Efficient Calculation of and Coil Shape Optimization for Lorentz Forces

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Introduction

- Lorentz forces on magnetic field coils pose practical and financial challenges to fusion reactor design due to
  - the potential for coil fatigue at high loads
  - the expense and bulk of support structures
  - unexpected device behavior at high loads, e.g., W7-X coil movement
- Forces are typically calculated using finite element analysis or direct evaluation of the Biot-Savart law
- These approaches are too slow to be practical for any physics design and/or optimization
- Importantly, a naive approach to use the 1D Biot-Savart law fails due to a logarithmic singularity in coil thickness!

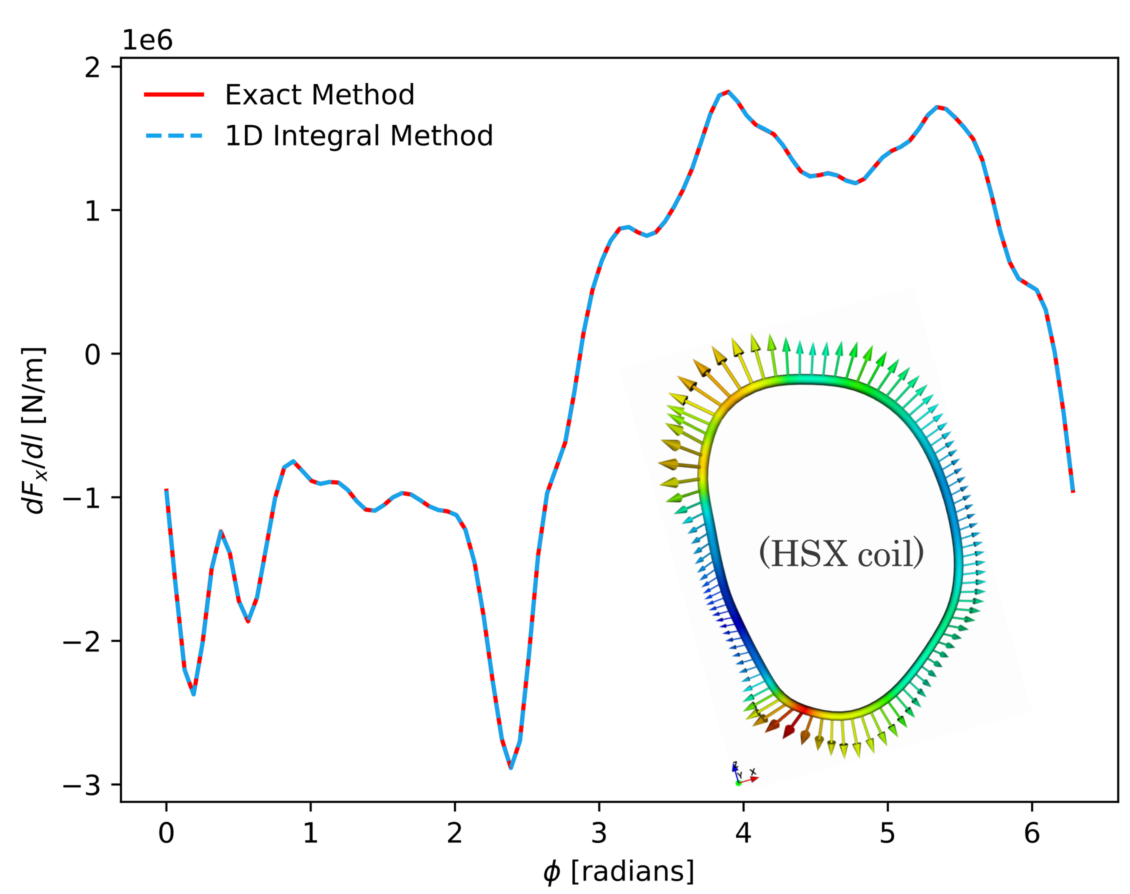
Reduced Force Model

- In previous work, we showed that coil force densities can be accurately and efficiently calculated with a reduced model

$$\frac{d\mathbf{F}}{d\ell} = I\mathbf{t} \times \mathbf{B}_{\text{reg}}$$

$$\mathbf{B}_{\text{reg}} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} d\tilde{\phi} \frac{\tilde{\mathbf{r}}_c' \times (\mathbf{r}_c - \tilde{\mathbf{r}_c})}{(|\Delta \mathbf{r}_c|^2 + \delta)^{3/2}}$$

where $\boldsymbol{r_c}(\boldsymbol{\phi})$ is the coil center-line parameterized by an angle $\boldsymbol{\phi}$, and δ is a constant that depends on the geometry of the cross-section

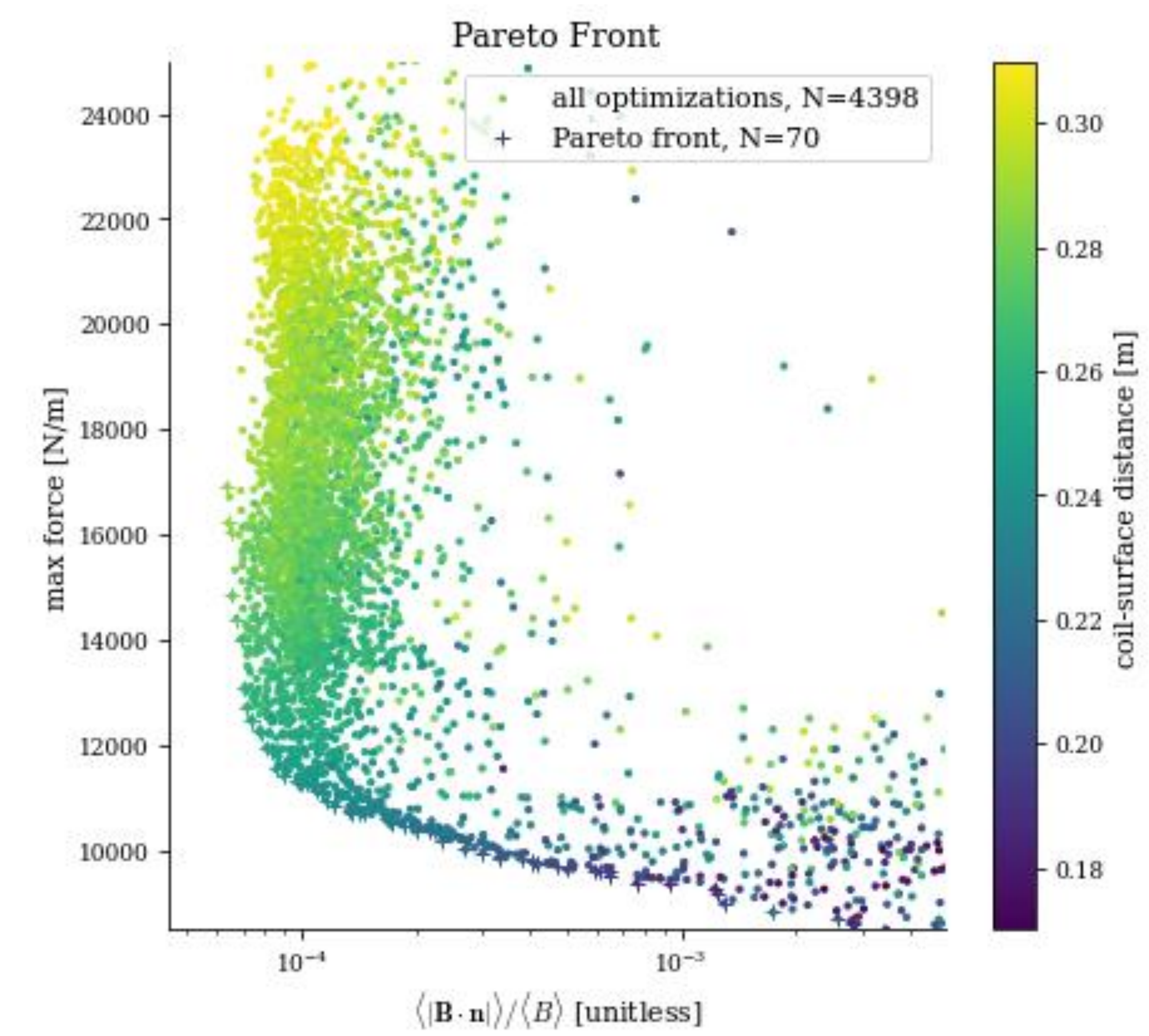


- This model is implemented in SIMSOPT (http://github.com/hiddenSymmetries/simsopt)

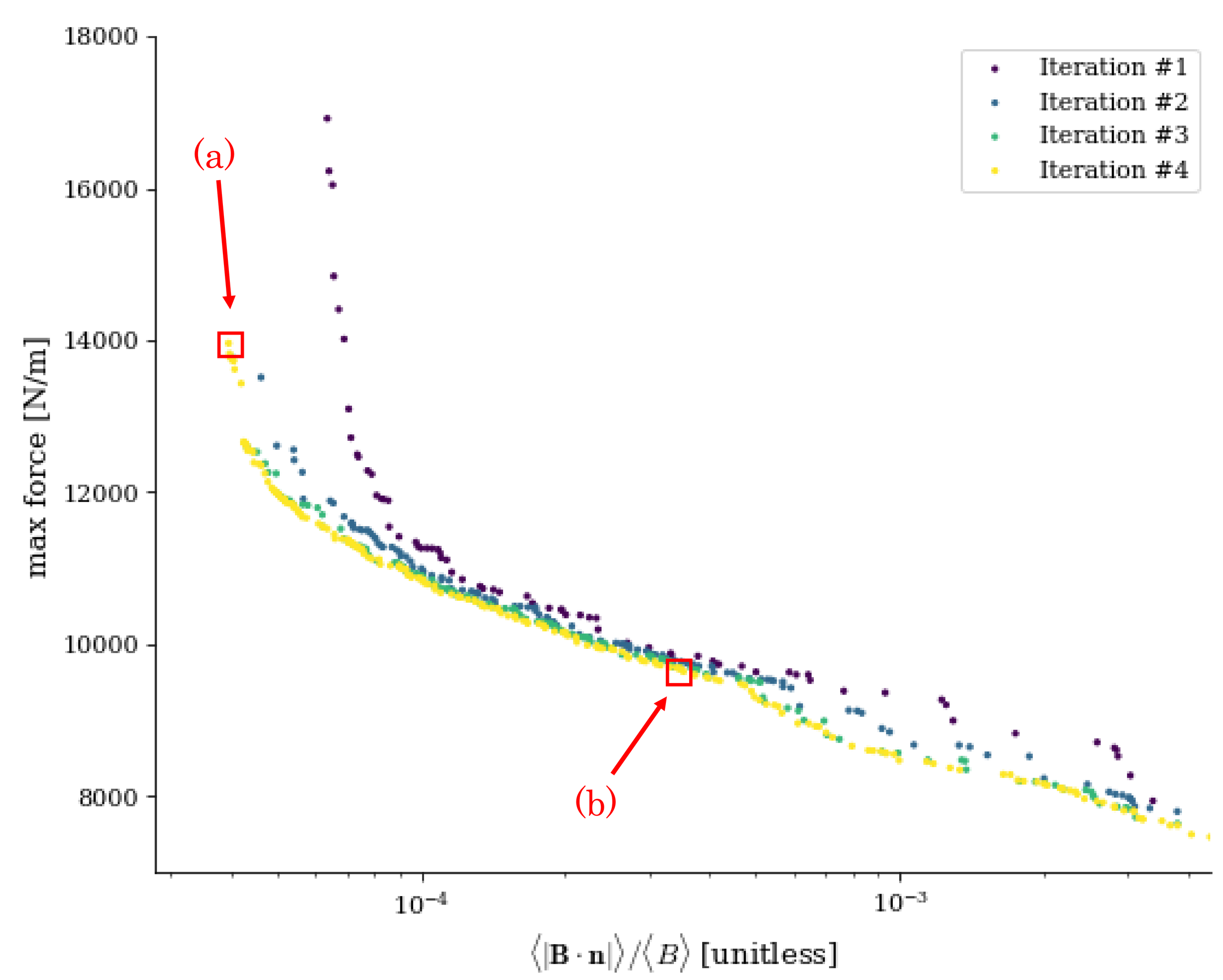
Coil Optimization

- We optimized coils for the 2021 Landreman-Paul precise QA configuration, $n_{\text{fp}} = 2$
- The objective function includes terms for (1) normalized flux, (2) Lorentz force density, (3) coil arc-length variation, (4) coil length, (5) minimum coil-coil distance, (6) minimum coil-surface distance, and (7) coil curvature

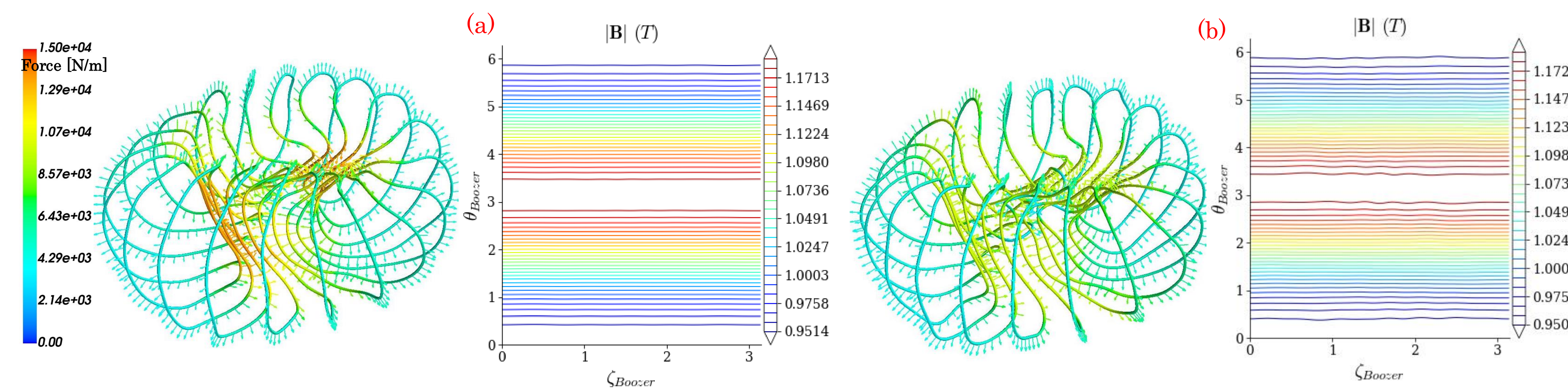
$$f = f_{\text{qf}} + f_{\text{F}} + f_{\sigma} + f_{\ell} + f_{\text{cc}} + f_{\text{cs}} + f_{\kappa} + f_{\kappa_{\text{MS}}}$$

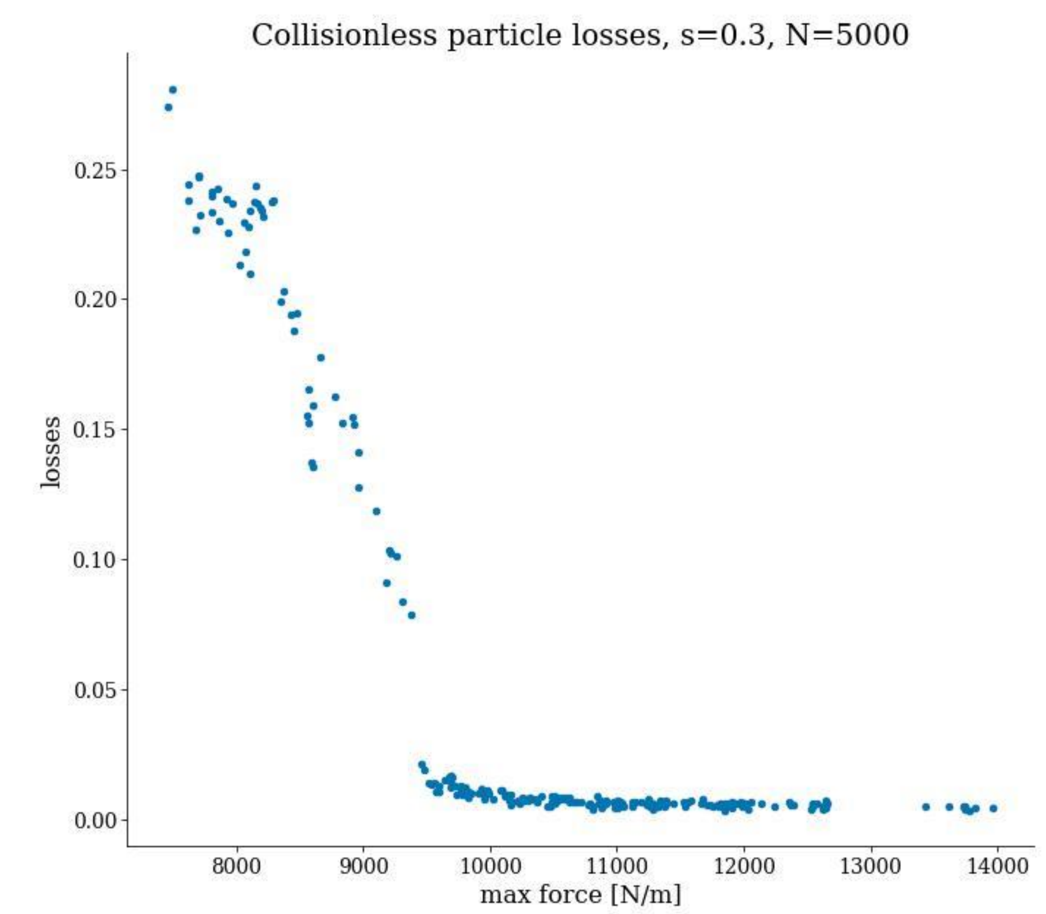


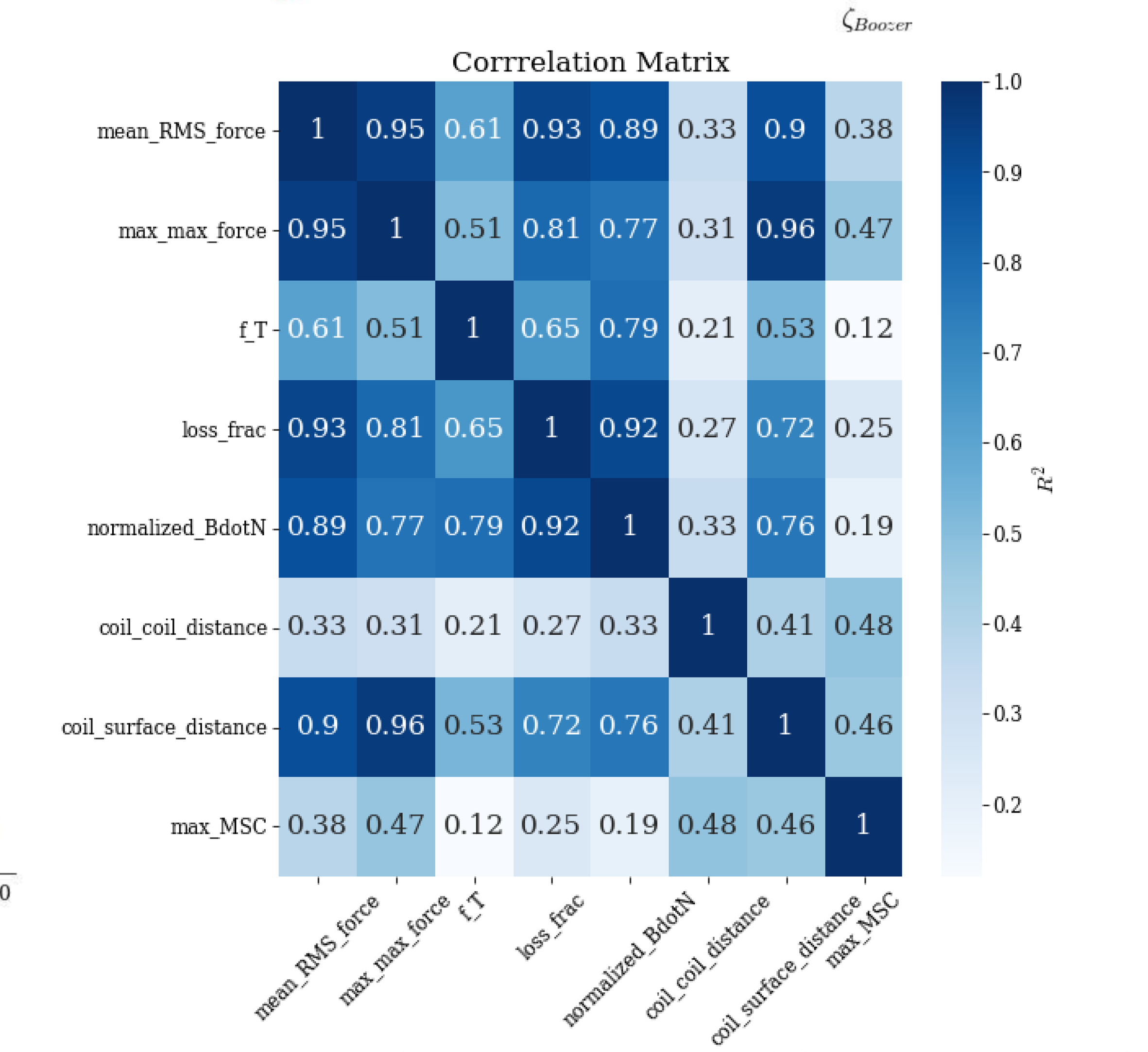
- We then perform a continuation method to explore the Pareto front in which optimizations are reperformed with slight variations to the initial hyperparameters



Results & Discussion







- Forces can be reduced with moderate detriment to quasisymmetry and losses; notably losses don't meaningfully increase until max forces pass a certain threshold
- Maximum coil force shows a high correlation with minimum coil-surface distance ($R^2 = 0.90$), though only a small correlation with minimum coil-coil distance ($R^2 = 0.33$), implying that reducing maximum coil forces is primarily dependent upon moving coil segments near the inboard side towards the plasma
- RMS coil force is less correlated with d_{cs} ($R^2 = 0.71$), implying that there is more flexibility in reactor design to reduce mean forces
- Maximum and RMS coil forces show moderate correlations ($R^2 = 0.57, 0.79$) with normalized flux, which can be explained by coil ripple
- This optimization tool allows for (a) significant reductions in mean forces with mild to moderate trade-offs with other elements, and (b) a clearer understanding of tradeoffs between max/mean forces and other design elements

References

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M. Landreman, S. Hurwitz, T. M. Antonsen, "Efficient calculation of self magnetic field, self-force, and self-inductance for electromagnetic coils with rectangular cross-section," *Nuclear Fusion* 65, 036008 (2025).

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