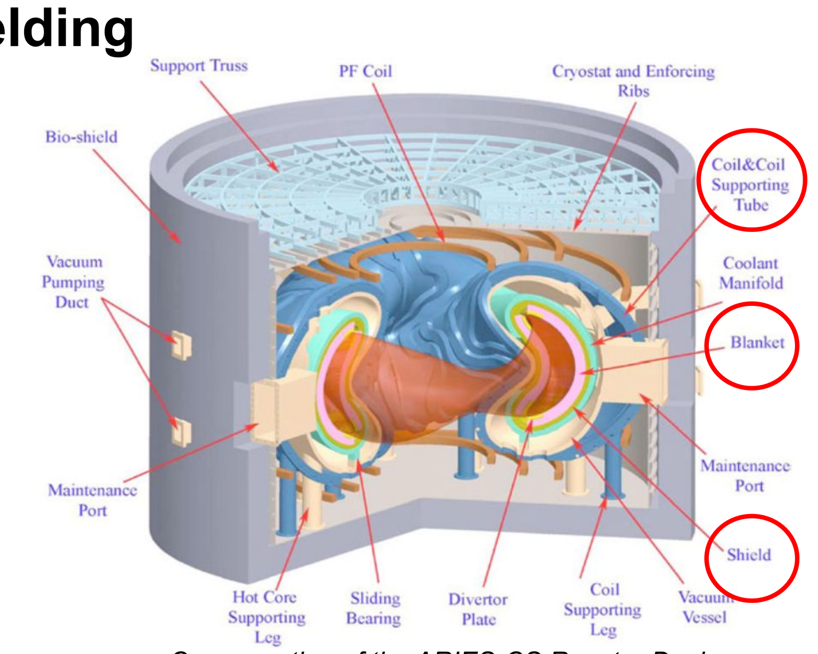
Stellarators Need Space for a Breeding Blanket & Neutron Shielding

During the design of ARIES-CS and W7-X, both configurations experienced engineering issues related to the space between the last closed flux surface and the external coils.[1][2]

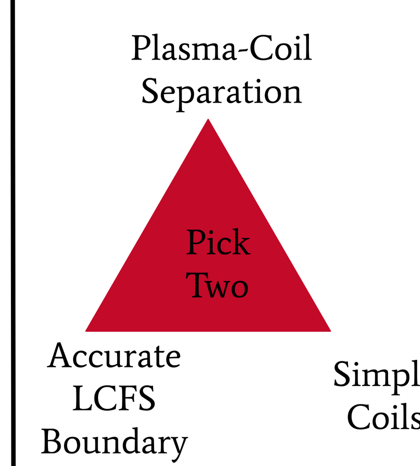
This "plasma-coil separation" must be > 1.5m to have enough room for neutron shielding and a blanket.

Larger plasma-coil separation reduces coil ripple, accommodates for shifts during startup and initialization, and can allow larger configurations to be scaled down.



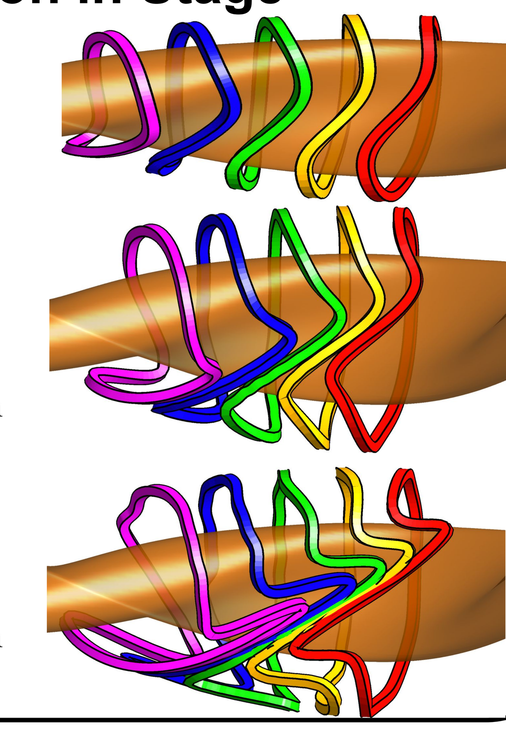
Cross-section of the ARIES-CS Reactor Design

Difficulty of Increasing Plasma-Coil Separation in Stage II Optimization



Moving coils further away from the plasma results in increased coil complexity (such as increased curvature, longer coils, and closer minimum coil-coil distance), as shown on the right.

Single-stage optimization[3] can be computationally challenging. **It is therefore valuable to develop an easy-to-calculate proxy for plasma-coil separation.**



REGCOIL[4] is a Useful Optimizer to Systematically Compare the Coils of Many Configurations

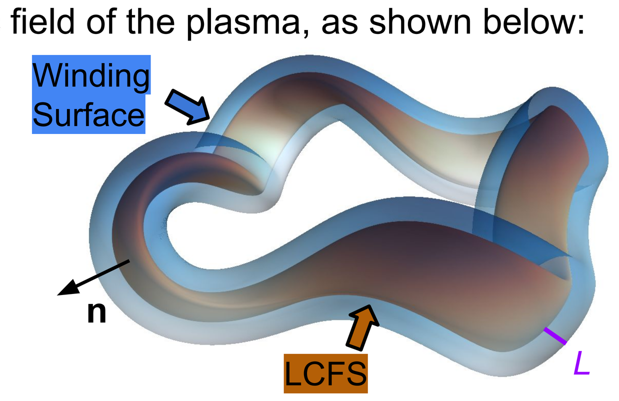
`REGCOIL`'s objective function **preserves convexity**, so any local minimum is a global minimum. It also has fewer tuning parameters than other codes.

`REGCOIL` calculates the surface current density on a **winding surface**, which is outside the LCFS at a constant distance *L*. This is used to find the magnetic field of the plasma, as shown below:

$$\Phi'(\theta',\zeta') = \sum_j \Phi_j \sin(m_j\theta' - n_j\zeta')$$

$$\mathbf{K}' = \mathbf{n}' \times \nabla\Phi'$$

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int d^2a' \frac{\mathbf{K}' \times (\mathbf{r}-\mathbf{r}')}{|r-r'|^3}$$



REGCOIL minimizes the following objective:

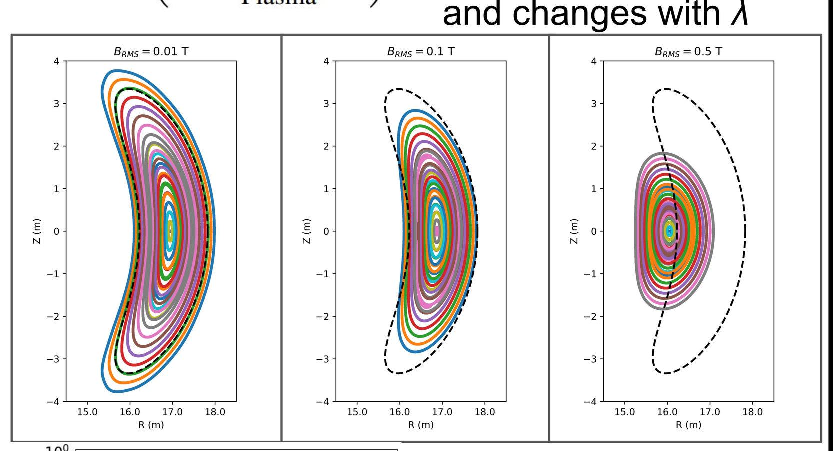
$$f = \int d^2a\,(\mathbf{B}(\theta,\zeta)\cdot\mathbf{n})^2 + \lambda\int d^2a'\,\|\mathbf{K}(\theta',\zeta')\|^2$$

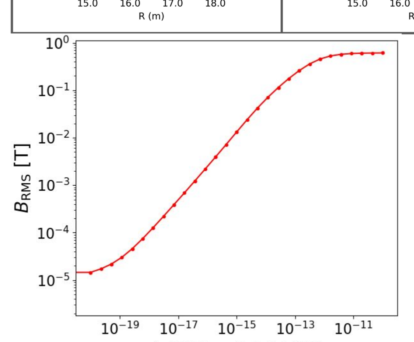
2 free parameters: *L* and *λ*. A unique solution requires 2 constraints:

1. $B_{RMS} = B^*_{RMS}$ 2. $\|K\|_\infty = \|K\|^*_\infty$

$$B_{RMS} = \left(\frac{\int d^2a\,(\mathbf{B}\cdot\mathbf{n})^2}{A_{\text{Plasma}}}\right)^{1/2}$$

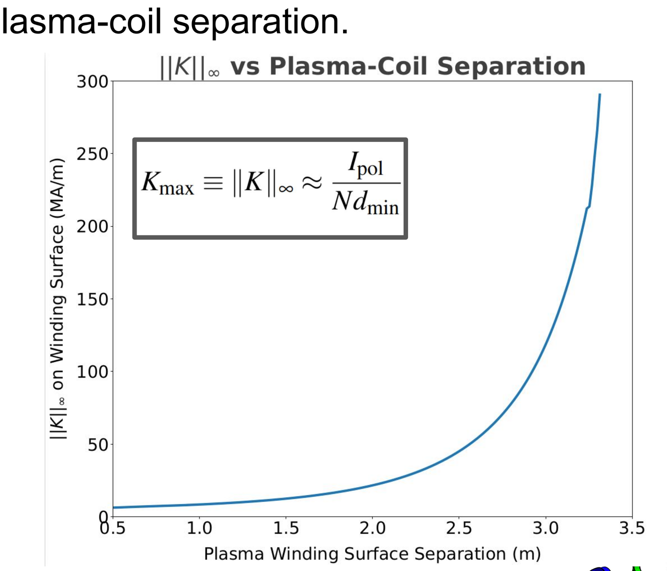
B_{RMS} is a measure of accuracy in the LCFS, and changes with *λ*

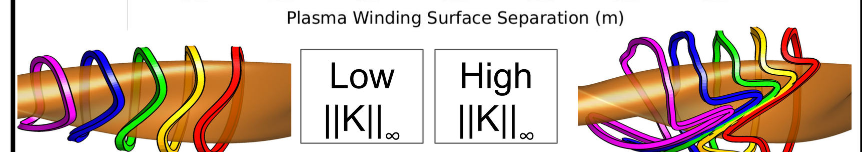




Ideally, B_{RMS} = 0, but the penalty for complex coils prevents this from being possible.

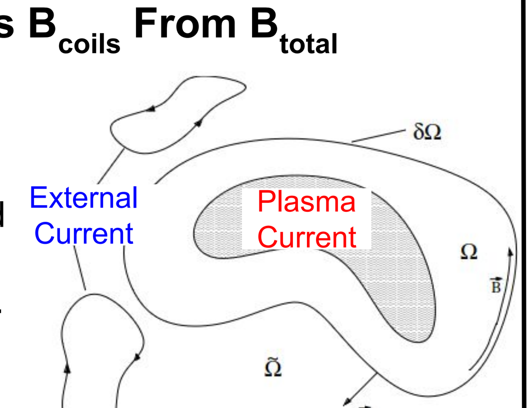
$\|K\|_\infty$ or K_{max} is the highest current density on the winding surface and uniquely defines plasma-coil separation.





Virtual Casing Decomposes B_{coils} From B_{total}

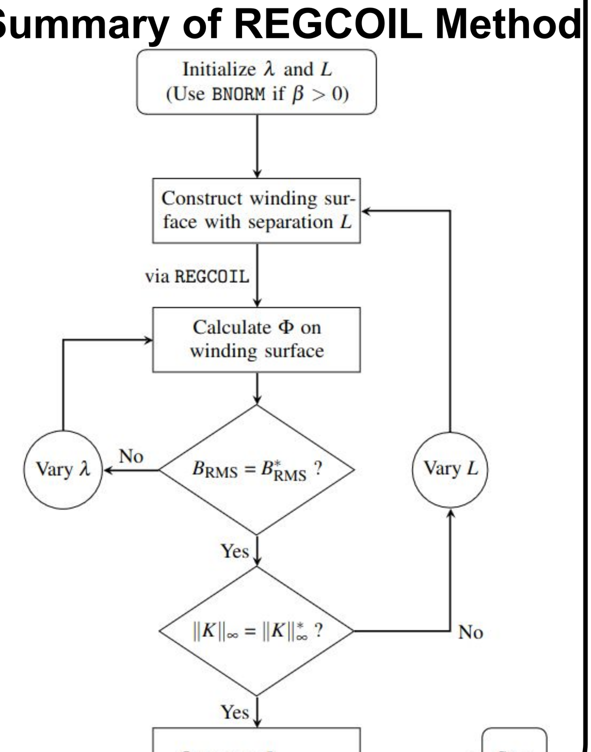
Using virtual casing, it is possible to find the magnetic field generated by only the external coils, as shown below. We utilized work by Dhairya Malhotra[5][6] to perform virtual casing when $\beta > 0$.



$$\mathbf{B}(\mathbf{x}) = \mathbf{B}_{\text{plasma}}(\mathbf{x}) + \mathbf{B}_{\text{coils}}(\mathbf{x})$$

$$\mathbf{B}^*_{\text{coils}}(\mathbf{x}) = -\frac{1}{4\pi}\oint_{\delta\Omega} d^2a\,\frac{(\mathbf{n}\times\mathbf{B}^*(\mathbf{p}))\times(\mathbf{x}-\mathbf{p})}{|x-p|^3}$$

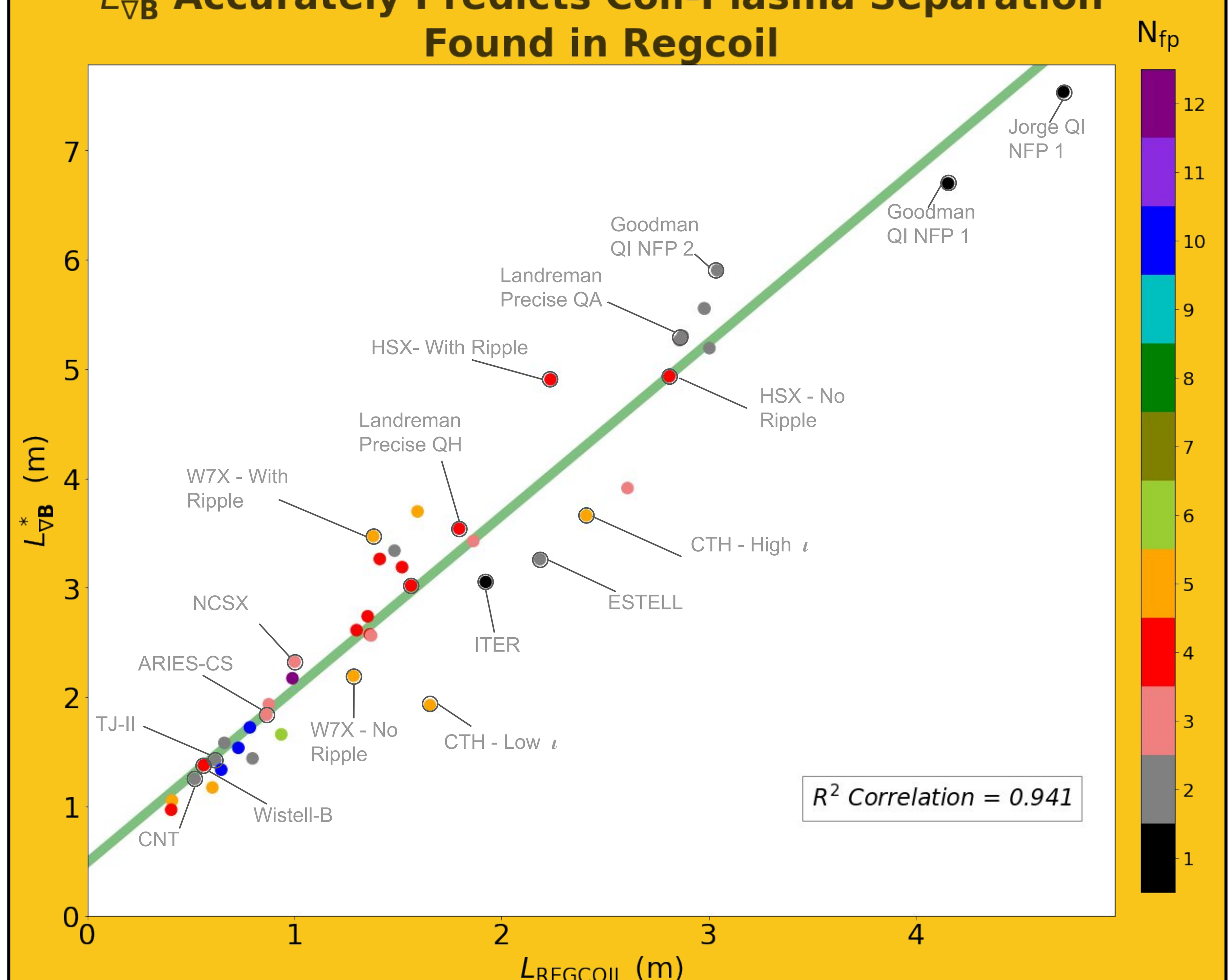
Summary of REGCOIL Method



References

1. F. Najmabadi et al., *Fusion Science and Technology* **54**, 655–672 (2008)
2. T. Klinger et al., *Nuclear Fusion* **12**, 599 (1972).
3. R. Jorge, A. Goodman, M. Landreman, J. Rodrigues, and F. Wechsung, *PPCF* **65**, 074003 (2023).
4. M. Landreman, *Nuclear Fusion* **57**, 046003 (2017).
5. D. Malhotra, A. J. Cerfon, M. O'Neil, and E. Toler, *PPCF* **62**, 024004 (2019).
6. D. Malhotra, "Boundary integral equation solver for taylor states", github.com/hiddenSymmetries/virtual-casing (2019).

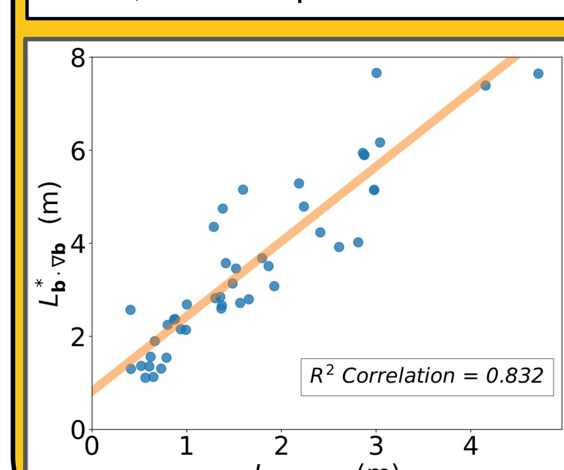
$L^*_{\nabla B}$ Accurately Predicts Coil-Plasma Separation Found in Regcoil

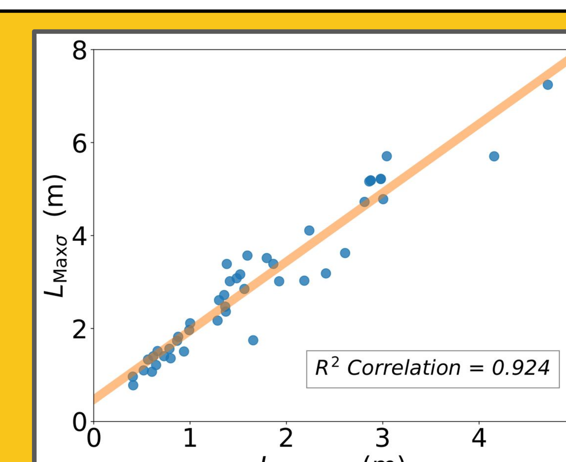


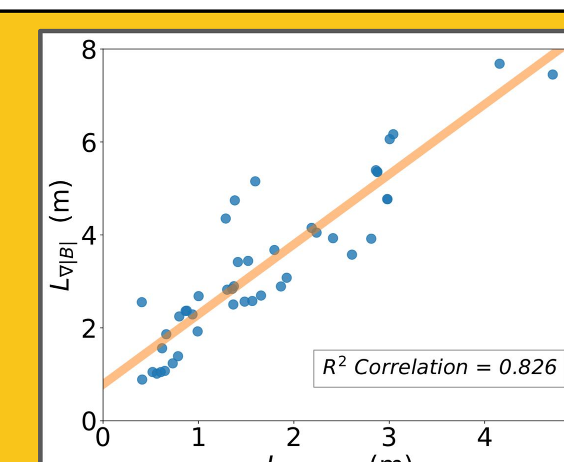
Parameters	$\|K\|^*_\infty$ = 17.16 MA/m	B_{Vol} = 5.865 T	m_θ & n_ζ = 96
B^*_{RMS} = 0.01 T	a = 1.704 m	mpol & ntor = 20	

We gathered database of > 40 stellarator and tokamak configurations. Within this database, the coil-to-plasma distance compared to the minor radius varies by over an order of magnitude. **The magnetic scale length** is well correlated to **the coil-to-plasma distance** of actual coil designs generated using the `REGCOIL` method.[4]

Below, we have plotted alternative scale lengths, which are also correlated with the coil-to-plasma distance.







Intuition for Magnetic Gradient Scale Length

Arguments of scale lengths are used in plasma physics to determine which effects are negligible versus significant.

A spatial gradient of the magnetic field encodes some information about the spatial distance from the coils to the plasma.

$$\nabla\mathbf{B} = \begin{bmatrix} \frac{\partial B_x}{\partial x} & \frac{\partial B_y}{\partial x} & \frac{\partial B_z}{\partial x} \\ \frac{\partial B_x}{\partial y} & \frac{\partial B_y}{\partial y} & \frac{\partial B_z}{\partial y} \\ \frac{\partial B_x}{\partial z} & \frac{\partial B_y}{\partial z} & \frac{\partial B_z}{\partial z} \end{bmatrix}$$

$$L_{\nabla\mathbf{B}} = \frac{\sqrt{2}B}{\|\nabla\mathbf{B}\|_F}$$

$$\|A\|_F = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n}|a_{ij}|^2}$$

Model Geometry: Infinite Straight Wire

For a current carrying infinite straight wire, $L_{\nabla\mathbf{B}}$ is equal to the distance between the magnetic field and the wire. Therefore, by measuring the magnetic field and its gradient, we can determine where the nearest wire must be located to create the magnetic field.

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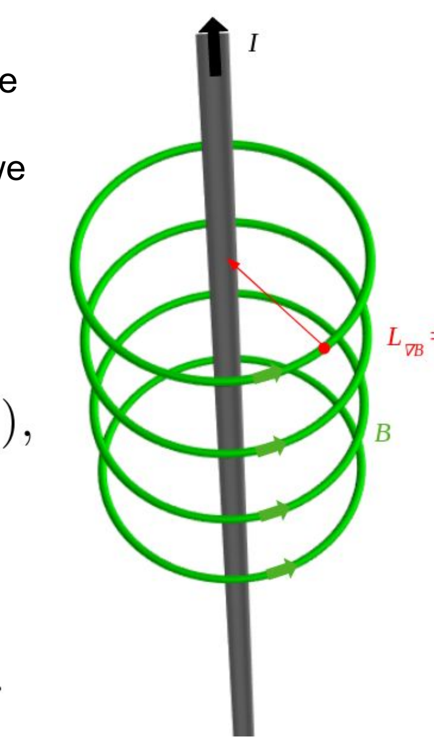
$$\mathbf{B}(R) = \frac{\mu_0 I}{2\pi R}\hat{\boldsymbol{\phi}}$$

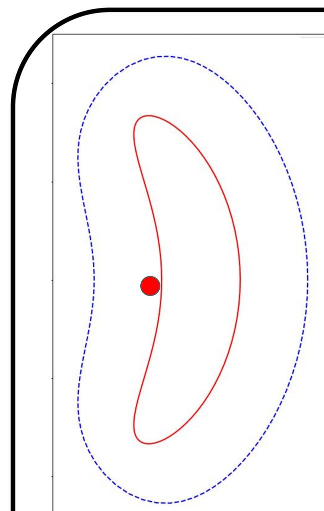
$$\nabla\mathbf{B} = -\frac{\mu_0 I}{2\pi R^2}(\hat{\boldsymbol{\phi}}\hat{\mathbf{R}} + \hat{\mathbf{R}}\hat{\boldsymbol{\phi}}),$$

$$\|\nabla\mathbf{B}\|_F = \frac{\sqrt{2}\mu_0 I}{2\pi R^2},$$

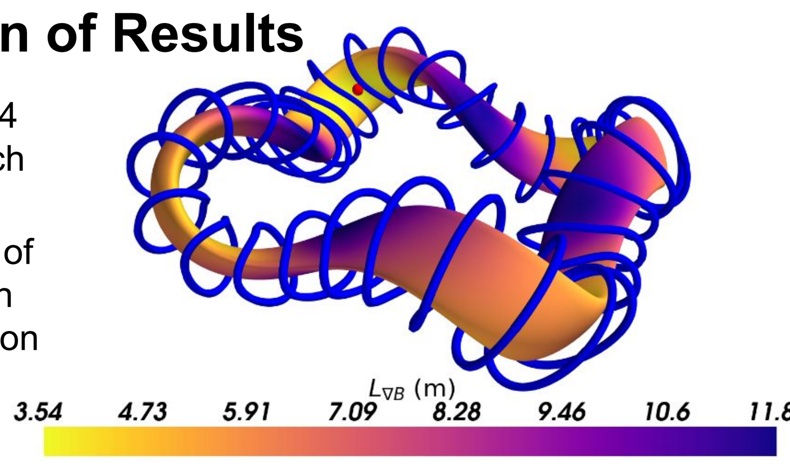
$$L_{\nabla\mathbf{B}} = \sqrt{2}\frac{B}{\|\nabla\mathbf{B}\|_F} = R.$$



Discussion of Results

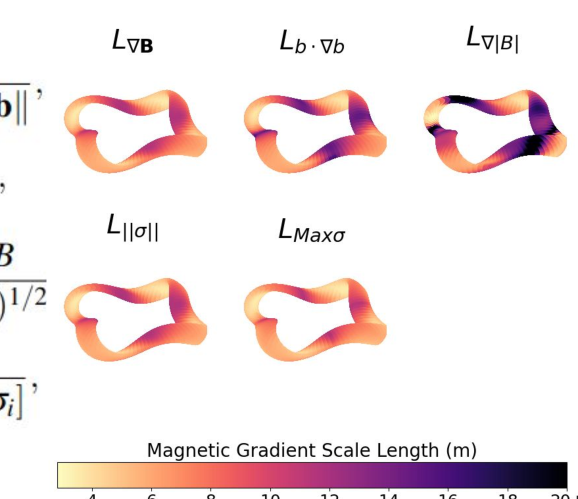


To the right is a NFP=4 QH stellarator on which $L_{\nabla\mathbf{B}}$ is plotted. It is shortest on the inside of the curve, or the "bean cross-section" shown on the left.

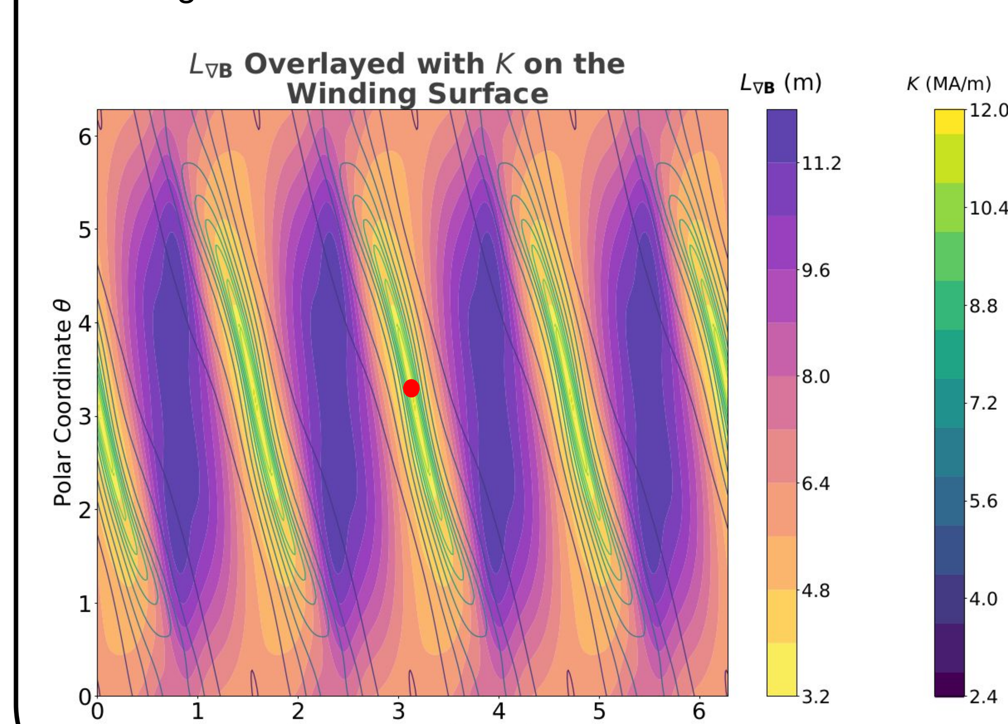


To the right, alternative scale lengths are shown on the surface of the NFP=4 QH stellarator. $L_{\nabla\mathbf{B}}$ matches $L_{\|\sigma\|}$ and is approximately equal to $L_{Max\sigma}$.

$$L_{\mathbf{b}\cdot\nabla\mathbf{b}} = \frac{1}{\|\mathbf{b}\cdot\nabla\mathbf{b}\|},\quad L_{\nabla|B|} = \frac{B}{\|\nabla B\|},\quad L_{\|\sigma\|} = \frac{\sqrt{2}B}{(\Sigma_i\sigma_i^2)^{1/2}},\quad L_{\text{Max}\sigma} = \frac{B}{\text{Max}[\sigma_i]},$$



Results of the main figure are insensitive to target $\|K\|_\infty$ and B_{RMS} within a plausible range. Configurations that lie off the line of best-fit tend to be configurations with high coil-ripple, axisymmetric, or their VMEC files do not converge.



There is good spatial correlation between $\|K\|_\infty$ and $L_{\nabla\mathbf{B}}{}^*$.

The smallest $L_{\nabla\mathbf{B}}$ and the largest $\|K\|_\infty$ are located in the same region.

Our future work is to implement $L_{\nabla\mathbf{B}}{}^*$ in the objective functions of Stage I optimizations (i.e., optimizing the plasma shape without explicitly considering the coil shapes).

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