2D MHD Equilibrium Solver using Physics-Informed Neural Networks

Byoungchan Jang[1], Alan Kaptanoglu[1], Matt Landreman[1], William Dorland[1] [1]Department of Physics, University of Maryland

Motivation

- Magnetohydrodynamic equilibrium calculations are a crucial tool for magnetic confinement.
- A fast solver for MHD equilibria is needed for real-time reconstructions and incorporation in optimization loops such as those that occur in stellarator shape optimization.
- Here we explored physics-informed neural networks (PINNs) as a solver for producing 2D MHD equilibria.

Grad-Shafranov Equation

- Grad-Shafranov Equation is a 2D ideal MHD equilibrium equation
- We are looking at a **particular set of analytic solutions:** *Solov'ev Profile*
- *The reason we use an analytic solution is not to **train** but to **check***

$$X\frac{\partial}{\partial X}\left(\frac{1}{X}\frac{\partial U}{\partial X}\right)+\frac{\partial^2 U}{\partial Y^2} = -\mu_0\frac{R_0}{\Psi_0^2}x^2\frac{dp}{d\psi}-\frac{R_0^2}{\Psi_0^2}F\frac{dF}{d\psi}$$

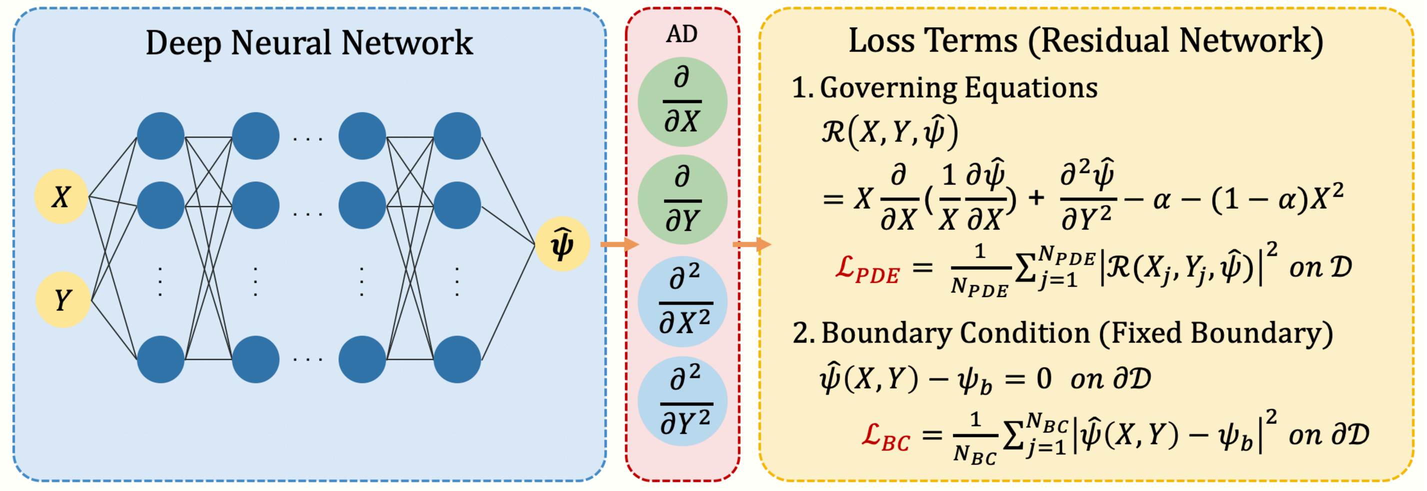
$$-\mu_0\frac{R_0}{\Psi_0^2}\frac{dp}{d\psi}=C$$

$$-\frac{R_0^2}{\Psi_0^2}F\frac{dF}{d\psi}=A$$

$$A+C=1$$

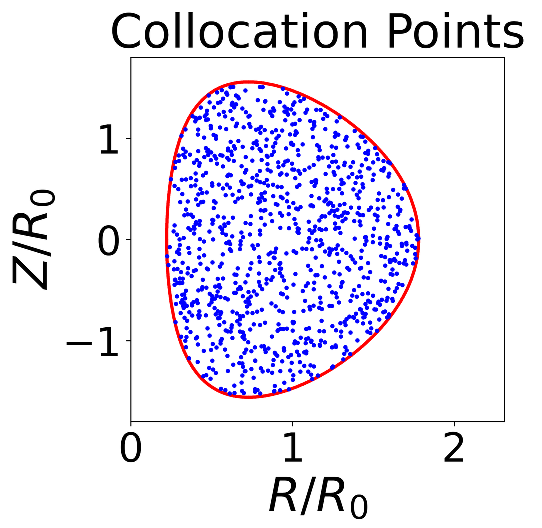
Physics-informed Neural Network

- PINN calculates residual of **PDE** and Boundary condition
- **Optionally**, we can add **data**(i.e. experimental or numerical solution)



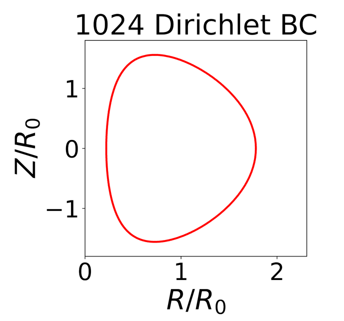
$$\mathcal{L}_{total}=\lambda_{PDE}\mathcal{L}_{PDE}+\lambda_{BC}\mathcal{L}_{BC}+(\lambda_{Data}\mathcal{L}_{Data})$$

Collocation Points



- Collocation points are where the PDE residuals are evaluated
- Possible to add data from numerical simulation or experiment

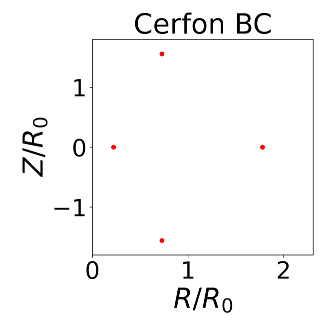
Boundary Conditions



$\psi(X_b,Y_b)=0$

$X_b = 1+\varepsilon\cos(\tau+\alpha\sin\tau)$

$Y_b = \varepsilon\kappa\sin(\tau)$

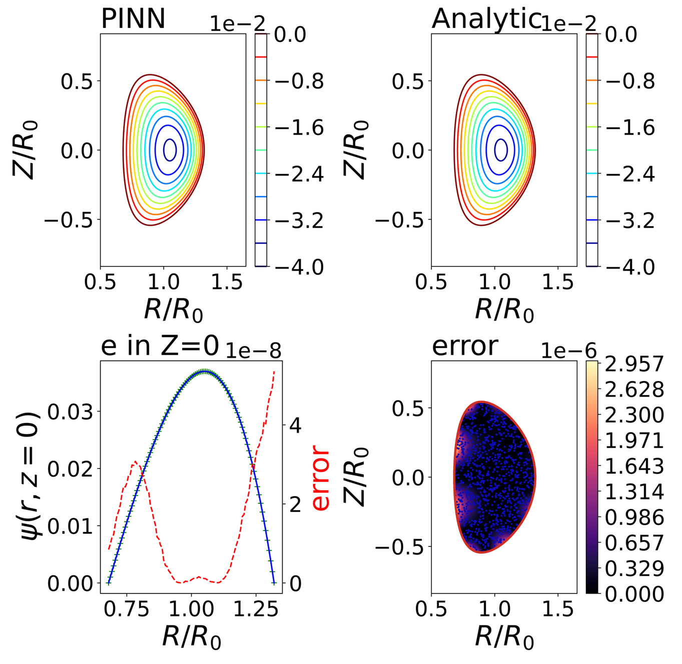


$\psi_Y(in)=0$		$in=(1+\varepsilon,0)$
$\psi_Y(out)=0$		$out=(1-\varepsilon,0)$
$\psi_X(high)=0$		$high=(1-\delta\varepsilon,\kappa\varepsilon)$
$\psi_X(low)=0$		$low=(1-\delta\varepsilon,-\kappa\varepsilon)$
$\psi(out)=0$	$\psi_{YY}(out)=-N_1\,\psi_X(out)$	
$\psi(in)=0$	$\psi_{YY}(in)=-N_2\,\psi_X(in)$	
$\psi(high)=0$	$\psi_{XX}(high)=-N_3\,\psi_Y(high)$	
$\psi(low)=0$	$\psi_{XX}(low)=N_3\,\psi_Y(low)$	

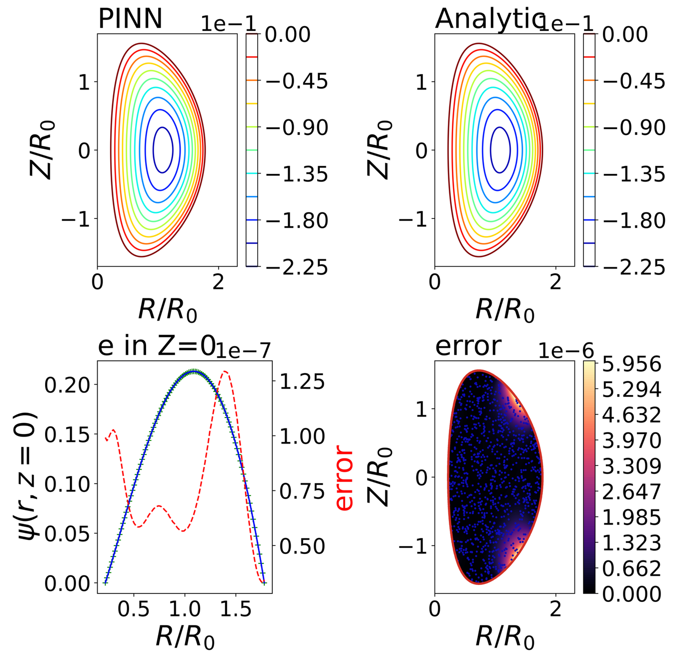
Up-Down Symmetric Configurations

Using 1024 Dirichlet Boundary Conditions

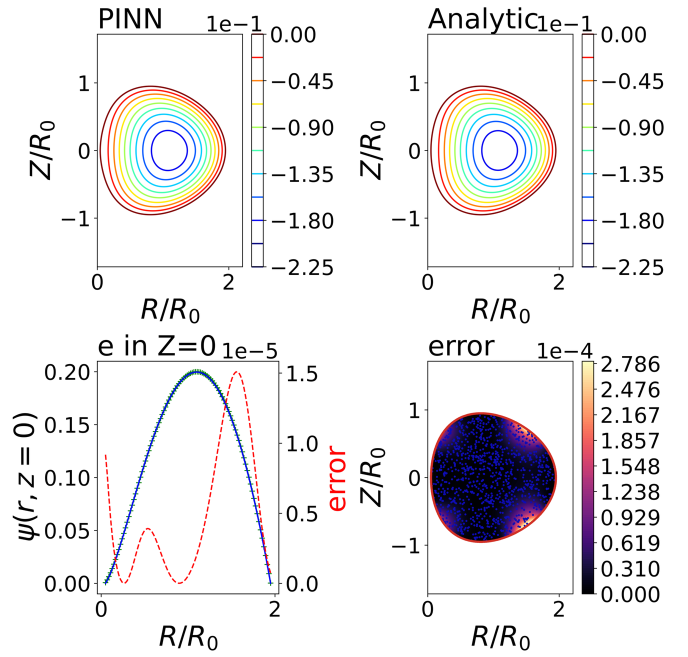
$$error=\left(\psi_{PINN}-\psi_{analytic}\right)^2/\psi_a^2 \quad \text{where } \psi_a=\psi_{PINN} \text{ in magnetic axis}$$



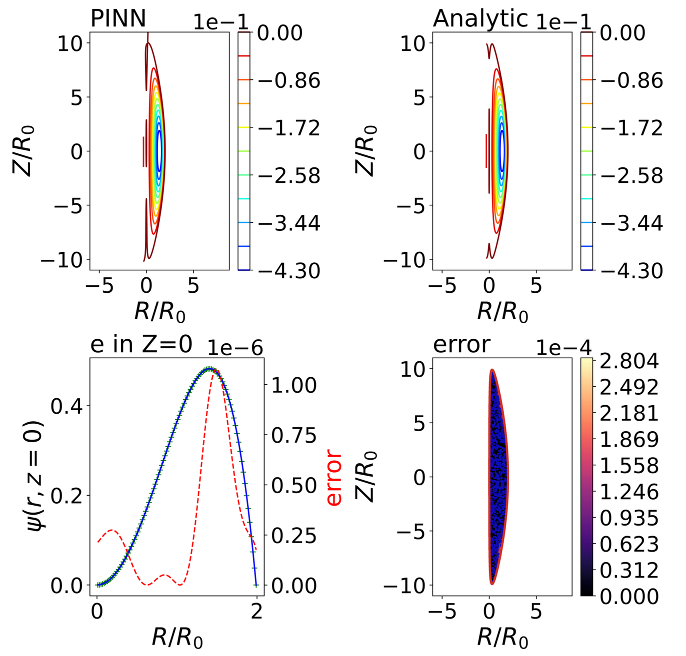
- ***ITER***
- A = −0.155
- 27.2 seconds to train



- ***NSTX***
- A = 1.0
- 23.2 seconds to train



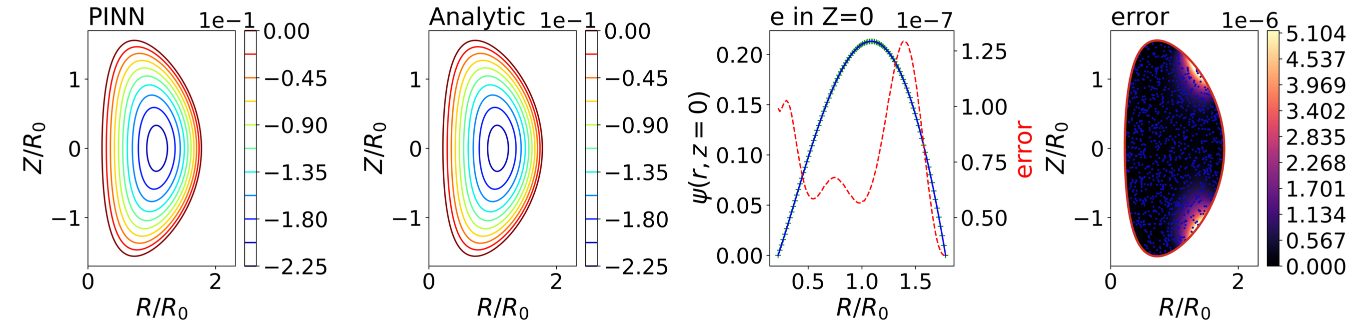
- ***Spheromak***
- A = 1.0
- 29.4 seconds to train



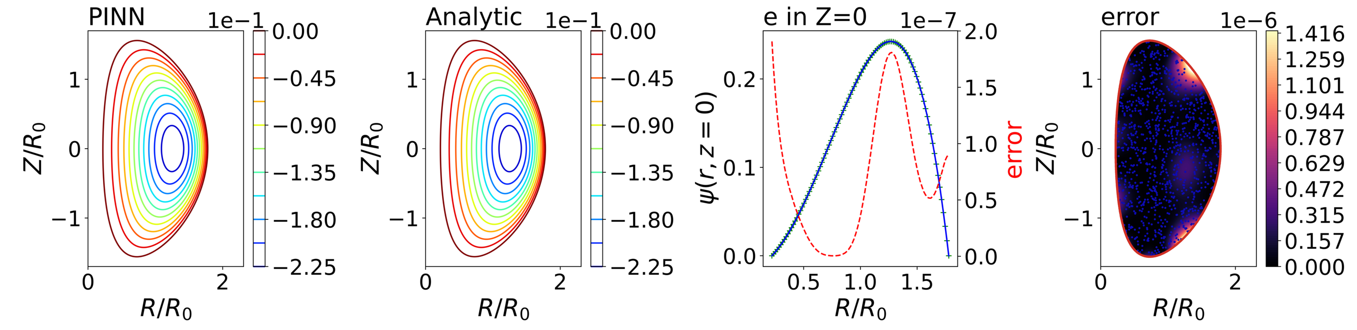
- ***FRC***
- A = 0.0
- 42.1 seconds to train

Shafranov Shift in NSTX

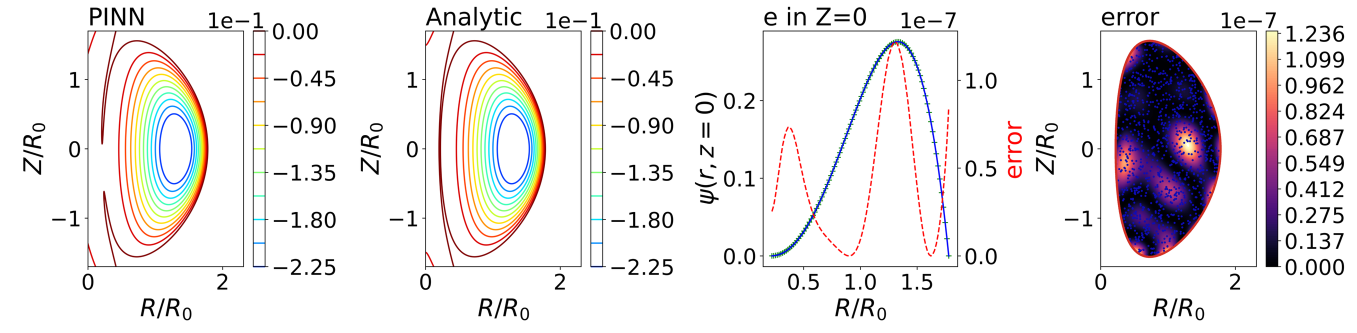
Using 1024 Dirichlet Boundary Conditions



- $\boldsymbol{A=1.0}$ (Force Free)
- 22.4 seconds to train



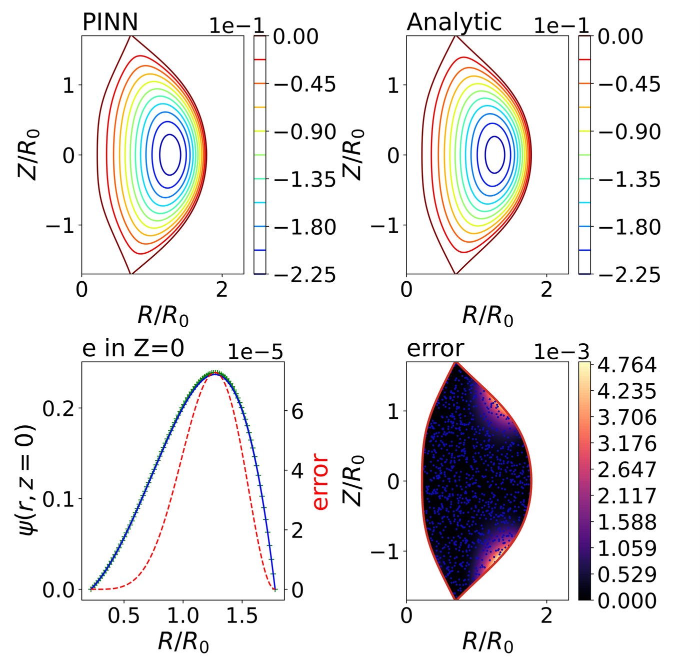
- $\boldsymbol{A=0.0}$ (Low-β)
- 21.6 seconds to train



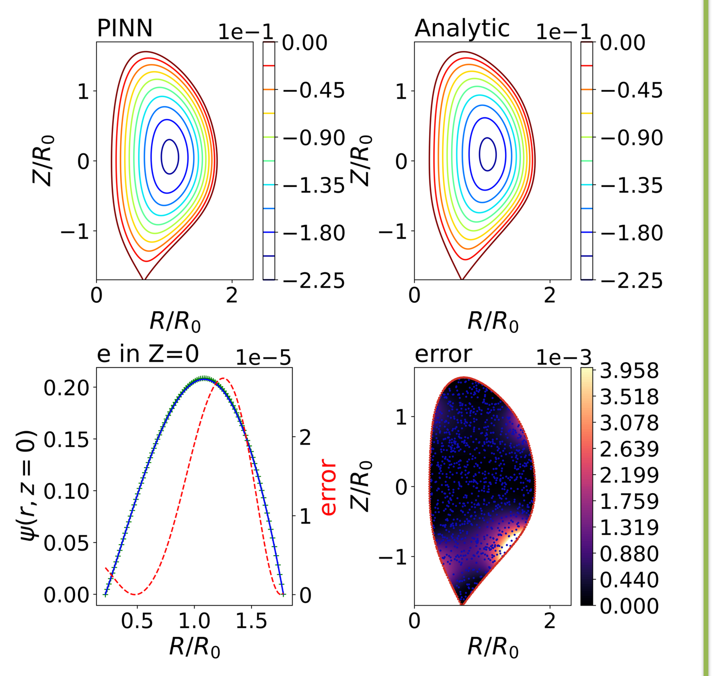
- $\boldsymbol{A=-0.7}$ (β limit)
- 26.4 seconds to train

Divertor Configuration – X-points

Using Cerfon Boundary Conditions

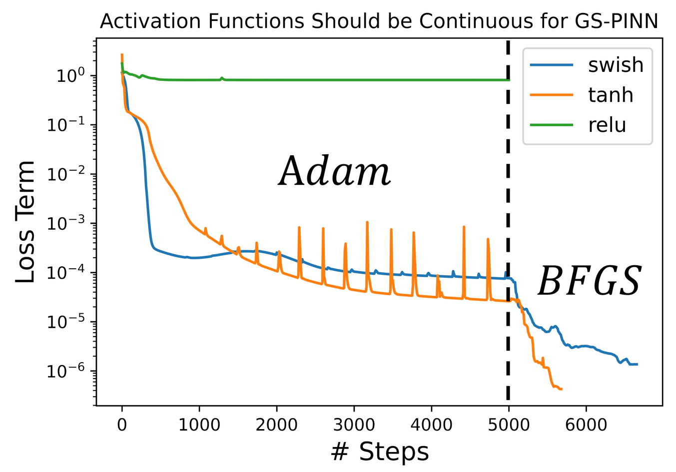


- **Double null divertor**
- 32.1 seconds to train

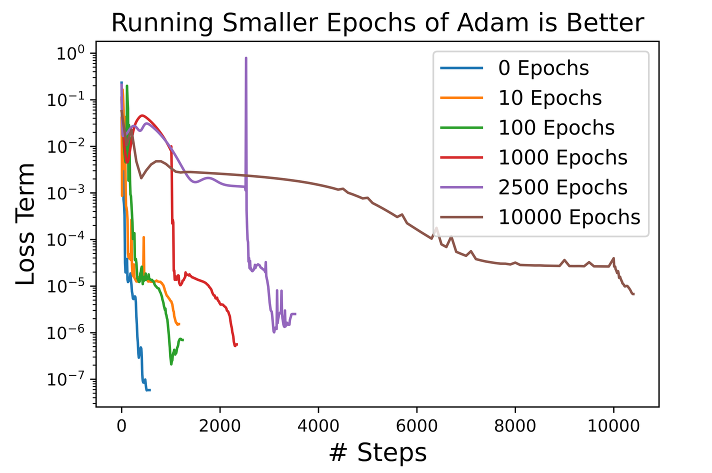


- **Single null divertor**
- 41.3 seconds to train

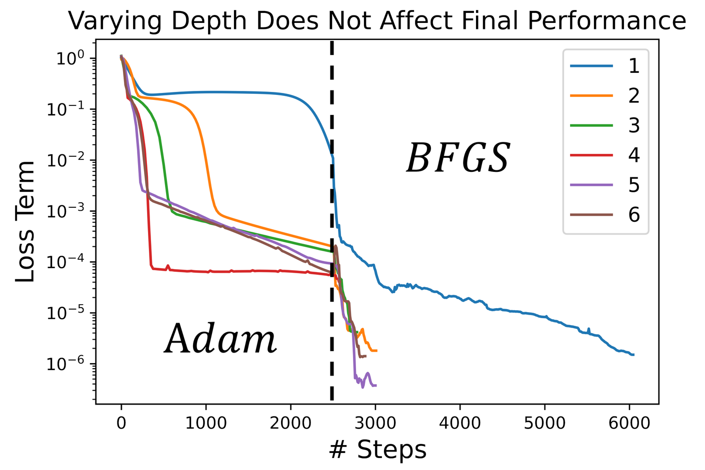
Hyperparameter Scan



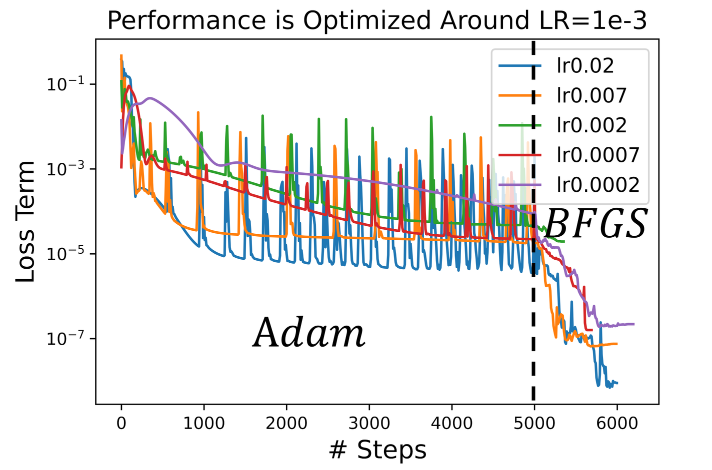
- **Activation Function**
- Activation functions should be differentiable to solve GS equation with PINN



- **BFGS vs. Adam**
- Number of Adam optimization steps does effect BFGS convergence

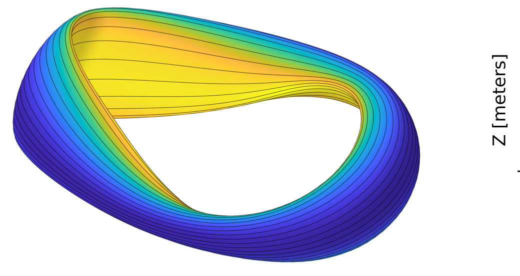


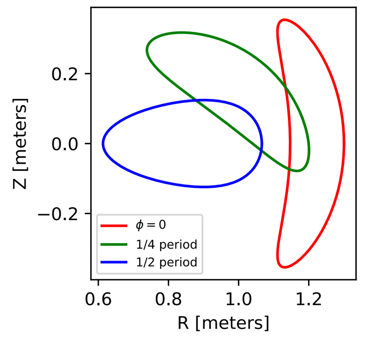
- **Network Architecture**
- For Adam, there is no significant improvement after depth=3
- For BFGS, it all converged nicely



- **Learning Rate**
- BFGS performs great regardless of previous learning rates of Adam

Future Works





- Expand to 3D MHD to reconstruct equilibria for stellarator optimization
- Parametric-PINN to expand our input to included *A*(current/pressure profile) and shape parameters (i.e. eps, kappa, and delta)

References

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[3] Lu, Lu, et al. *SIAM Review* **63.1**:208-228 (2021).
[4] D. A. Kaltsas et al. *Physics of Plasmas*, **29**(2), 022506 (2022)
[5] S. Markidis. *Frontiers in Big Data* **92**(2021).
[6] M Landreman , B Medasani, & C Zhu, *Phys Plasmas* 28, 092505 (2021).

All calculations were done with a 2019 MacBook Pro (2.6 GHz 6-Core Intel Core i7)