Visualizing Stellarator Objective Functions

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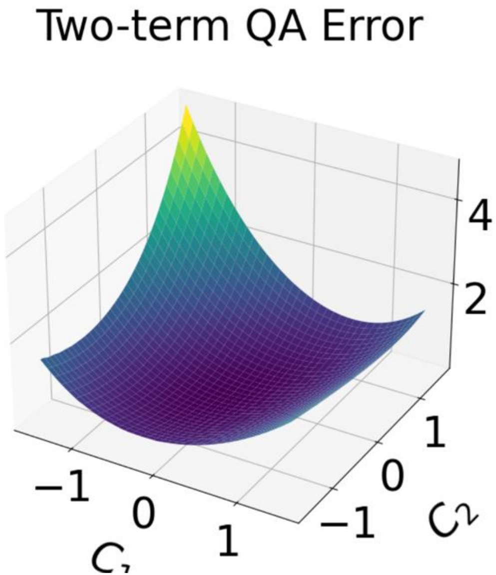

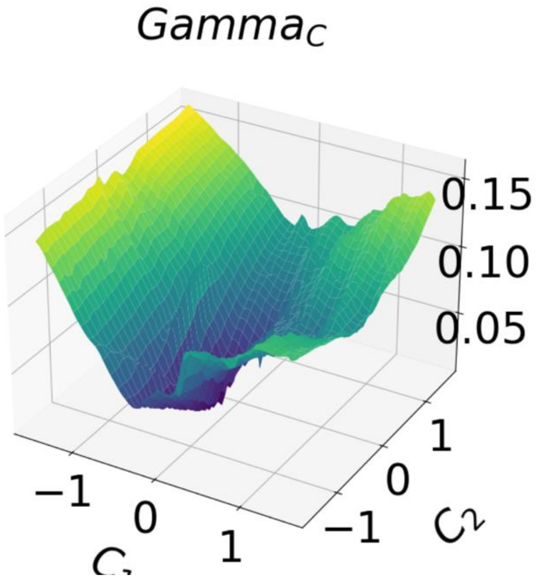

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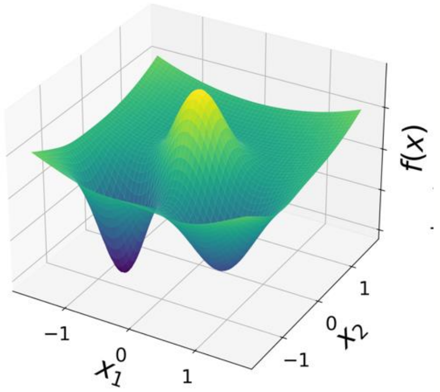
Motivation

- In optimization for stellarators, the optimization algorithms(i.e. optimizers) can get stuck, and the solution depends on initial condition.
- This limits us from finding all solutions from the parameter space.

To avoid local minima and ultimately to explore more of the parameter space

➡ **Need to understand the landscape of the stellarator optimization problems**



Stellarator Optimization

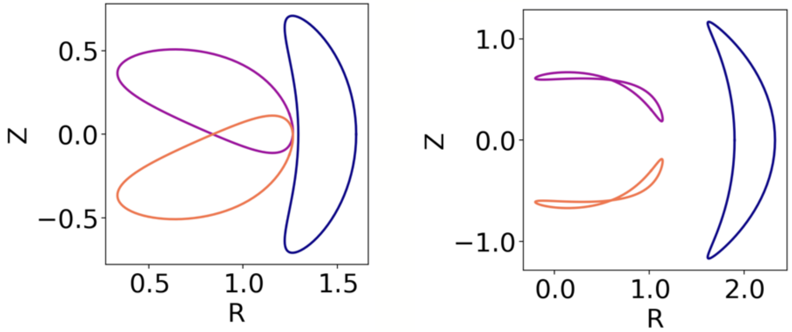
Given a "objective function" $f: \mathbb{R}^n \rightarrow \mathbb{R}$, minimize $f(\boldsymbol{x})$

(aka "loss function", "cost function")

Parameter space: $\boldsymbol{x}$

$\boldsymbol{x}$ = shape of toroidal surface (25 - 121 dofs)

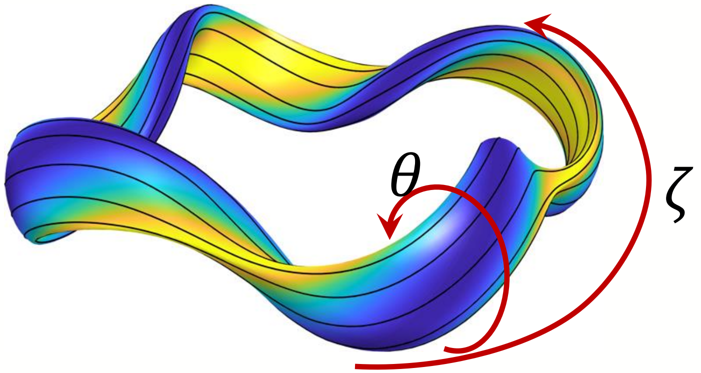
- Part of the parameter space corresponds to unphysical self-intersecting shapes

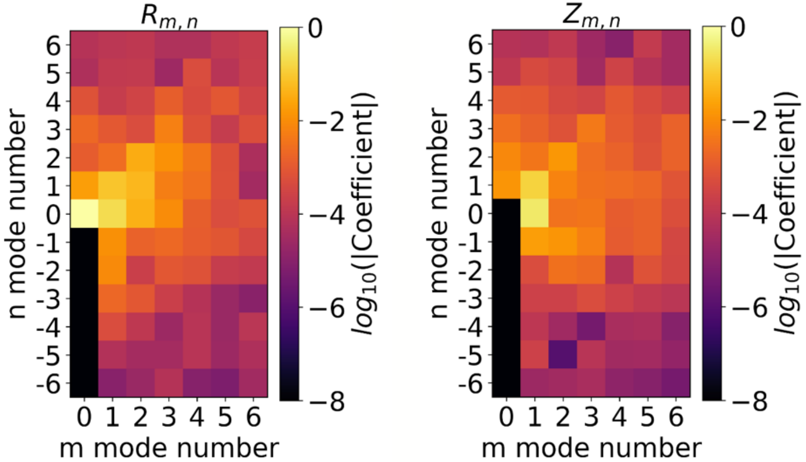


Objective functions: $\boldsymbol{f}$

- Large volume of good magnetic surfaces (no islands and chaos)
- Rotational transform (i.e. inverse safety factor)
- Good confinement of particle trajectories
- Low neoclassical transport
- Low turbulent transport
- Magnetohydrodynamic (MHD) stability

Parameter space





Parametrization of boundary surface:

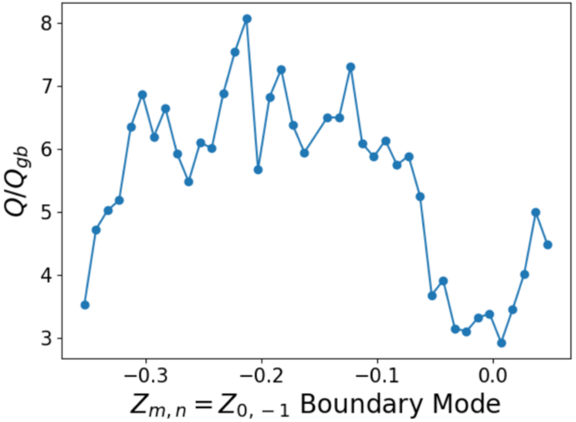
$$R(\theta,\zeta) = \sum_{m,n} R_{m,n} \cos(m\theta - n_{fp} n\zeta) \qquad Z(\theta,\zeta) = \sum_{m,n} Z_{m,n} \sin(m\theta - n_{fp} n\zeta)$$

ζ = toroidal angle, θ = poloidal angle, n_{fp} = number of field periods

Parameter space for optimization: $x = [R_{m,n}, Z_{m,n}]$

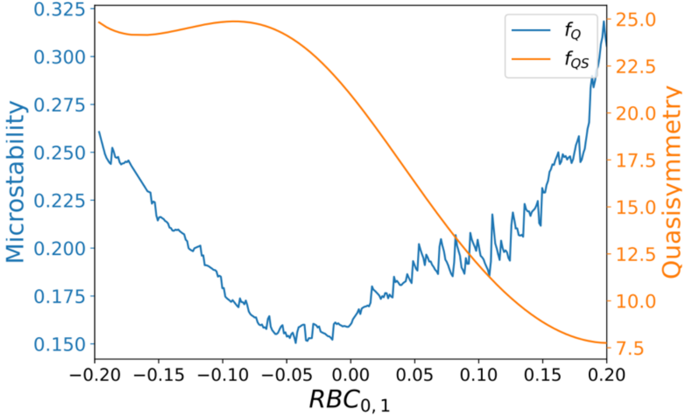
1D landscape

1D Interpolation: Changing one of the modes ➡ $x = [R_{m,n}, Z_{m,n}]$ $R_{0,1} \in [-0.2, 0.2]$

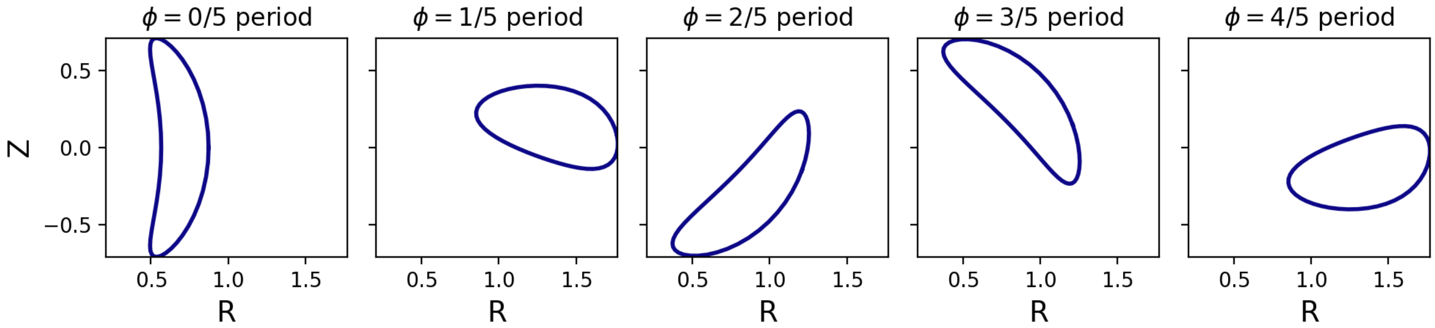


Kim JPP (2024)

Jorge arXiv (2023)

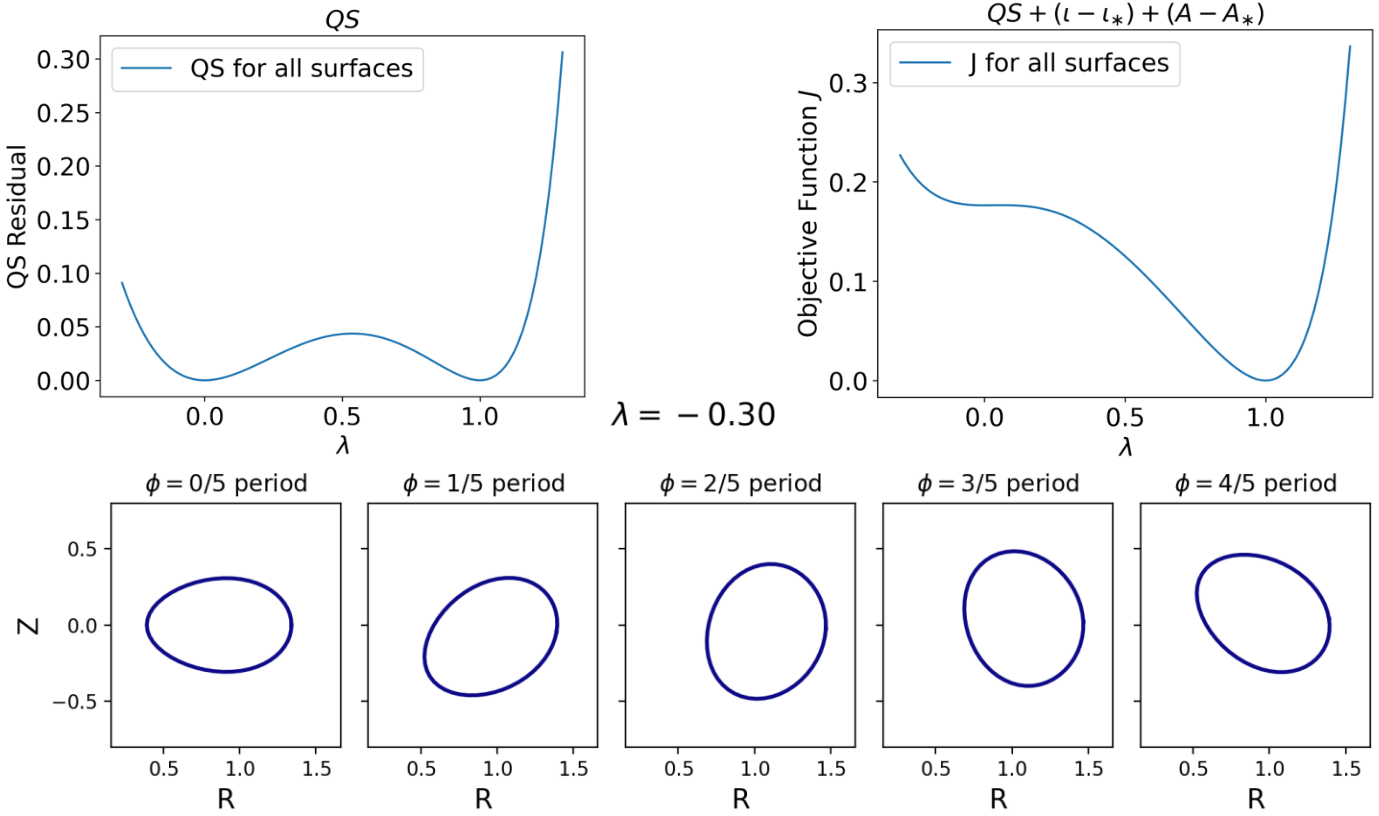


$RC_{0,1} = -0.30$

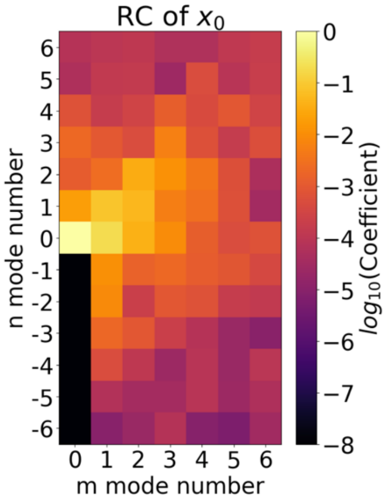


1D landscape

1D Interpolation between two configurations ➡ $x' = (1-\lambda)x_1 + \lambda x_2$



Method of Visualization



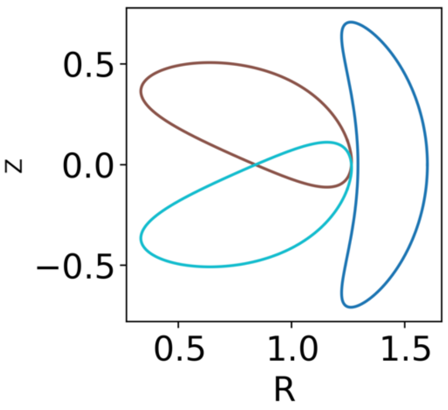
x_0

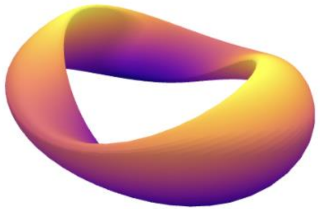
Array of Flattened Fourier Coefficients

$$R(\theta,\zeta) = \sum_{m,n} R_{m,n} \cos(m\theta - n\zeta)$$

$$Z(\theta,\zeta) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\zeta)$$

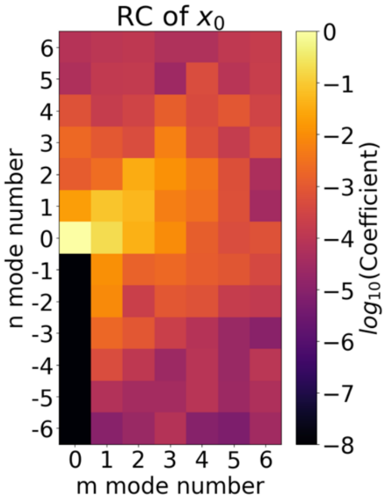
$x' = x_0$





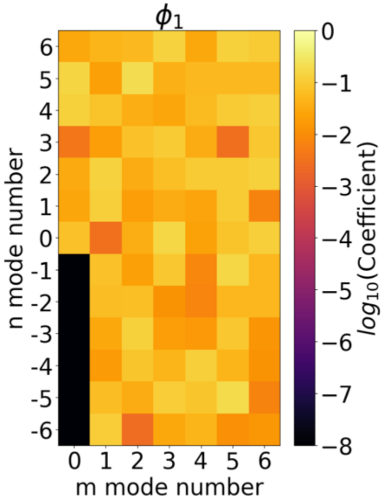
Method of Visualization

$$x' = x_0 + C_1\phi_1 + C_2\phi_2$$



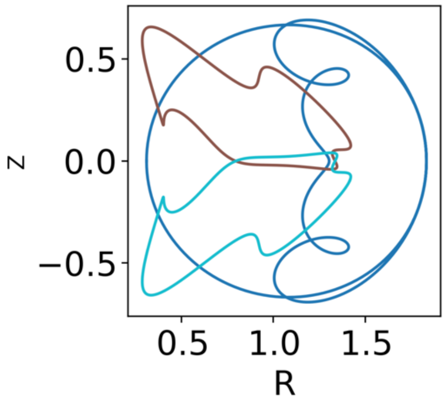
x_0

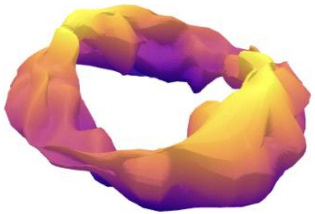
Array of Flattened Fourier Coefficients



ϕ_i

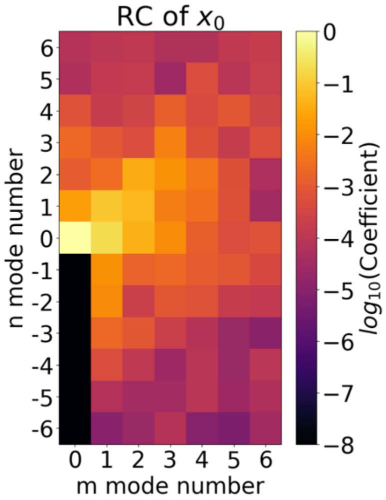
Random Sample from Gaussian Distribution





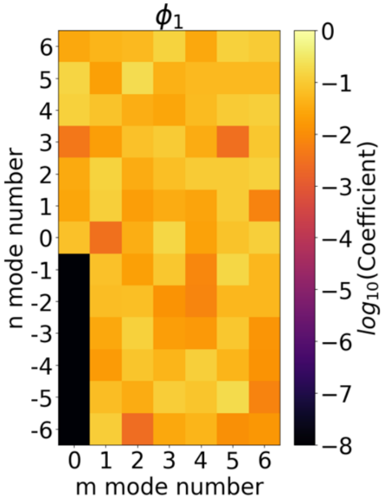
Method of Visualization

$$x' = x_0 + C_1 q_1 + C_2 q_2$$



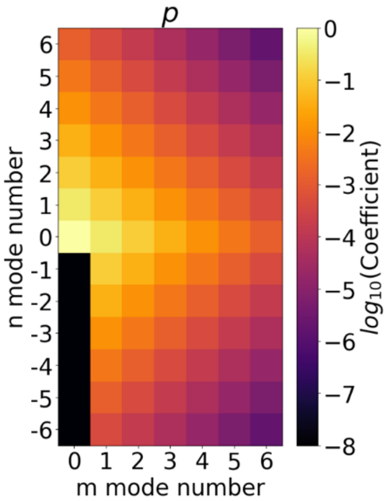
x_0

Array of Flattened Fourier Coefficients



ϕ_i

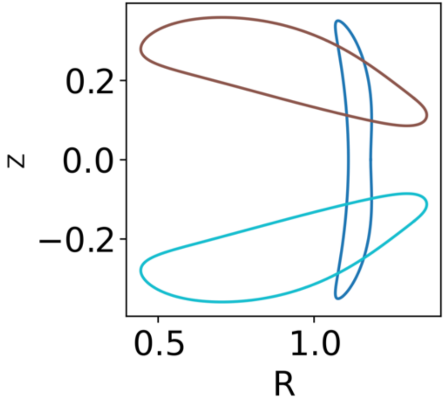
Random Sample from Gaussian Distribution



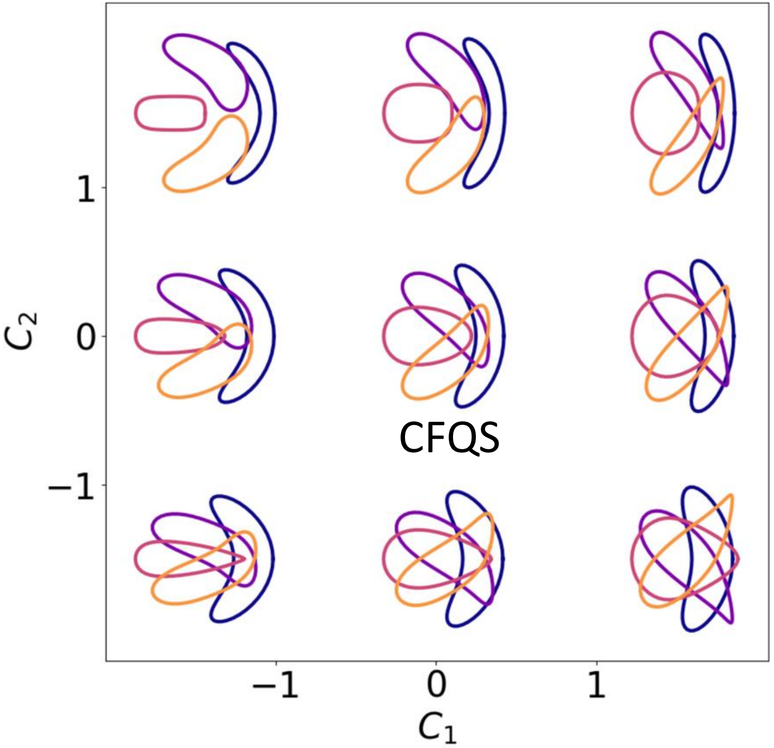
p

Exponential Weighting for Smooth Surfaces

$$q_i = p \odot \phi_i$$



Comparing different proxies for neoclassical confinement

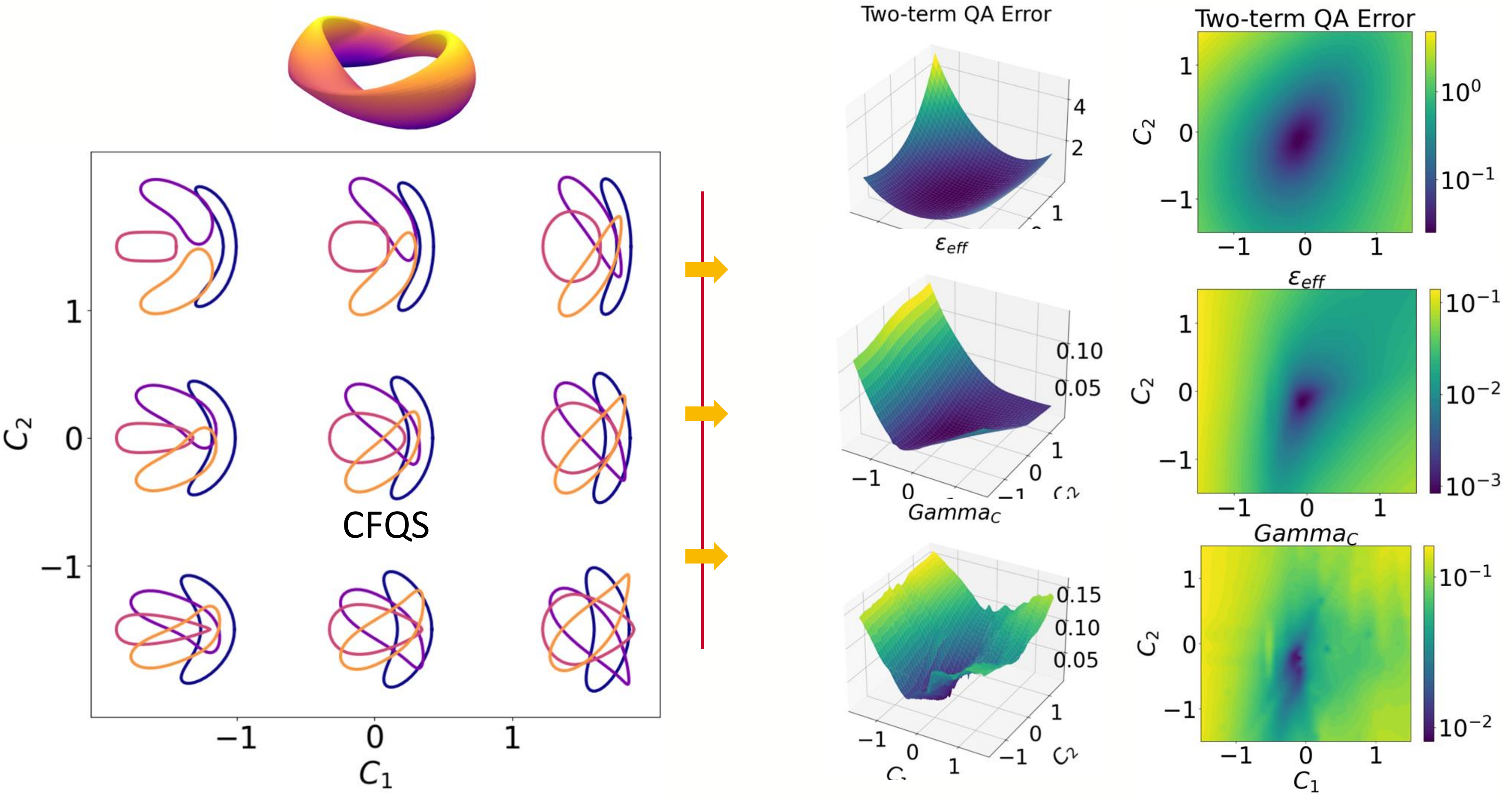


Two-term quasisymmetry

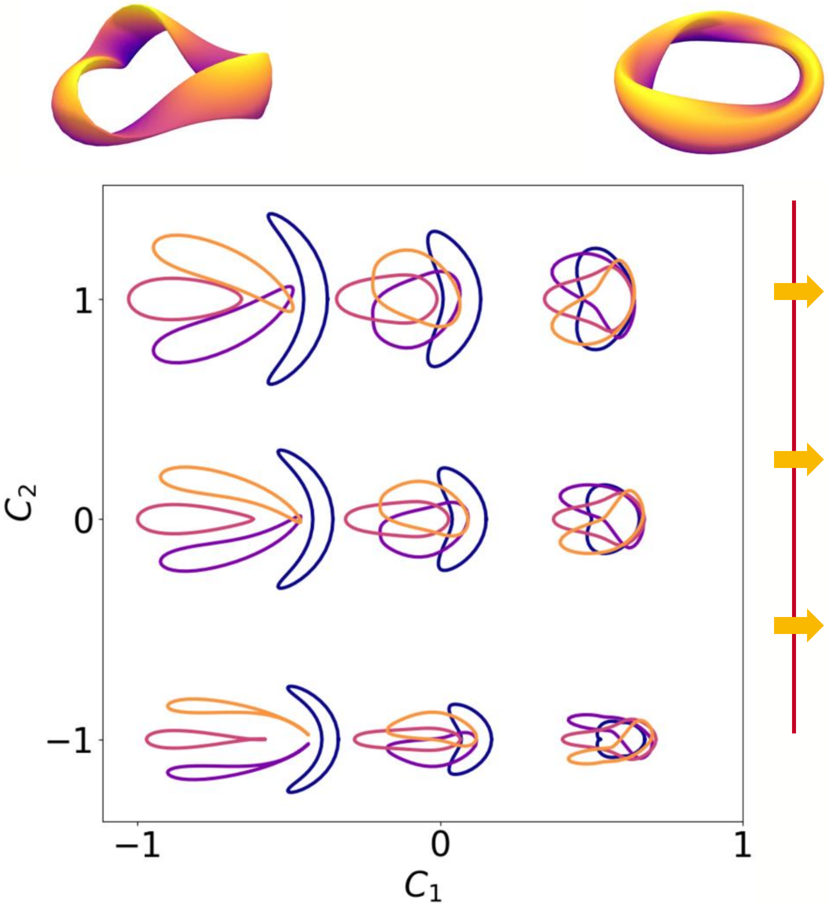
ϵ_{eff}: effective ripple

$Gamma_c$

Comparing different proxies for neoclassical confinement



1D interpolation + 1D random direction



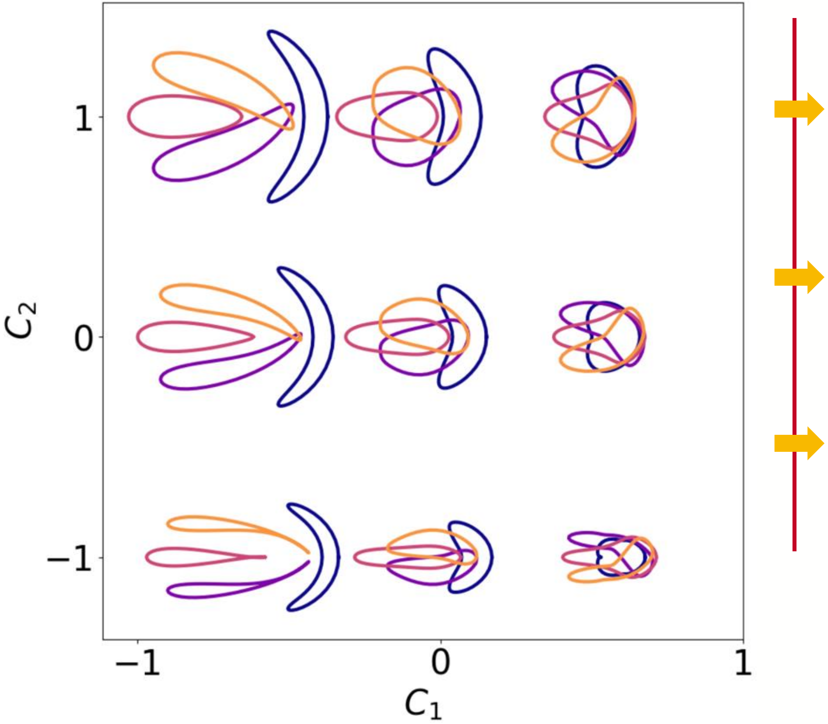
$$x' = x_1 + C_1(x_2 - x_1) + C_2 q_2$$

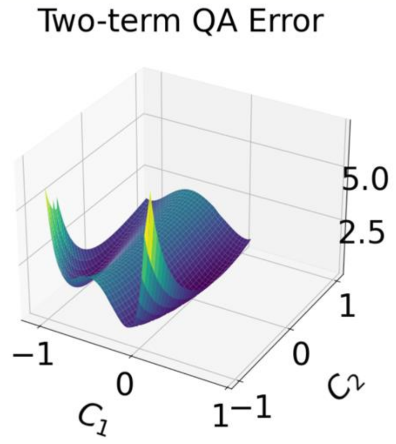
Two-term quasisymmetry

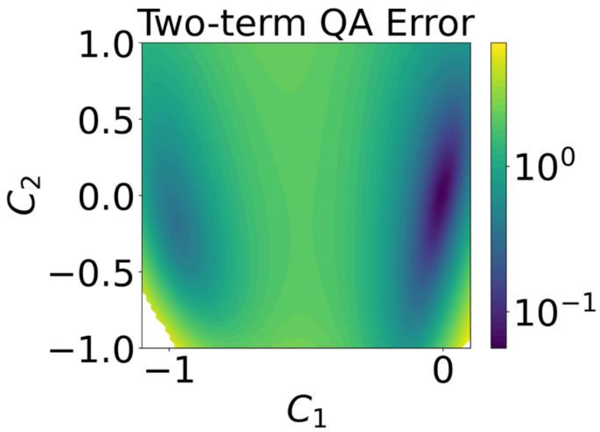
ϵ_{eff}: effective ripple

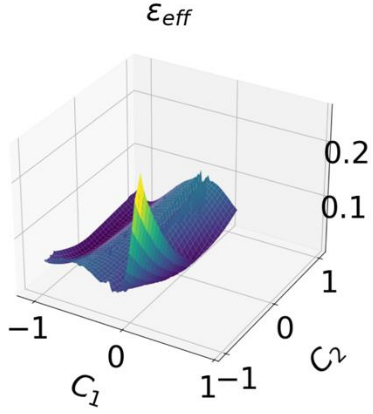
$Gamma_c$

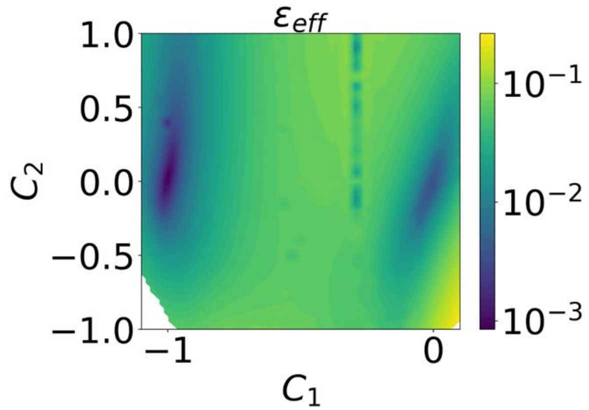
1D interpolation + 1D random direction











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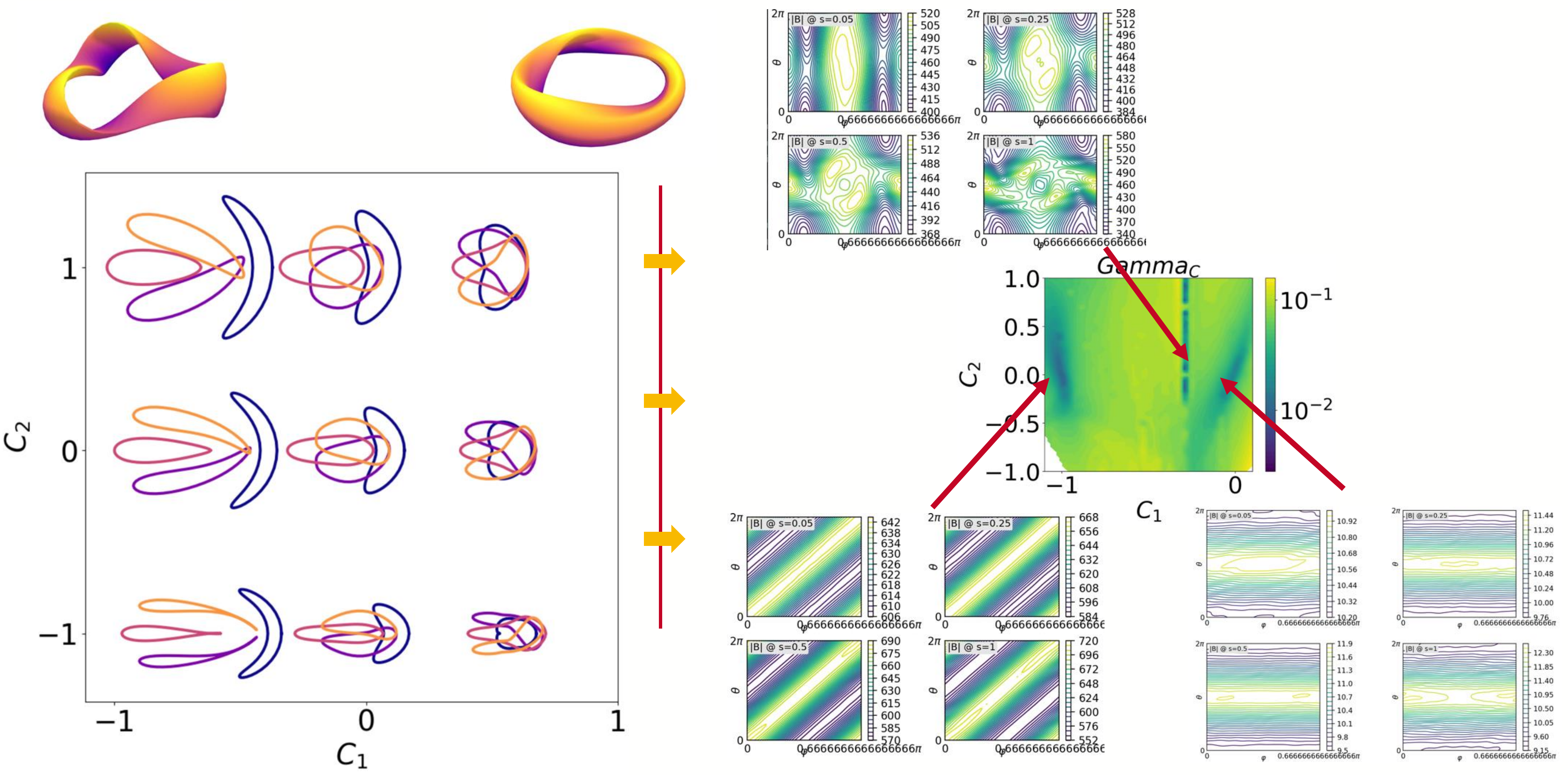
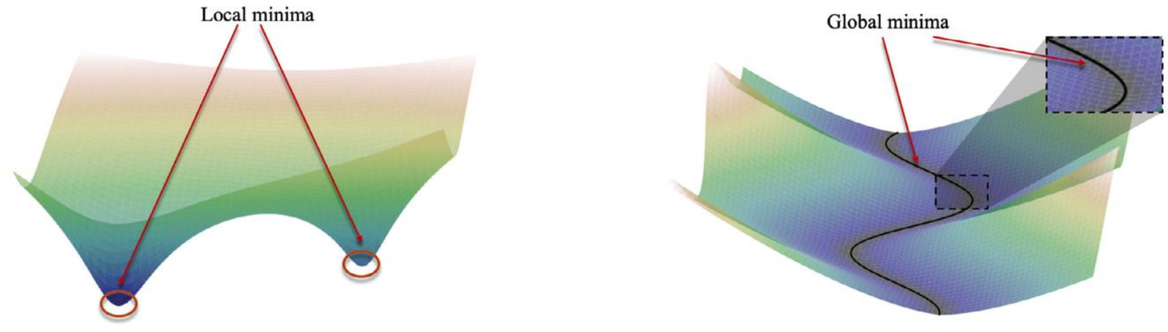


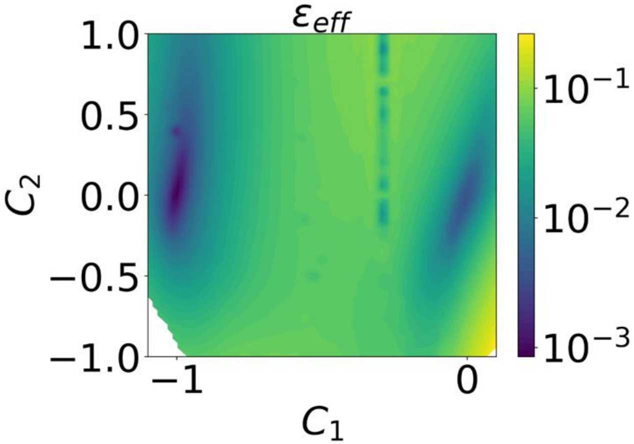
Ongoing work: Closing the gap



(a) Loss landscape of under-parameterized models

(b) Loss landscape of over-parameterized models

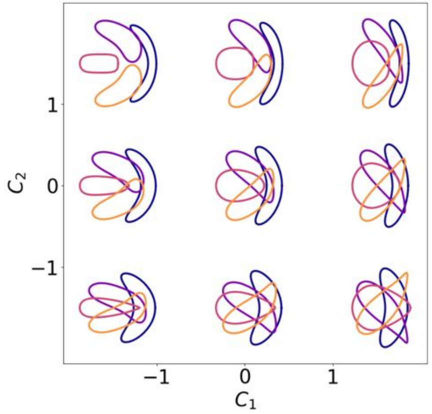
Liu, C., Zhu, L. and Belkin, M., 2022 *Applied and Computational Harmonic Analysis*.

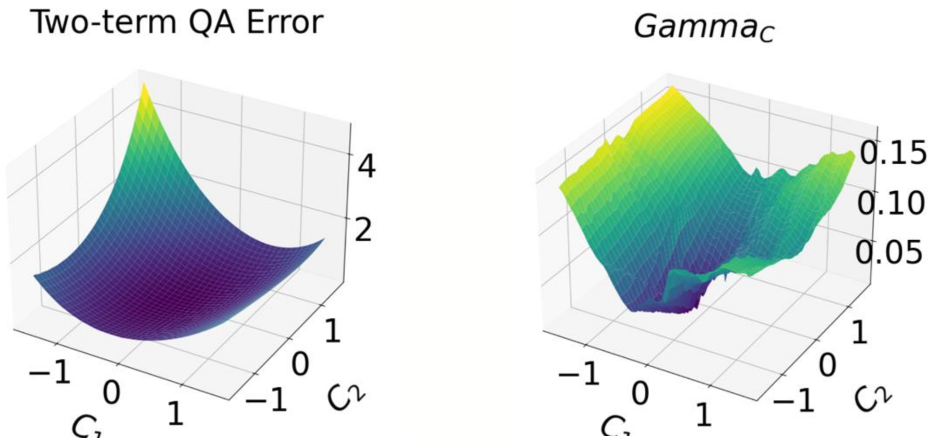


Garipov, Timur, et al. 2018. NeurIPS

Closing thoughts

- The new visualization methods can be used to explore objective functions in stellarator optimization
- There seems to be a clear favorite for optimizer for certain objective functions due to its smoothness.
- We can further build qualitative intuition about the stellarator objective function landscape.





Thank you

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