Optimizing stellarators to maximize turbulent impurity transport over energy transport

Stefan Buller, Rahul Gaur and Matt Landreman

University of Maryland

Overview

- Neoclassically optimized stellarators like Wendelstein 7-X may need turbulent transport to avoid impurity accumulation.
- Performed quasi-linear calculations of impurity flux (Γ_z) and energy flux (Q) in **ARIES-CS**, **NCSX** and **TJ-II** geometries.
- Used SIMSOPT and STELLA to optimize ARIES-CS for maximal Γ_z/Q in an ion temperature gradient (ITG) turbulence scenario.

Simulation scenarios

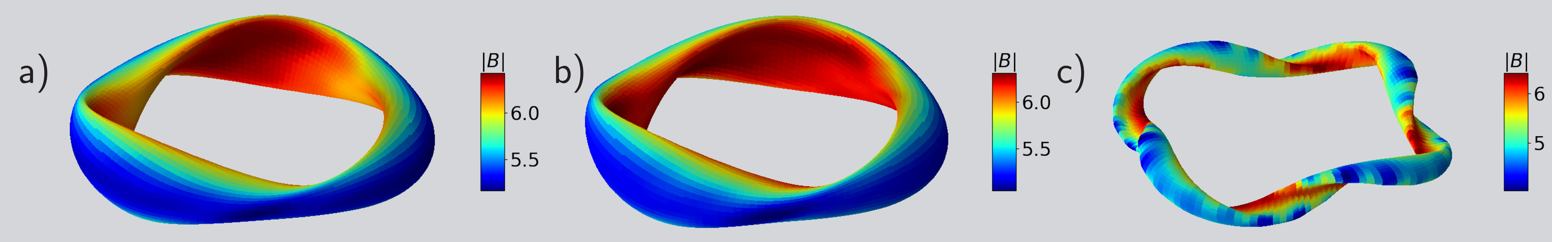

Figure 1:a) ARIES-CS b) NCSX c) TJ-II. All at $s = \psi_t/\psi_{t,\mathrm{LCFS}} = 0.49$

- All geometries scaled to ARIES-CS minor radius $a = 1.7\,\mathrm{m}$ and $B_{00} = 5.68$.
- Three sets of gradients for the bulk species are considered ("ITG", "TEM", "ETG")

	a/L_{Ti}	a/L_n	a/L_{Te}
ITG	4.0	0.0	0.0
TEM	0.0	4.0	0.0
ETG	0.0	0.0	4.0

Table 1:The three different gradient scenarios

- Trace Ar+16 impurity ($Z = 16$, $A = 40$) with gradients $a/L_{Tz} = 2.75$, $a/L_{nz} = 0.42$.

Definitions

Electrostatic gyrokinetics with kinetic electrons.

- Gyrokinetic equation

$$\frac{\partial g_s}{\partial t} + v_\parallel \vec{b}\cdot\nabla z\left(\frac{\partial g_s}{\partial z} + \frac{Z_s e}{T_s}\frac{\partial \langle\varphi\rangle_{\vec{R}}}{\partial z}F_s\right) - \frac{\mu_s}{m_s}\vec{b}\cdot\nabla B\frac{\partial g_s}{\partial v_\parallel}$$
$$+ \vec{v}_{Ms}\cdot\left(\nabla_\perp g_s + \frac{Z_s e}{T_s}\nabla_\perp\langle\varphi\rangle_{\vec{R}}F_s\right) + \langle\vec{v}_E\rangle_{\vec{R}}\cdot\nabla_\perp g_s + \langle\vec{v}_E\rangle_{\vec{R}}\cdot\nabla_E F_s = 0, \quad (1)$$

- Quasi-neutrality

$$0 = \sum_s Z_s\delta n_s = \sum_s Z_s\left(g_s - Z_s\frac{Z_s e}{T_s}F_s[\varphi - \langle\varphi\rangle_{\vec{R}}]\right), \quad (2)$$

- (Raw) fluxes

$$\Gamma_s = \frac{1}{n_s}\int \mathrm{d}^3v(\vec{v}_E\cdot\nabla r)g_s, \quad (3)$$
$$Q_s = \int \mathrm{d}^3v\frac{mv^2}{2}(\vec{v}_E\cdot\nabla r)g_s. \quad (4)$$

- Expanded $g = g_{k_x,k_y}\exp(ik_x x + ik_y y)$ and linearize. ($k_x = 0$ here)
- Quasi-linear fluxes

$$\Gamma^{\mathrm{QL}}_{k_y} = \frac{\Gamma_{k_y}\gamma_{k_y}}{\left\langle|\phi_{k_y}|^2\right\rangle k_y^2}, \quad (5)$$
$$Q^{\mathrm{QL}}_{k_y} = \frac{Q_{k_y}\gamma_{k_y}}{\left\langle|\phi_{k_y}|^2\right\rangle k_y^2}. \quad (6)$$

Growth rates and quasi-linear fluxes

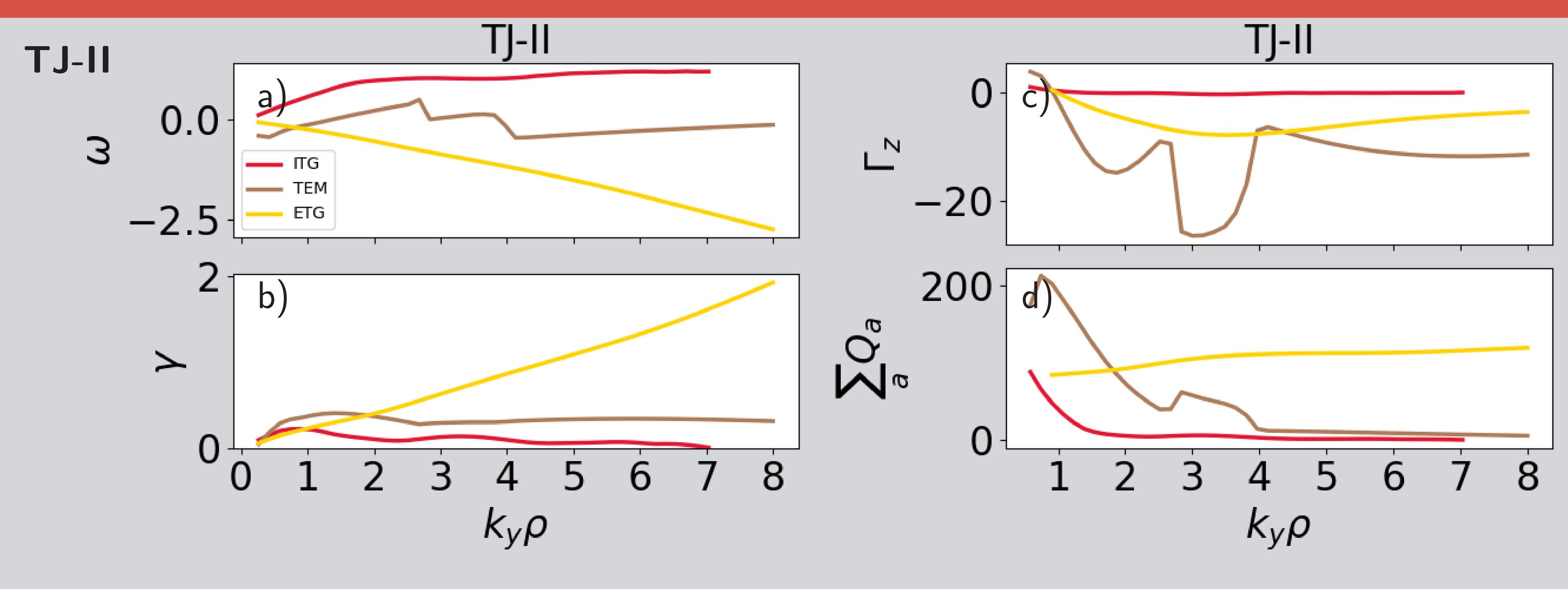

Figure 2:a) Frequences. b) Growthrates. c) QL impurity flux. d) QL heat flux

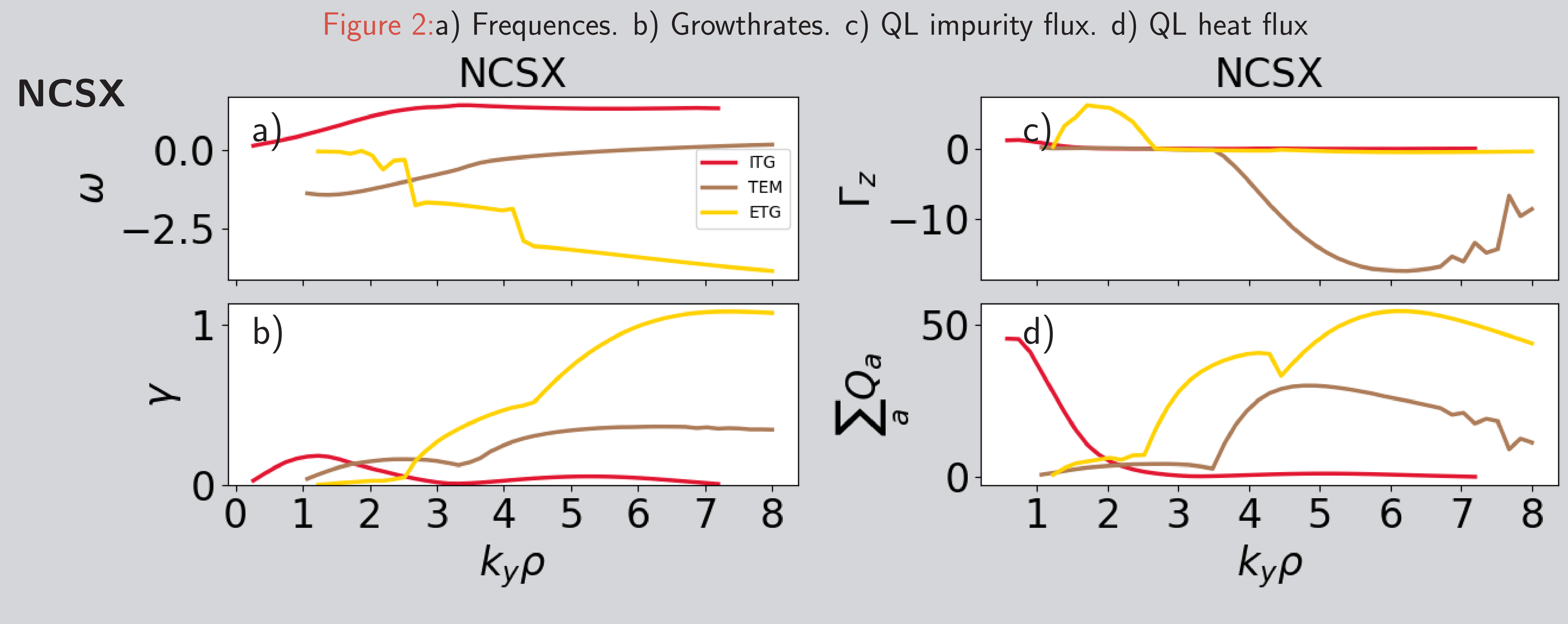

Figure 3:a) Frequences. b) Growthrates. c) QL impurity flux. d) QL heat flux

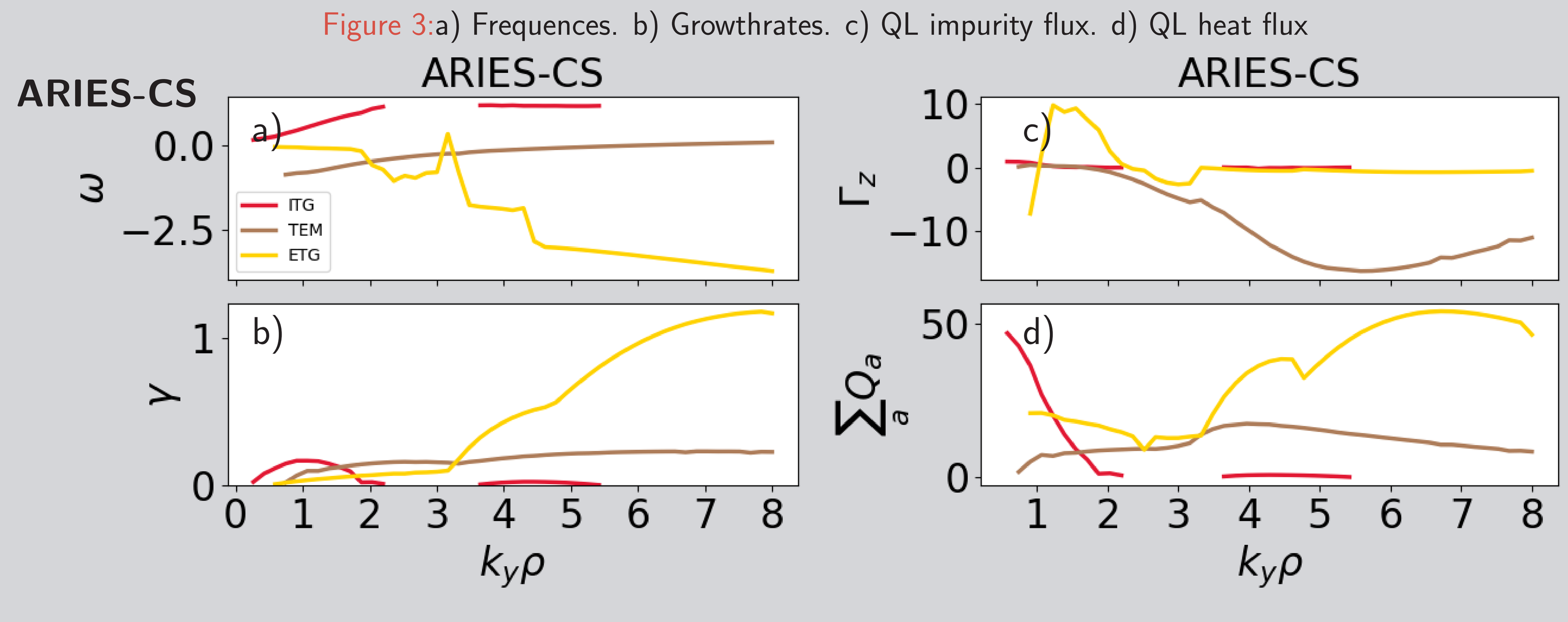

Figure 4:a) Frequences. b) Growthrates. c) QL impurity flux. d) QL heat flux

Scalar optimizable

Want to have fastest/cheapest measure of Γ_z/Q.

- Evaluate Γ_z/Q ratio for $k_x = 0$ and the fastest growing mode at low k_y.
- Not suitable for ETG.
- Find lowest resolution for which ratio at this k_y is converged.

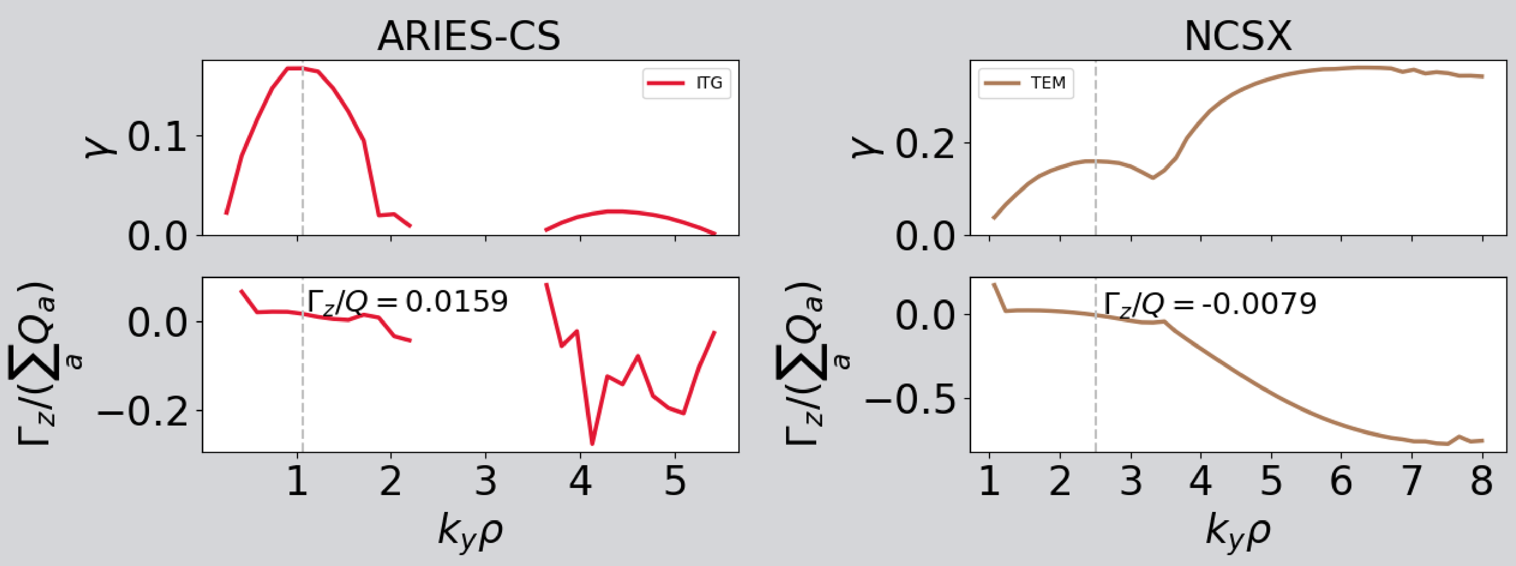

Figure 5:(a) ITG example. (b) TEM example.

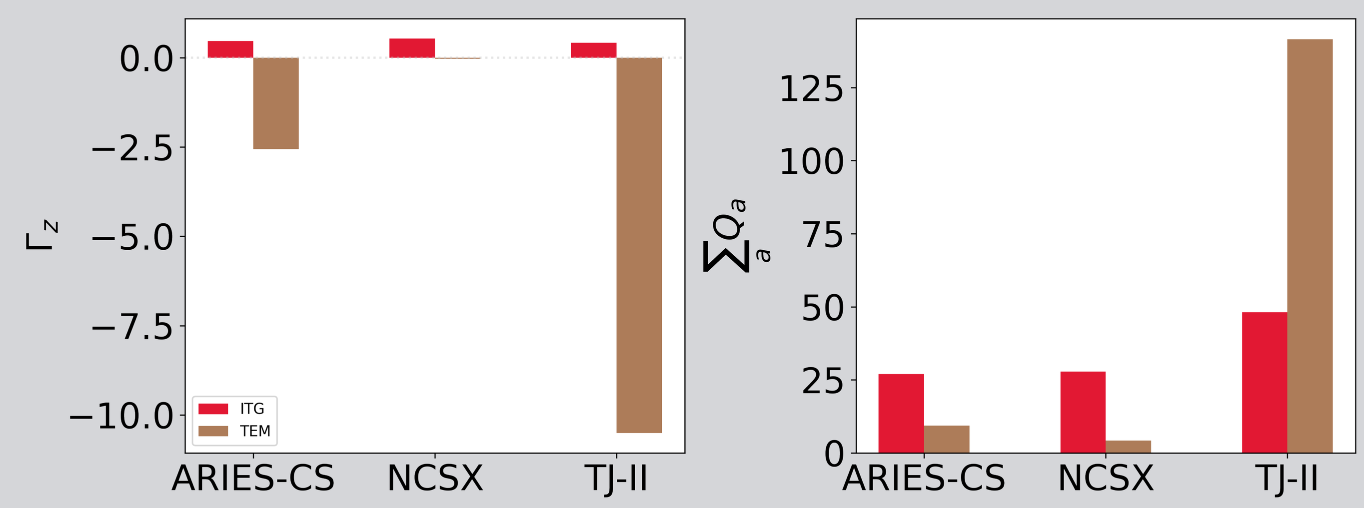

Figure 6:Scalar impurity and heat flux for the different configurations

		Γ_z	Q	Γ_z/Q
ARIES-CS	ITG	0.4305	27.02	0.01593
NCSX	ITG	0.5436	27.83	0.01953
TJ-II	ITG	0.4154	48.09	0.008636
ARIES-CS	TEM	-2.58	9.32	-0.2765
NCSX	TEM	-0.0333	4.23	-0.007874
TJ-II	TEM	-10.51	141.5	-0.07431

Table 2:Ratio of impurity and heat flux in the different configurations

Optimizing ARIES-CS' boundary shape in the ITG scenario

- Optimize boundary shape for Fourier modes up to $|n| = m = 1$, keeping $r_{c,0,0}$ fixed. Preserve stellarator symmetry. $\implies$ 8 degrees of freedoms.

$$R(\theta,\phi) = \sum_{m=0}^{1}\sum_{n=-1}^{1} r_{c,m,n}\cos(m\theta - n_{\mathrm{fp}}n\phi)$$
$$Z(\theta,\phi) = \sum_{m=0}^{1}\sum_{n=-1}^{1} z_{s,m,n}\sin(m\theta - n_{\mathrm{fp}}n\phi) \quad (7)$$

- Using VMEC + STELLA, run with SIMSOPT.

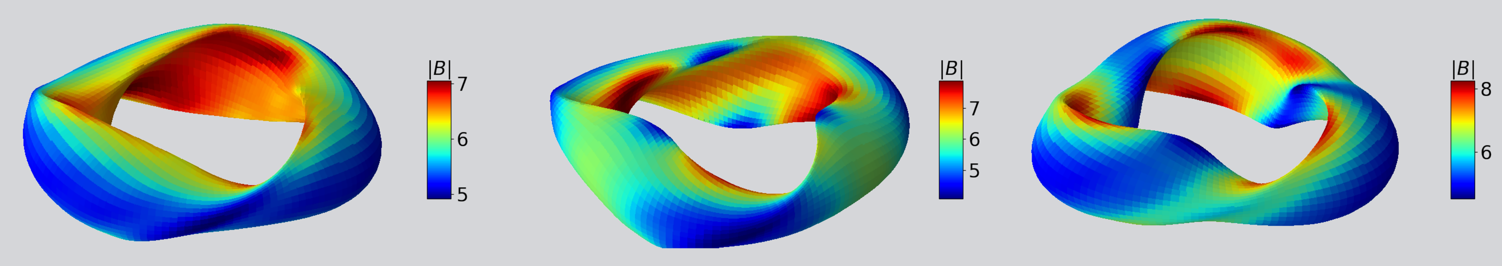

Figure 7:ARIES-CS (a) Initial boundary. (b) Optimized boundary. (c) Pessimized boundary.

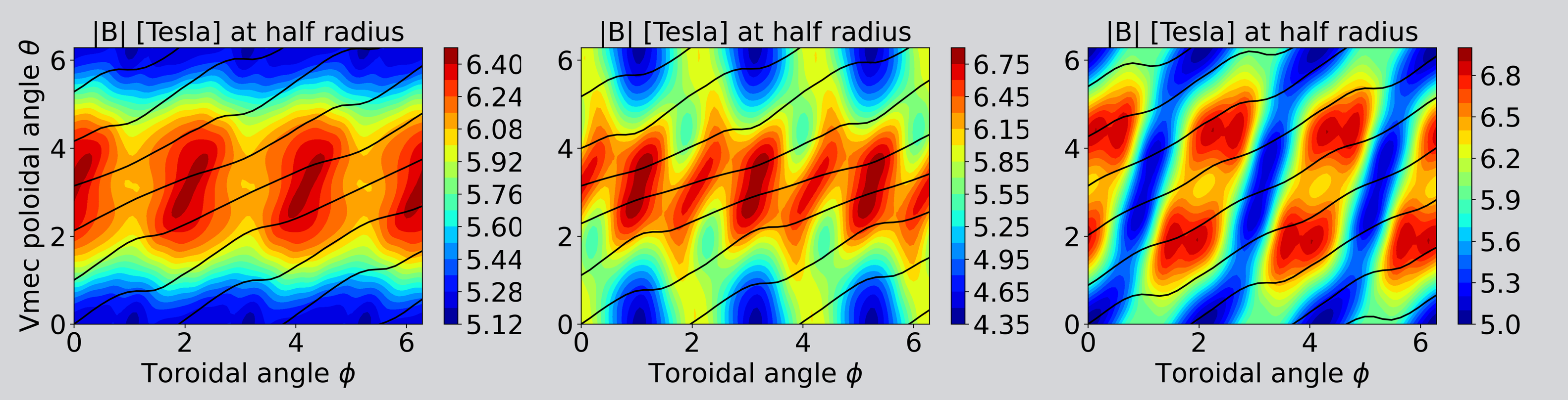

Figure 8:ARIES-CS (a) Initial at $s = 0.49$. (b) Optimized at $s = 0.49$. (c) Pessimized at $s = 0.49$.

	Original	Optimized	Pessimized
Γ_z	0.4305	0.5226	0.001858
Q	27.02	25.68	20.255
Γ_z/Q	0.01593	0.02035	9.175×10^{-5}
$\epsilon_{\mathrm{eff}}^{3/2}$(edge)	0.00049	0.0856	0.033
Fast-particle loss fraction	0.26	0.33	0.32

Table 3:ARIES-CS ITG scenario before and after Γ_z/Q-optimization (pessimization)

Outlook

- Check against $k_x = 0$ and $alpha = 0$ (field-line label).
- Comparison to nonlinear GX simulations ongoing.
- Global optimizations tricky since most points in boundary representation are non-sensible.

References

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