Topology optimization for designing stellarator coils

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Introduction

Stellarator optimization

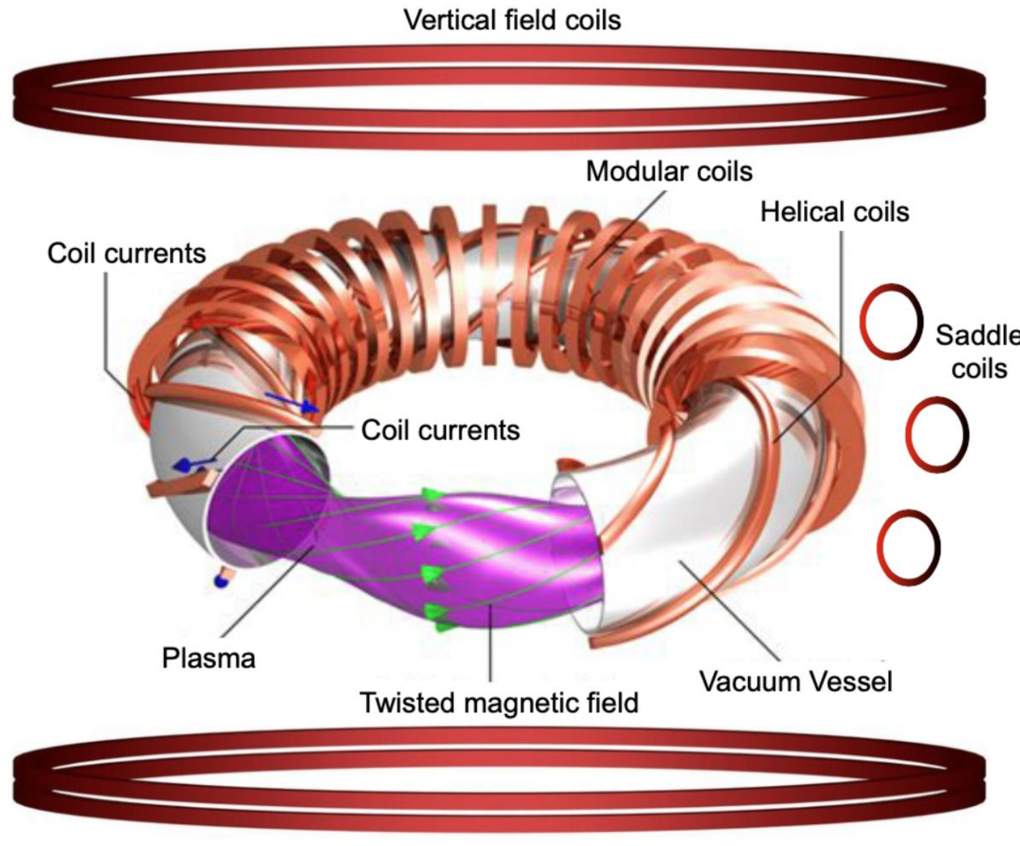
- Stellarators are plasma devices considered as possible future nuclear fusion reactors.
- Stellarators magnetic fields are optimized to obtain beneficial physics properties.
- Coils are then designed to produce these fields.

Stellarator coil optimization

- By changing the coil currents and coil shapes, coil optimization minimizes:

$$\min_{J} \int_S \|(\boldsymbol{B}_{\text{coil}} - \boldsymbol{B}_{\text{target}})\cdot \hat{\boldsymbol{n}}\|^2 dr$$

- Common methods are the "winding surface" (surface currents on a toroidal surface) and filaments (zero-thickness 3D curves in space).
- Both methods require restrictions on possible output coil topologies.
- *New voxel method: freedom in all three spatial dimensions like filaments, but with the convexity & linearity of the winding surface.*



A classical stellarator, adapted from Proll (2014) and courtesy of C. Brandt. A combination of modular, helical, saddle, and vertical field coils (red & orange) are used to match a target $\boldsymbol{B}\cdot\boldsymbol{n}$ *at the plasma surface S.*

New Current Voxels Method

- Volumetric grid of current-carrying "voxels", and decompose Biot-Savart contribution of each grid cell:

$$\boldsymbol{B}_{\text{coil}}(\boldsymbol{r})\cdot\hat{\boldsymbol{n}} = -\frac{\mu_0}{4\pi}\sum_{k=1}^{D}\int_{V_k'} \frac{\hat{\boldsymbol{n}}\times(\boldsymbol{r}-\boldsymbol{r}_k')}{\|\boldsymbol{r}-\boldsymbol{r}_k'\|^3}\cdot \boldsymbol{J}_k(\boldsymbol{r}_k')d\boldsymbol{r}_k'$$

- Divergence-free, linear basis functions

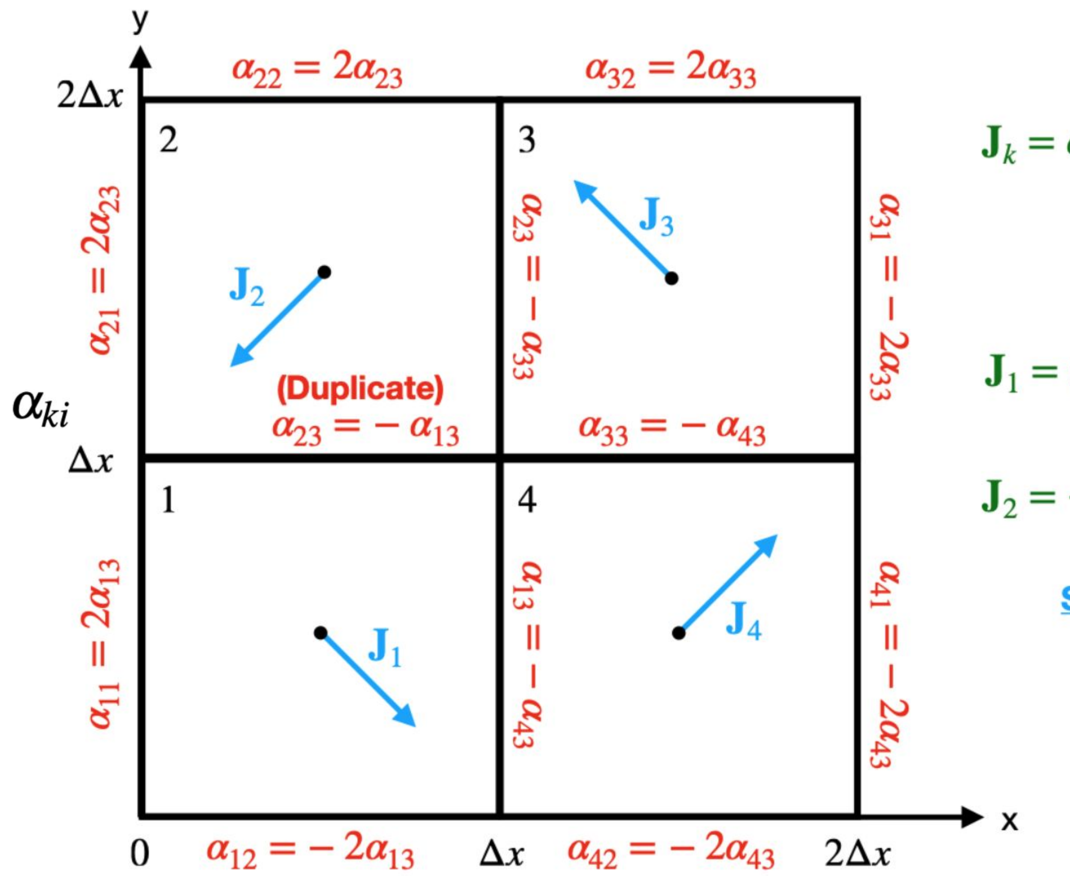
$$\boldsymbol{J}_k \equiv \boldsymbol{\alpha}_k\cdot\boldsymbol{\phi}_k(\boldsymbol{r}_k') = \sum_{i=1}^{N}\alpha_{ki}\boldsymbol{\phi}_{ki}.$$

centers of the cell at (x_k, y_k, z_k) and $\bar{\Delta} = (\Delta x_k \Delta y_k \Delta z_k)^{\frac{1}{3}}$,

$$X_k \equiv \frac{x - x_k}{\bar{\Delta}}, \quad Y_k \equiv \frac{y-y_k}{\bar{\Delta}}, \quad Z_k \equiv \frac{z-z_k}{\bar{\Delta}},$$

$$\boldsymbol{\phi}_k(\boldsymbol{r}_k') = \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}X_k\\-Y_k\\0\end{bmatrix}, \begin{bmatrix}X_k\\0\\-Z_k\end{bmatrix}$$

Solution *for four 2D square voxels. There are 12 free parameters and 11 unique constraints from flux matching at cell interfaces.*



Basis expansion

$$\mathbf{J}_k = \alpha_{k1}\begin{bmatrix}1\\0\\0\end{bmatrix} + \alpha_{k2}\begin{bmatrix}0\\1\\0\end{bmatrix} + \alpha_{k3}\begin{bmatrix}X_k\\-Y_k\\0\end{bmatrix}$$

Solution

$$\mathbf{J}_1 = \varphi\begin{bmatrix}2+X_k\\-2-Y_k\end{bmatrix} \quad \mathbf{J}_3 = \varphi\begin{bmatrix}-2+X_k\\2-Y_k\end{bmatrix}$$

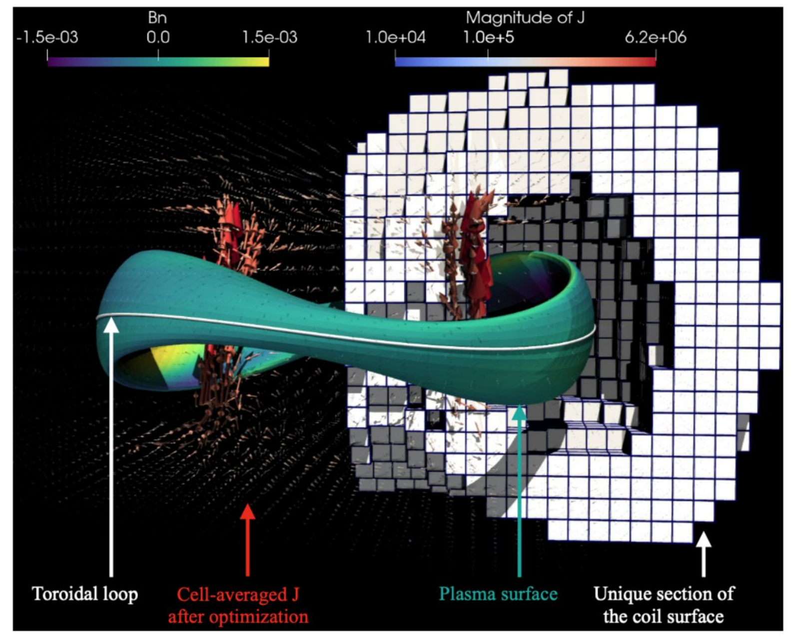
$$\mathbf{J}_2 = -\varphi\begin{bmatrix}2+X_k\\2-Y_k\end{bmatrix} \quad \mathbf{J}_4 = -\varphi\begin{bmatrix}-2+X_k\\-2-Y_k\end{bmatrix}$$

Solution at midpoints $(X_k, Y_k) = (0,0)$

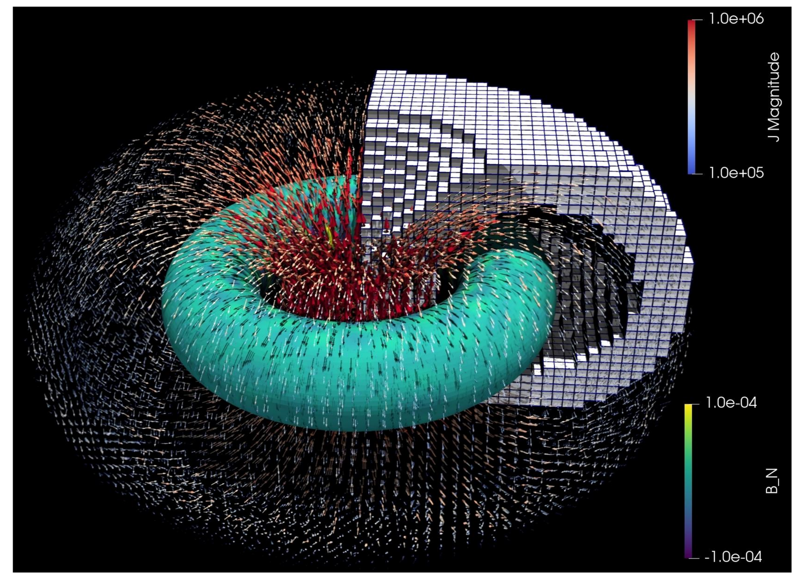
$$\mathbf{J}_1 = 2\varphi\begin{bmatrix}1\\-1\end{bmatrix} \quad \mathbf{J}_3 = 2\varphi\begin{bmatrix}-1\\1\end{bmatrix}$$

$$\mathbf{J}_2 = 2\varphi\begin{bmatrix}-1\\-1\end{bmatrix} \quad \mathbf{J}_4 = 2\varphi\begin{bmatrix}1\\1\end{bmatrix}$$

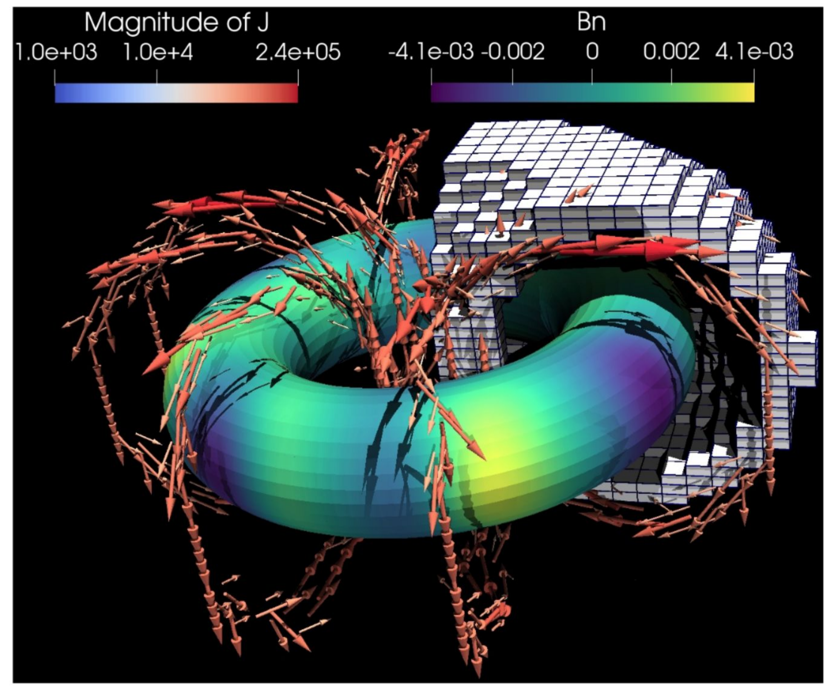
Optimization formulation



Full geometry for the Landreman & Paul QA stellarator, $(\lambda = 0)$*. The unique quarter of the voxel grid is pictured.* $\boldsymbol{B}\cdot\boldsymbol{n}$ *errors are shown on the plasma surface S and the cell-averaged* $\boldsymbol{J}$ *solution vectors are color-coded by* $\|\boldsymbol{J}\|$*.*



Full geometry & solution for an axisymmetric torus $(\lambda = 0)$*.*



Full geometry & solution for an axisymmetric torus $(\lambda > 0)$*.* $\boldsymbol{B}\cdot\boldsymbol{n}$ *errors are shown on the plasma surface S and the cell-averaged* $\boldsymbol{J}$ *solution vectors are color-coded by* $\|\boldsymbol{J}\|$*.*

- Coil optimization in the voxel method reduces to optimizing the basis coefficients of the $\boldsymbol{\alpha}$:

Match the target field | Tikhonov regularization | Avoid the trivial solution | Zero out most voxels

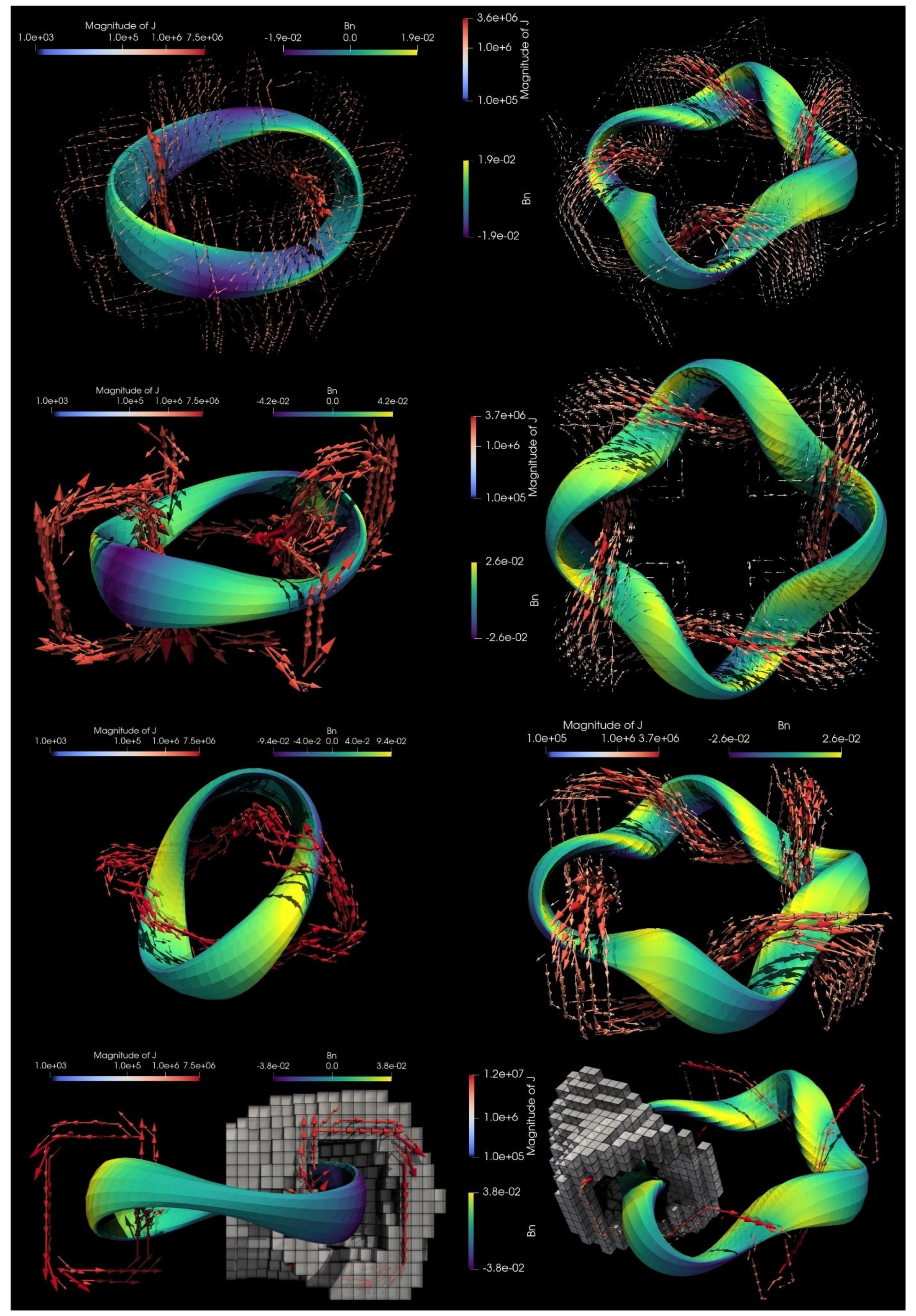
$$\min_{\boldsymbol{\alpha}}\left\{f_B(\boldsymbol{\alpha}) + \kappa f_K(\boldsymbol{\alpha}) + \sigma f_I(\boldsymbol{\alpha}) + \lambda\|\boldsymbol{\alpha}\|_0^G\right\}$$

$$s.t.\quad \boldsymbol{C\alpha} = \boldsymbol{0}.$$

Enforces div($\mathbf{J}$) = 0 everywhere

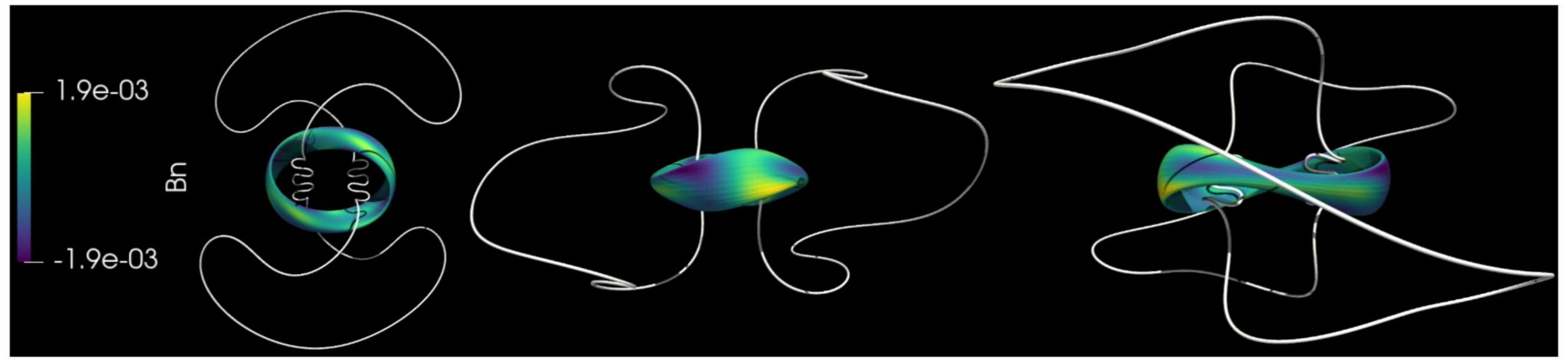
Voxel method results

Left below: *Landreman-Paul QA coil solutions of increasing sparsity* $(\lambda > 0)$ *from top to bottom (unique quarter of the voxel grid pictured at bottom).*
Right below: *Landreman-Paul QH coil solutions of increasing sparsity* $(\lambda > 0)$ *from top to bottom (unique eighth of the voxel grid pictured at bottom).*
$\boldsymbol{B}\cdot\boldsymbol{n}$ *plasma surface errors are shown and the cell averaged* $\boldsymbol{J}$ *solution is illustrated and color-coded by* $\|\boldsymbol{J}\|$*.*

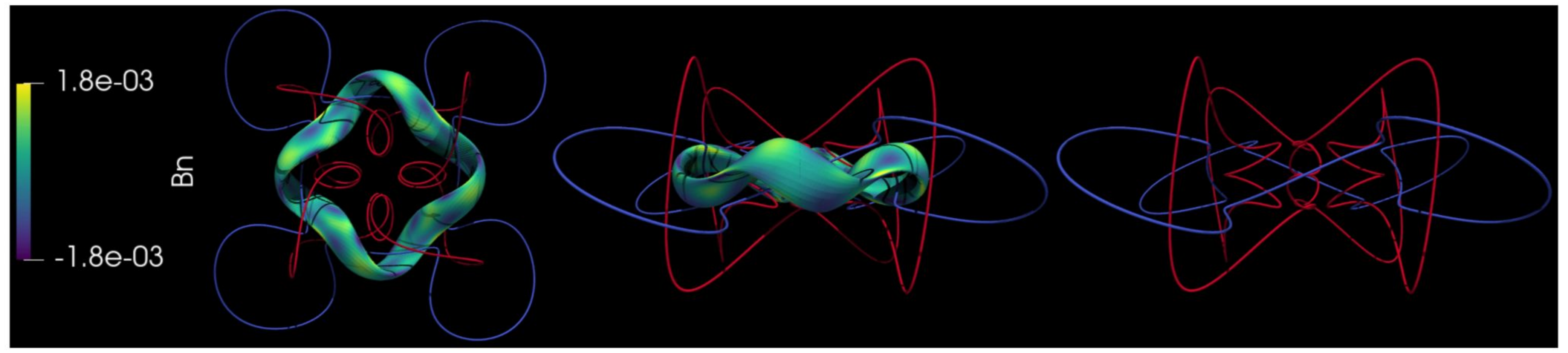


Initializing filament optimization

- Voxel method provides new topologies that can be further optimized as filaments.
- Using the figure-eight voxel QA solution, a *single helical coil* solution was found with acceptable field errors.
- Using the helical voxel QH solution, a *double helical coil* solution was found with acceptable field errors.



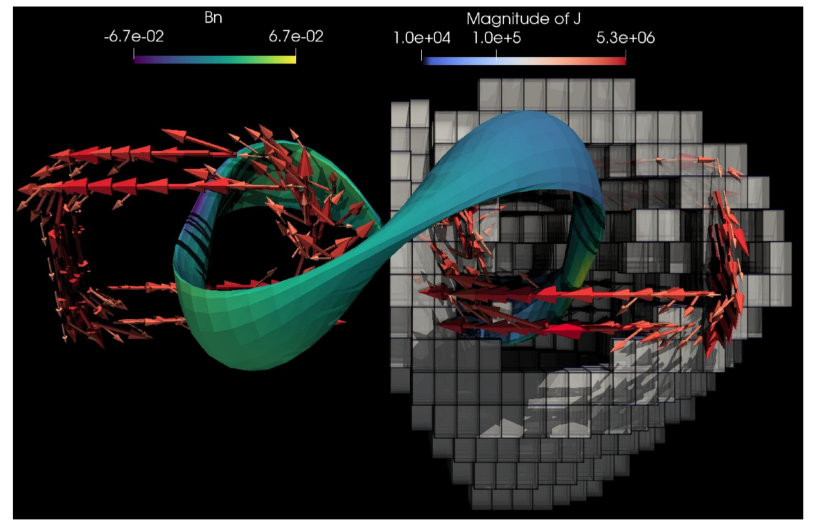
Three views of a 40 meter filament coil generated from a voxel solution for the Landreman-Paul QA stellarator.



Three views of the helical coils with combined 24 + 29 = 53 meter length, generated from a voxel solution for the Landreman-Paul QH stellarator. The right-most panel only shows the coils for a better look at the geometry.

Discussion & future work

- **Introduced a new method for coil optimization without topological assumptions on the coils.**
- This work provides principled topology for initializing more complex filament optimization for stellarators.
- Future work:
  - Implementation of higher-order basis functions.
  - Tetrahedral meshes.
  - Algorithmic speedups through improved iterative solvers and preconditioners.
  - Additional loss terms in the optimization for reducing coil forces or coil curvatures.
  - A reformulation may be possible that has current conservation by construction.
  - Initial conditions can bias the solutions towards producing a particular topological structure or a certain number of identifiable coils.



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