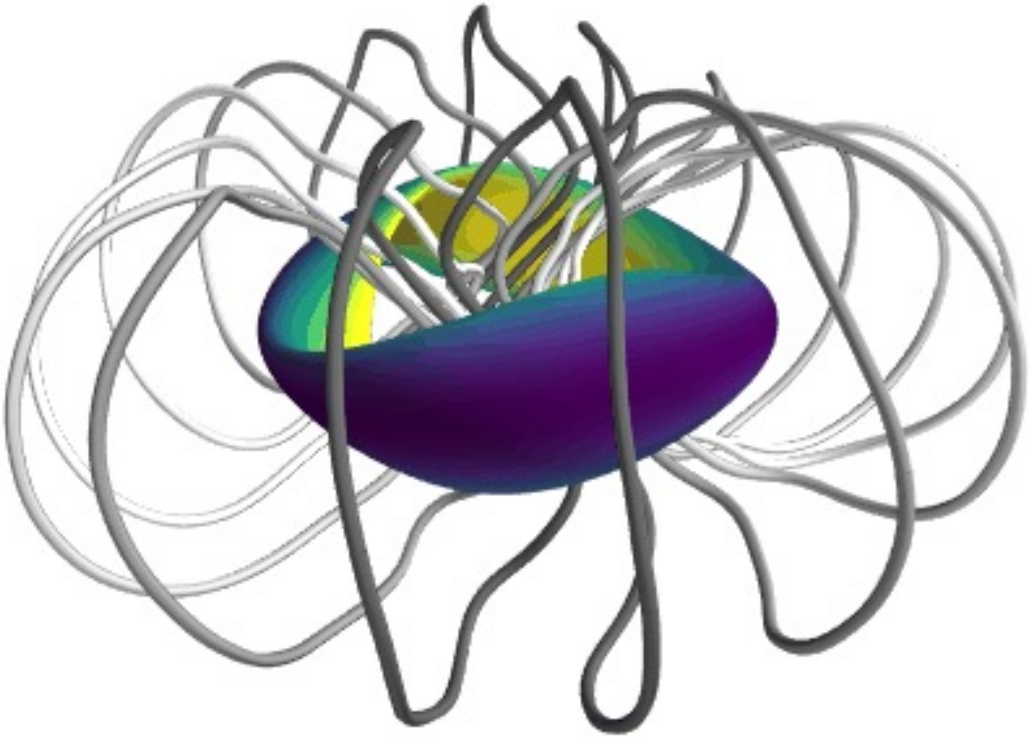
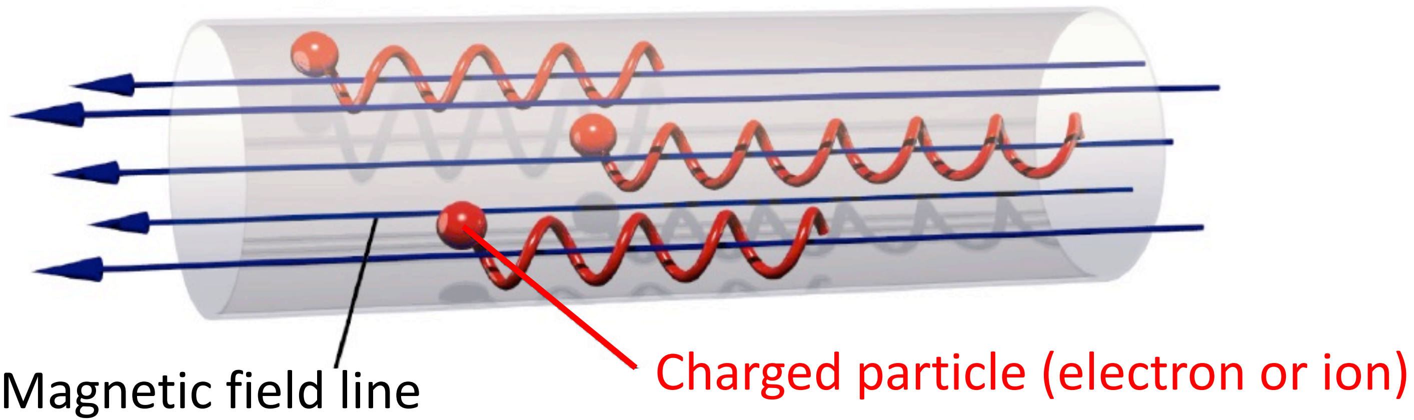
Some numerical topics in stellarator optimization

Matt Landreman, UMD, Institute for Research in Electronics & Applied Physics (IREAP)



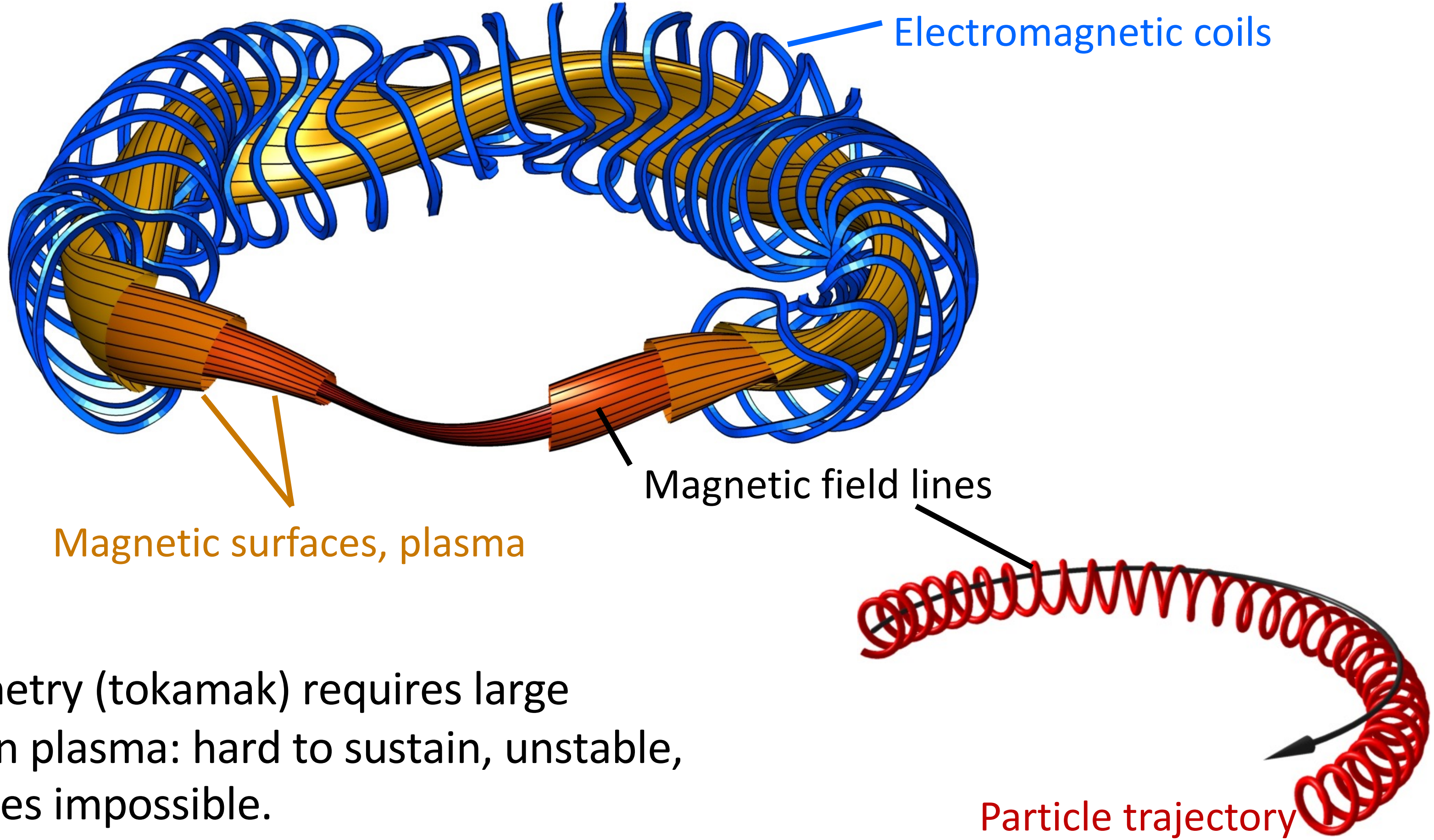
David Bindel[1], Stefan Buller[2], Bill Dorland[2], Rogerio Jorge[3], Alan Kaptanoglu[2], Patrick Kim[2], Noah Mandell[4], Misha Padidar[1], Elizabeth Paul[5]
1 Cornell, 2 UMD, 3 IST Lisbon, 4 Princeton Plasma Physics Laboratory, 5 Columbia

Magnetic fields can be used to confine charged particles & plasma



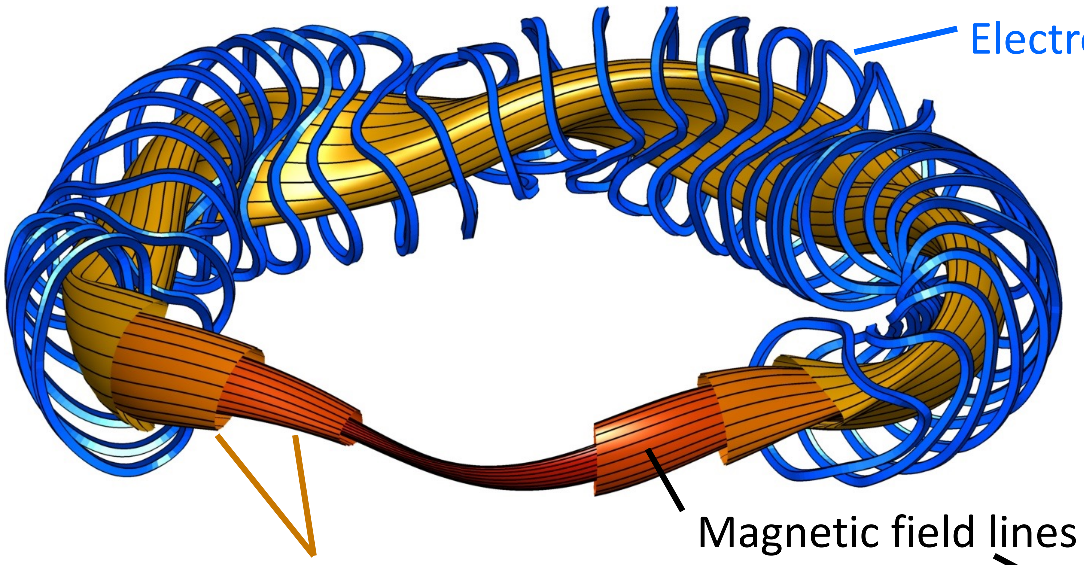
There is no confinement along the magnetic field,
so bend it into a torus.

A *stellarator* is a configuration of magnets for confining plasma without continuous rotation symmetry



Axisymmetry (tokamak) requires large current in plasma: hard to sustain, unstable, sometimes impossible.

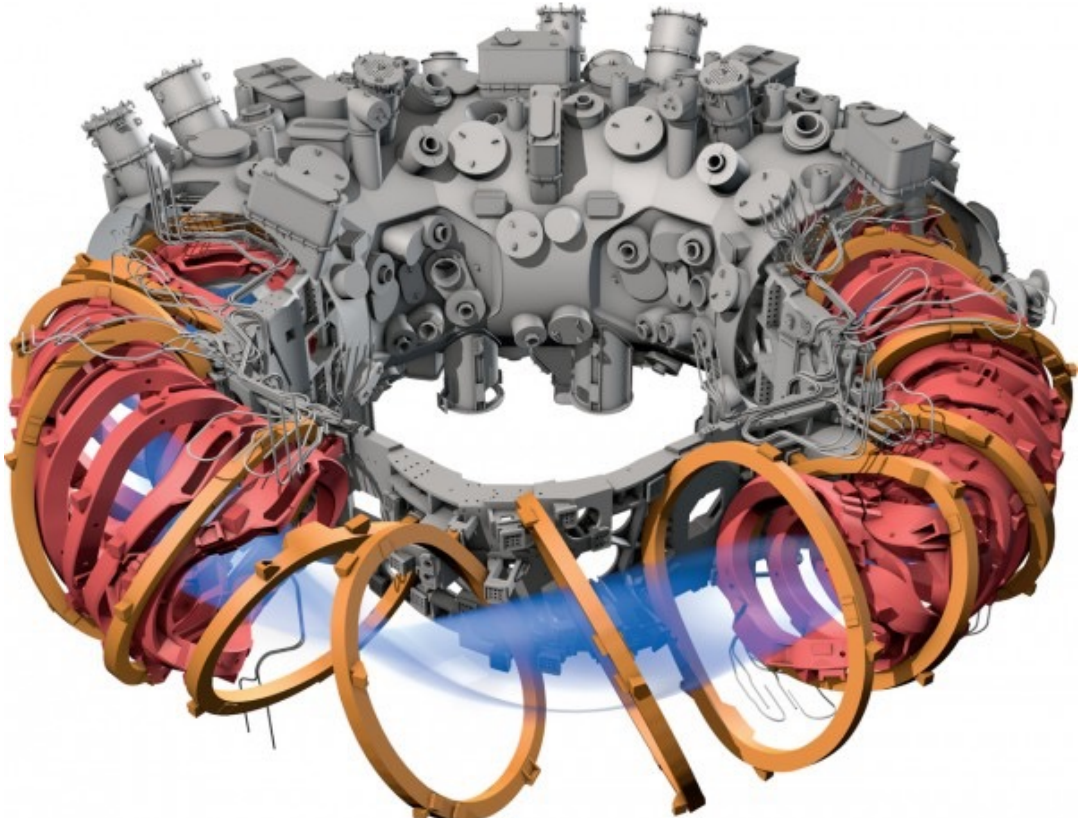
A *stellarator* is a configuration of magnets for confining plasma without continuous rotation symmetry



Applications:

- Fusion energy
- Particle trap for basic physics

Example: W7-X (Germany)



Clery, Science (2022)

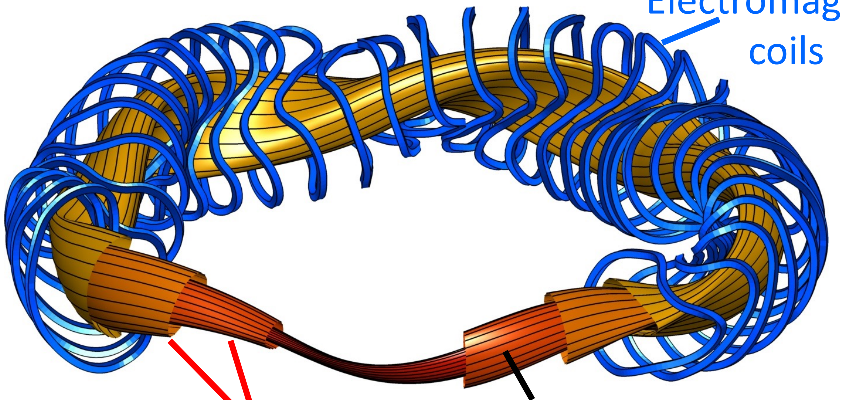
TechCrunch+ Fundraising

Fusion startup Type One Energy gets $29M seed round to fast-track its reactor designs

Tim De Chant @tdechant / 8:00 AM EDT • March 28, 2023

Stellarator plasma & coil shapes can be optimized for several objectives

- Field lines lie on nested 2D toroidal surfaces, not volume-filling
- Good confinement of particle trajectories
- Buildable coil shapes
- Any plasma current doesn’t modify **B** too much
- Magnetohydrodynamic stability
- Reduced turbulence



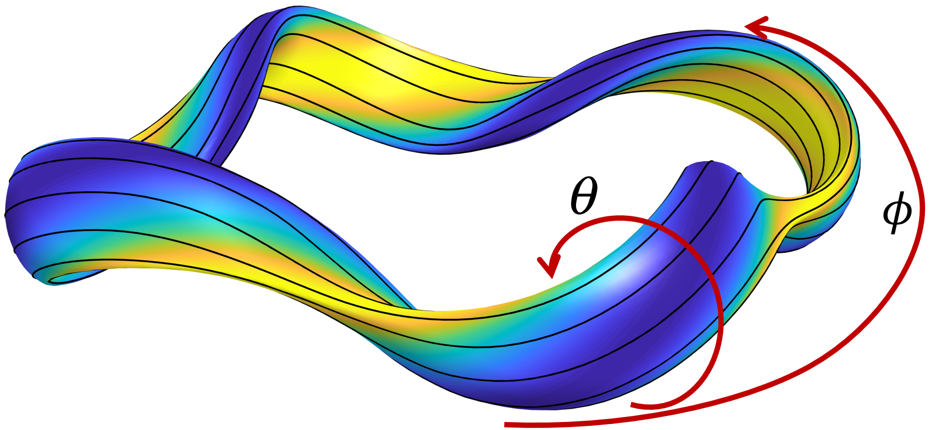
Outline

- Optimizing plasma shape, directly targeting particle confinement.
- Optimizing plasma shape using a surrogate ("quasisymmetry")
- Optimizing plasma shape to reduce turbulence
- Topology optimization for magnets

Outline

- Optimizing plasma shape, directly targeting particle confinement.
- Optimizing plasma shape using a surrogate (“quasisymmetry”)
- Optimizing plasma shape to reduce turbulence
- Topology optimization for magnets

For plasma shape optimization, parameter space is usually Fourier amplitudes of the boundary surface



$$R(\theta,\phi)=\sum_{m,n} R_{m,n}\cos(m\theta-n\phi),$$

$$Z(\theta,\phi)=\sum_{m,n} Z_{m,n}\sin(m\theta-n\phi)$$

Room for innovation here…

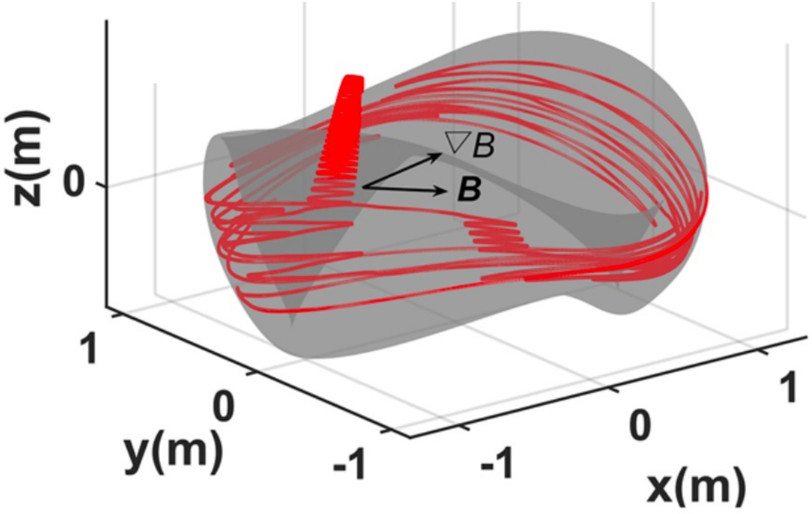
Shape of boundary surface determines the magnetic field $\boldsymbol{B}$ inside (up to scale factor).

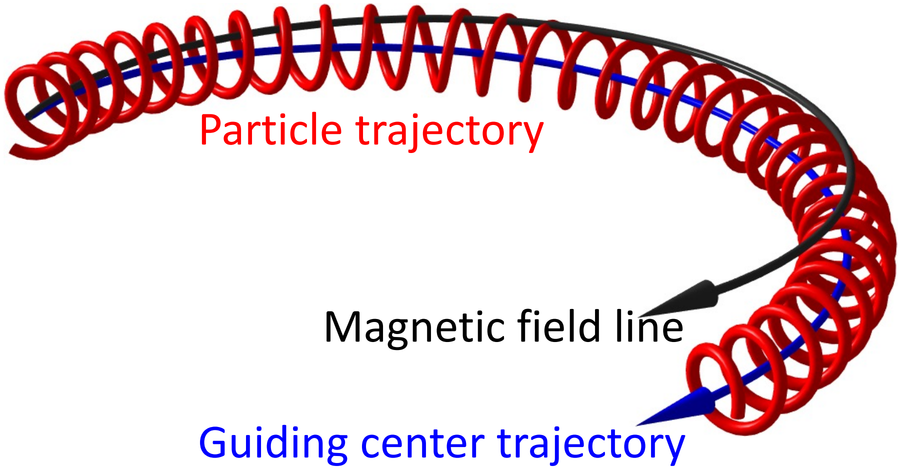
- Without plasma: $\boldsymbol{B} = \nabla V,\ \nabla^2 V = 0$ in Ω, $\boldsymbol{n}\cdot\nabla V = 0$ on $\partial\Omega$.
- With plasma: $\nabla\times(\nabla\times\boldsymbol{B}) = \nabla p$. Also need to specify pressure p & current on each interior surface.

Charged particle trajectories can be simulated accurately using "guiding center" ODEs

$$\frac{d\mathbf{R}}{dt} = v_{\|}\mathbf{b} + \frac{m v_{\|}^2}{qB^3}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\mathbf{B}\times\nabla B,$$

$$\frac{dv_{\|}}{dt} = -\frac{v_{\perp}^2}{2B}\mathbf{b}\cdot\nabla B.$$



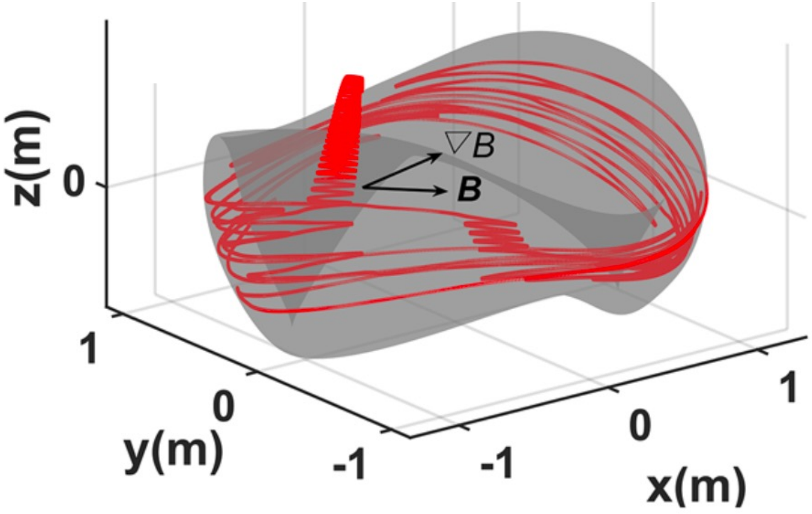


Su et al (2020)

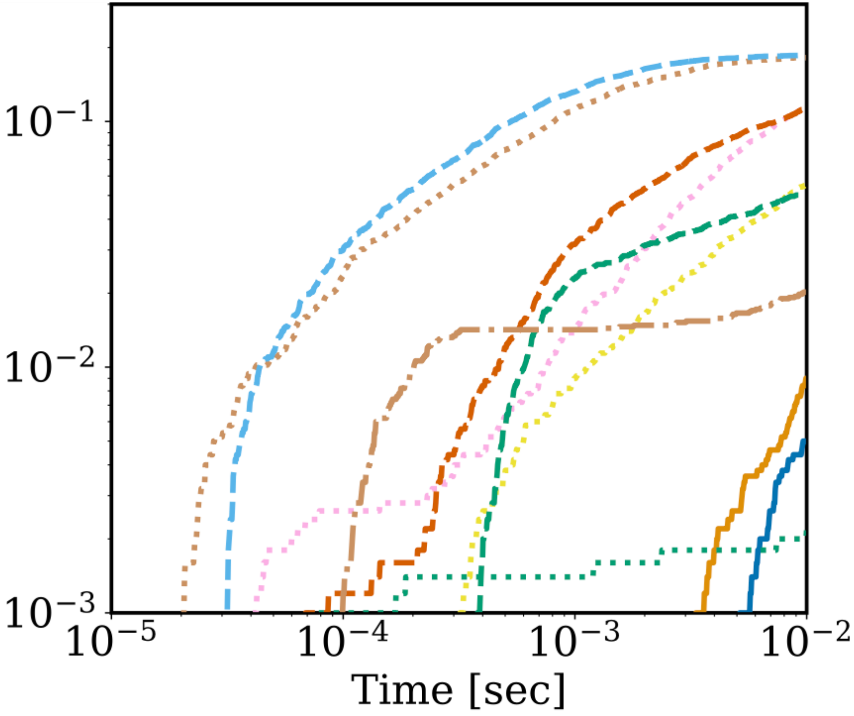
Charged particle trajectories can be simulated accurately using "guiding center" ODEs

$$\frac{d\mathbf{R}}{dt} = v_{\|}\mathbf{b} + \frac{mv_{\|}^2}{qB^3}\left(v_{\|}^2 + \frac{v_{\perp}^2}{2}\right)\mathbf{B}\times\nabla B,$$

$$\frac{dv_{\|}}{dt} = -\frac{v_{\perp}^2}{2B}\mathbf{b}\cdot\nabla B.$$



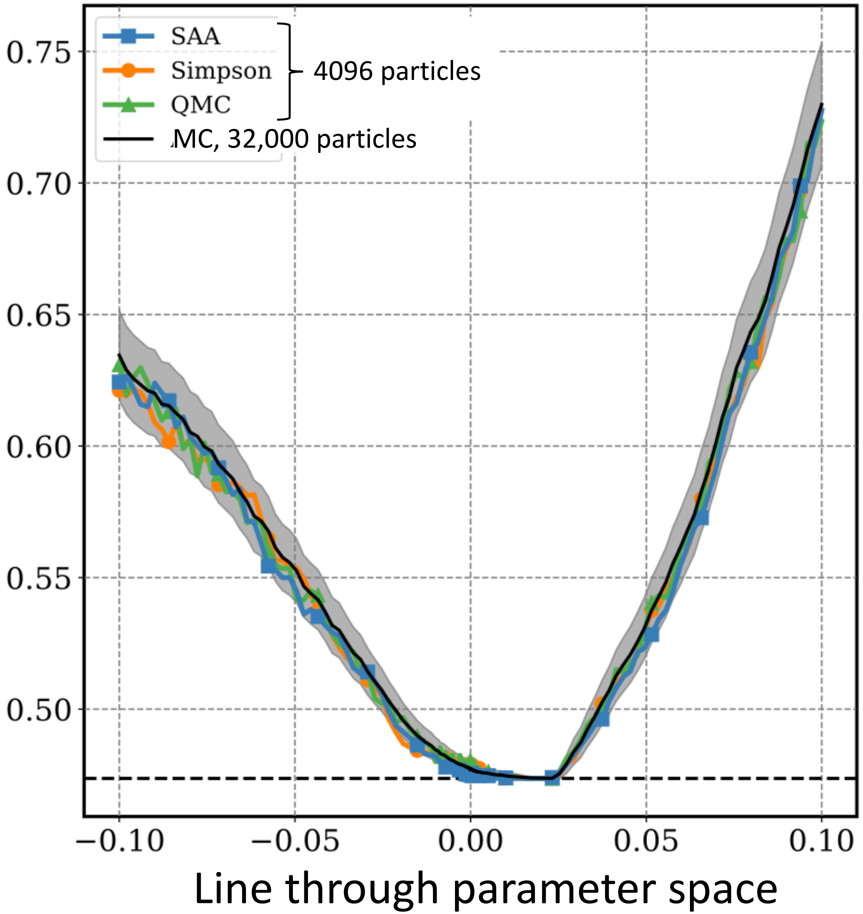
Fraction of particles lost in various stellarators



Su et al (2020)

Direct optimization of particle confinement is challenging due to computational cost & noise

Energy loss, integrating over initial conditions (objective function)

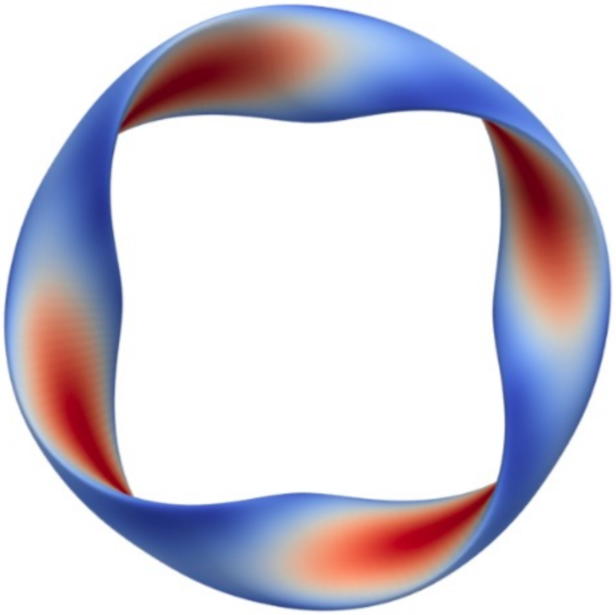


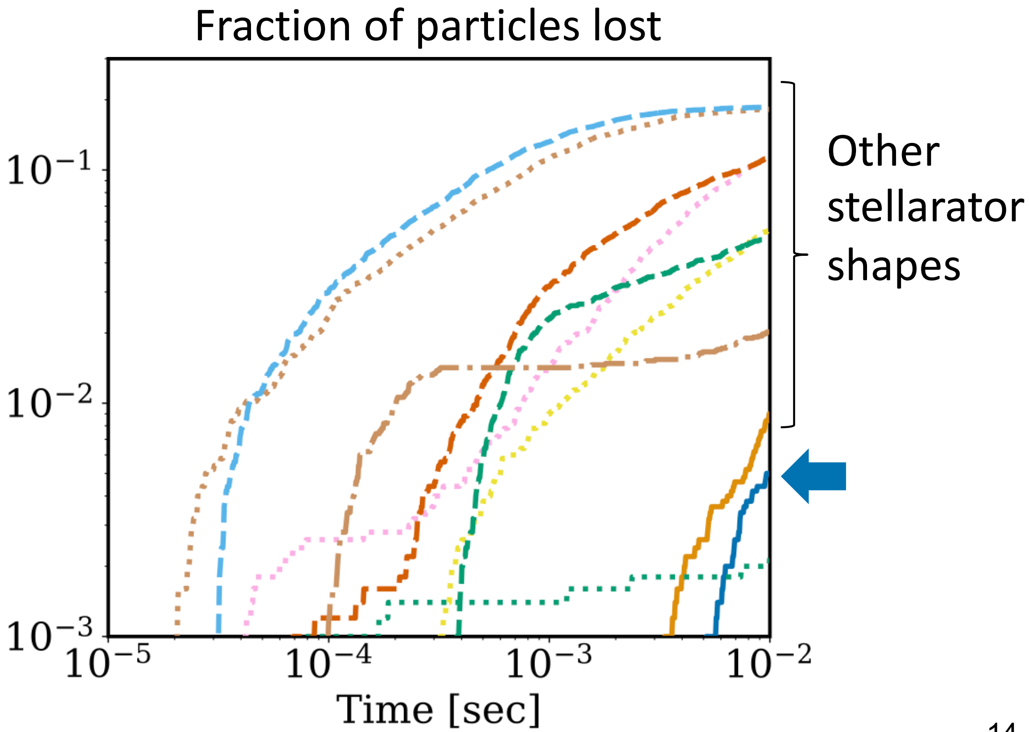
Bindel, ML, Padidar, arXiv (2023)

Improvement in confinement can be achieved with this direct approach

Bindel, ML, Padidar, arXiv (2023)

$$\underset{\mathbf{w}}{\text{minimize}} \quad \mathcal{J}_{\text{penalty}}(\mathbf{w}) := \underbrace{\mathcal{J}(\mathbf{w})}_{\text{Energy losses}} + \sum_{i=1}^{n_{\text{grid}}} \max(B(\mathbf{x}_i) - B_+^*, 0)^2 + \max(B_-^* - B(\mathbf{x}_i), 0)^2$$

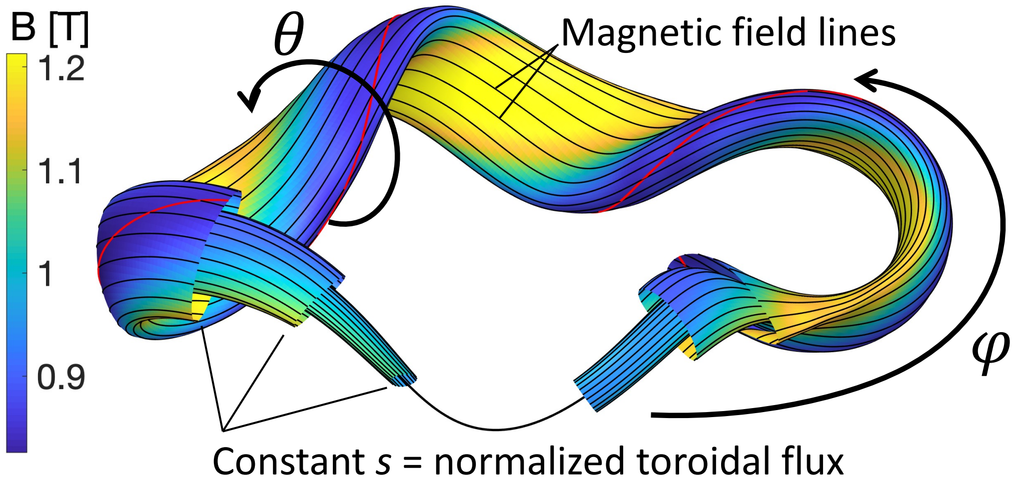




Outline

- Optimizing plasma shape, directly targeting particle confinement.
- Optimizing plasma shape using a surrogate (“quasisymmetry”)
- Optimizing plasma shape to reduce turbulence
- Topology optimization for magnets

Quasisymmetry is a sufficient (though not necessary) condition for confinement, & a useful surrogate



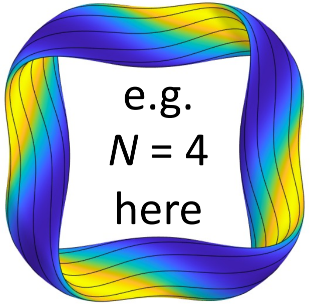
$B = B(s, \theta - N\,\varphi)$

"Boozer angles"

Objective: $$f_{QS} = \int d^3x \left(\frac{1}{B^3}\left[\left(N - \iota\right)\mathbf{B}\times\nabla B\cdot\nabla\psi - \left(G + NI\right)\mathbf{B}\cdot\nabla B \right] \right)^2$$

For quasi-axisymmetry,
N = 0.

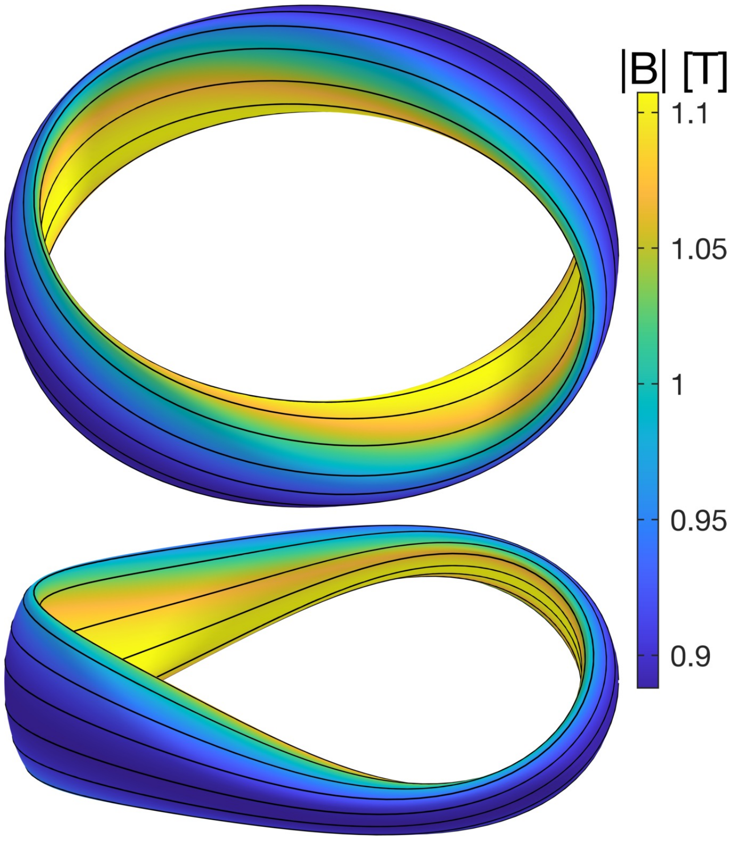
For quasi-helical symmetry,
N is the number of field periods,



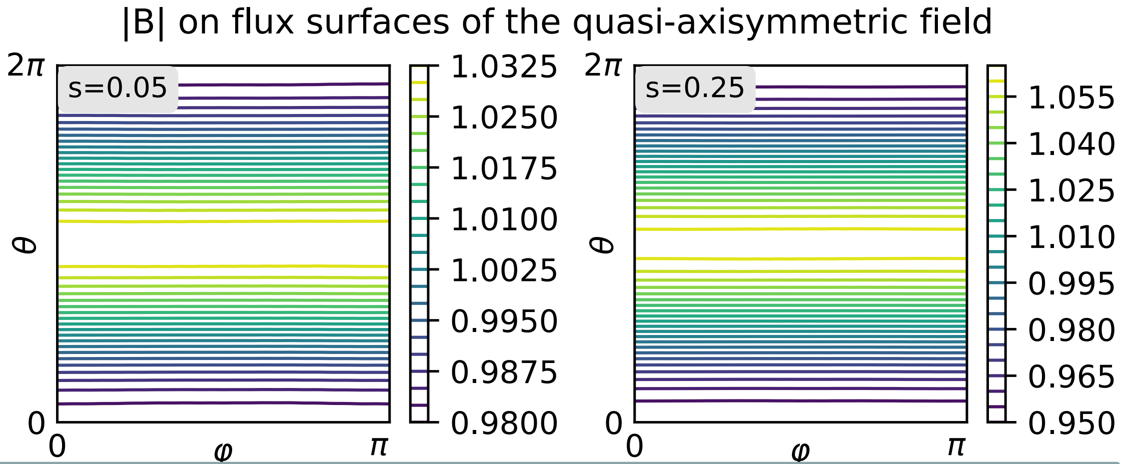
$$f_{QH} = \left(A - A_*\right)^2 + f_{QS}$$

Boundary aspect ratio

Example of quasi-axisymmetry optimization



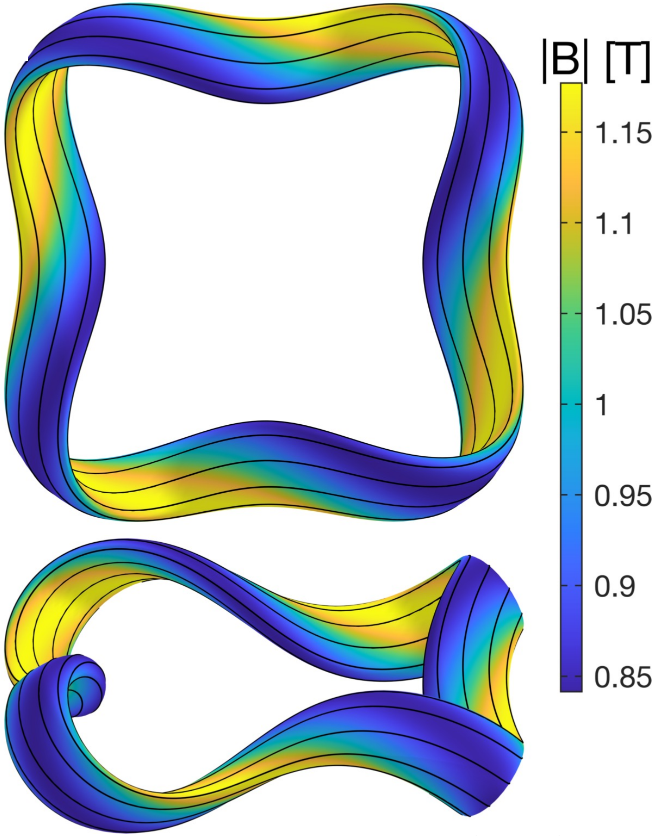
ML & Paul, Phys Rev Lett (2022)



Optimization details:

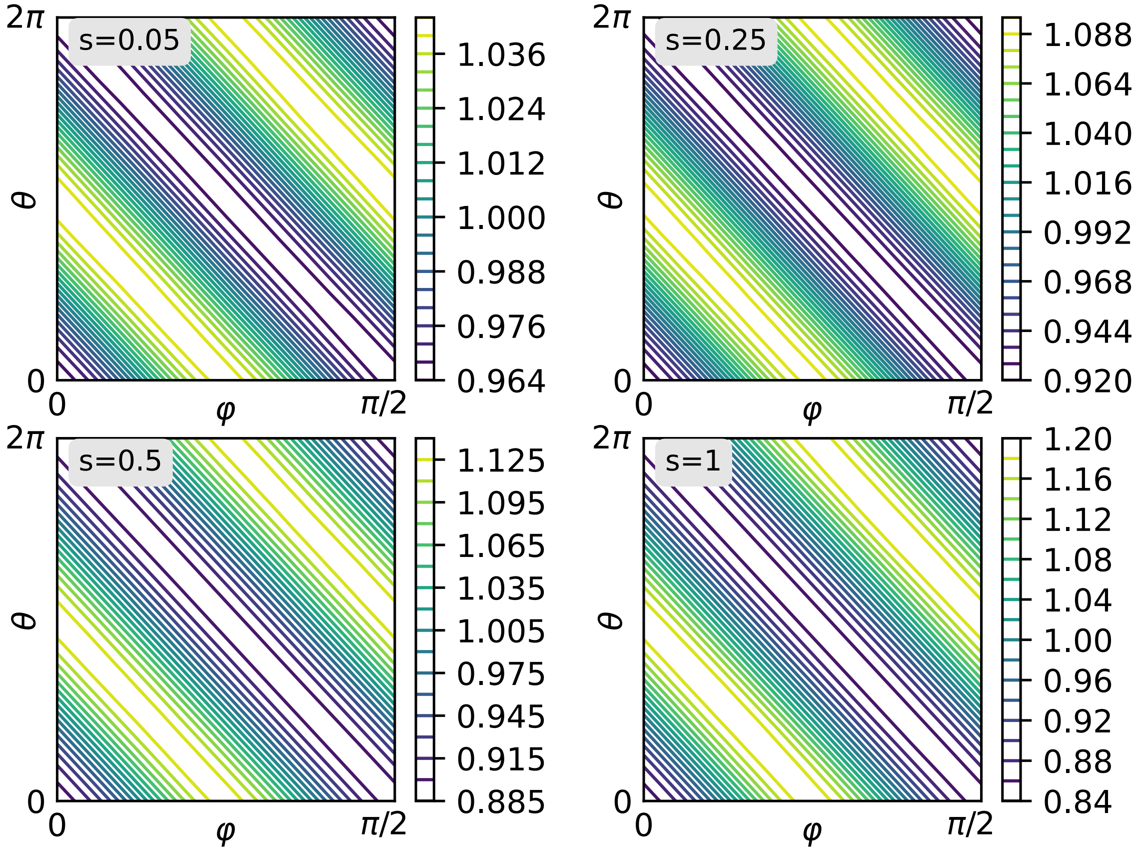
- Default algorithm in scipy for nonlinear least-squares ("trust region reflective")
- Initialize with circular cross-section torus.
- Gradients from finite differences or automatic differentiation.
- Expand the number of Fourier modes in the parameter space in steps. 8-120 design variables.

Example of quasi-helical symmetry optimization



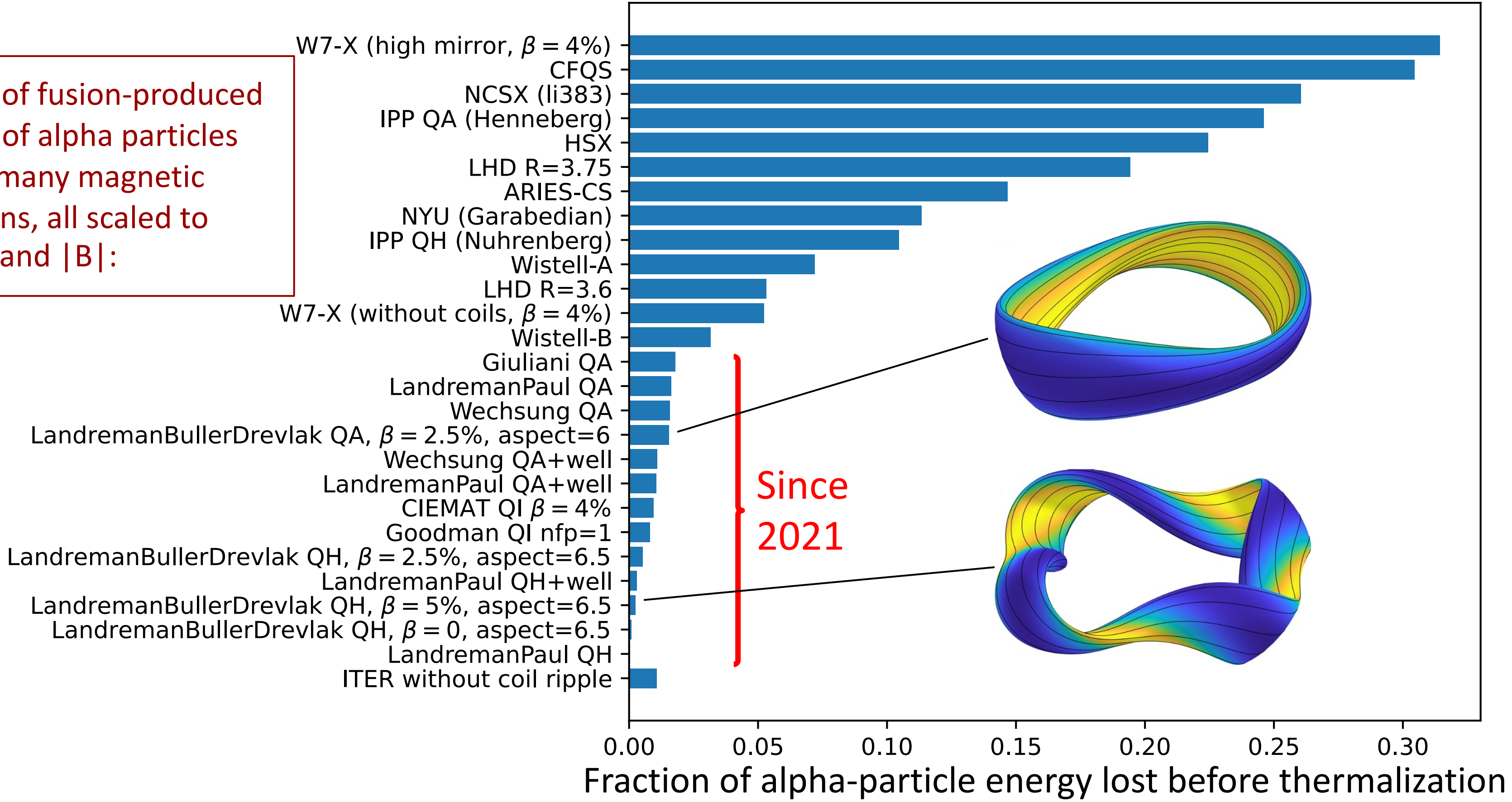
ML & Paul, Phys Rev Lett (2022)

|B| on flux surfaces of the quasi-helically symmetric field



Quasisymmetry has been an effective surrogate for finding plasma shapes with good confinement, though it might be too limiting.

Trajectories of fusion-produced distribution of alpha particles followed in many magnetic configurations, all scaled to reactor size and |B|:



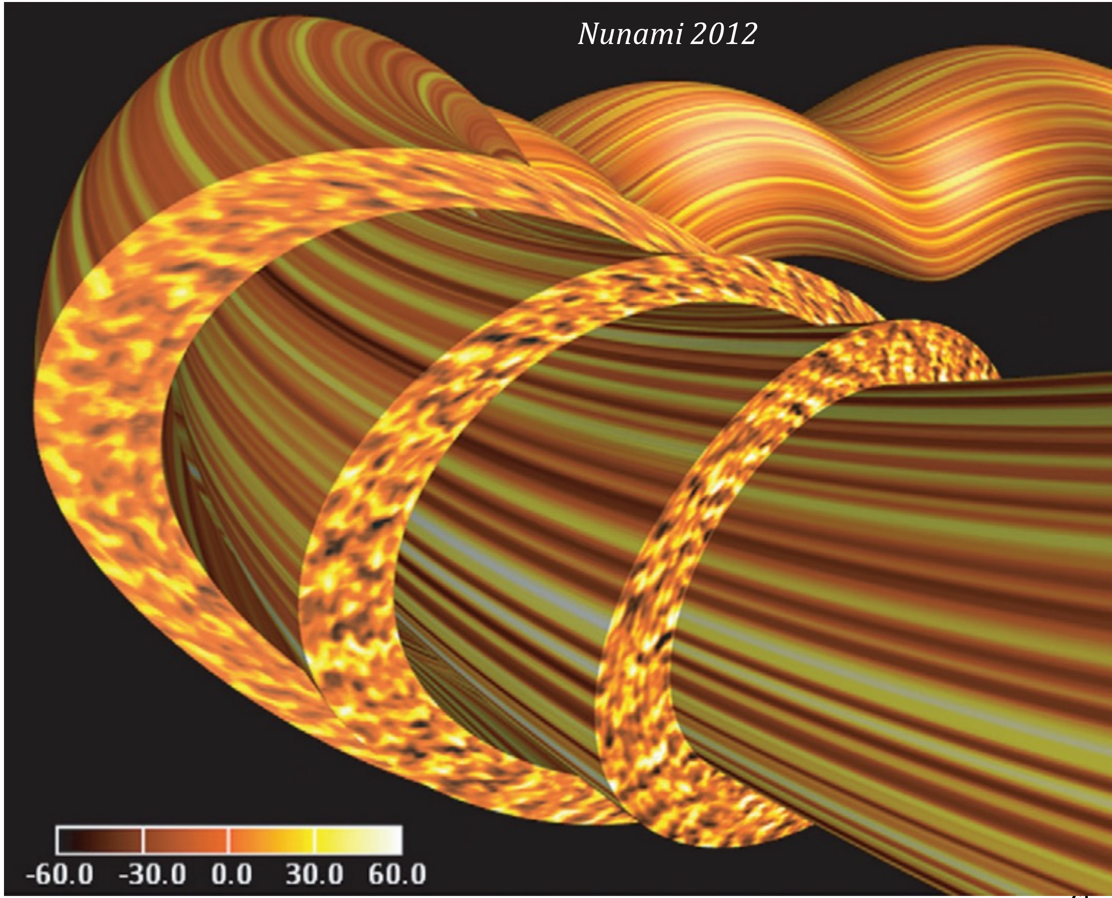
Multi-fidelity method with quasisymmetry + direct ODE integration?

Outline

- Optimizing plasma shape, directly targeting particle confinement.
- Optimizing plasma shape using a surrogate (“quasisymmetry”)
- Optimizing plasma shape to reduce turbulence
- Topology optimization for magnets

Shape optimization to reduce plasma turbulence would be valuable but is hard

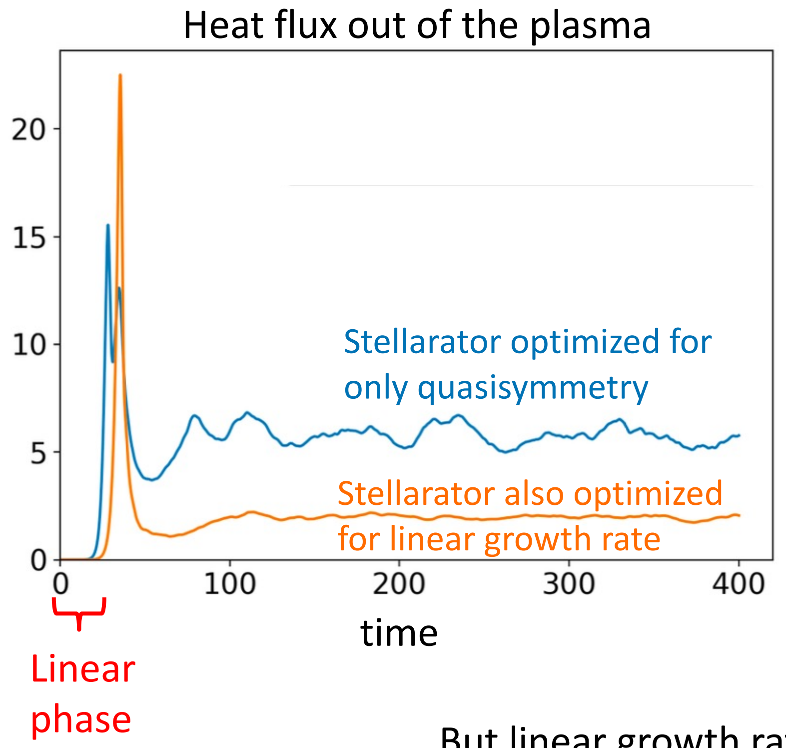
- Gradients of plasma density & temperature cause instabilities & turbulence.
- Causes heat to leak out.
- Known to depend on plasma shape.
- Turbulence simulations: ~10 GPU-min to 10^6s of CPU-hours.
- 5D + time
- No agreed-upon surrogate like Reynolds-averaged Navier-Stokes + closure yet.

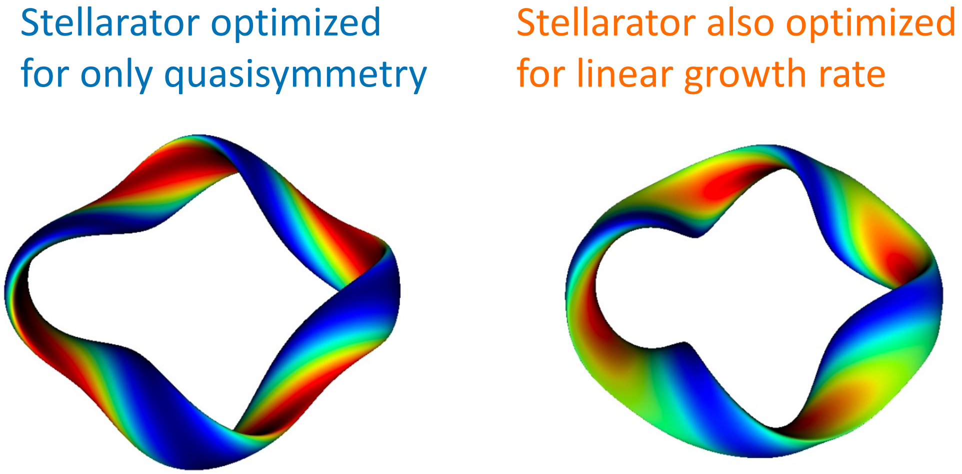


One approach to optimizing turbulence: minimize linear growth rate of instabilities

Linear growth rate gives a smooth, deterministic, non-chaotic objective

Jorge et al, arXiv (2023)



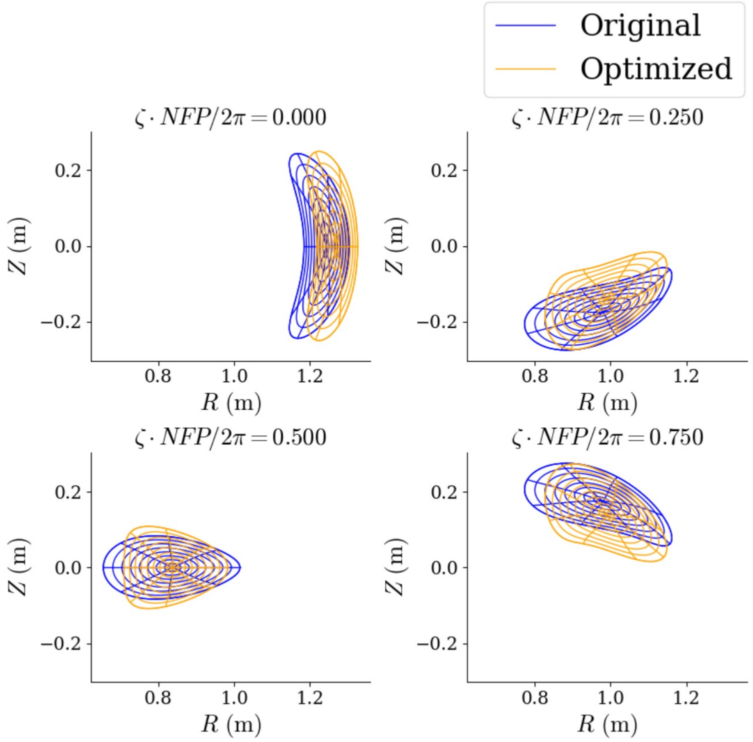


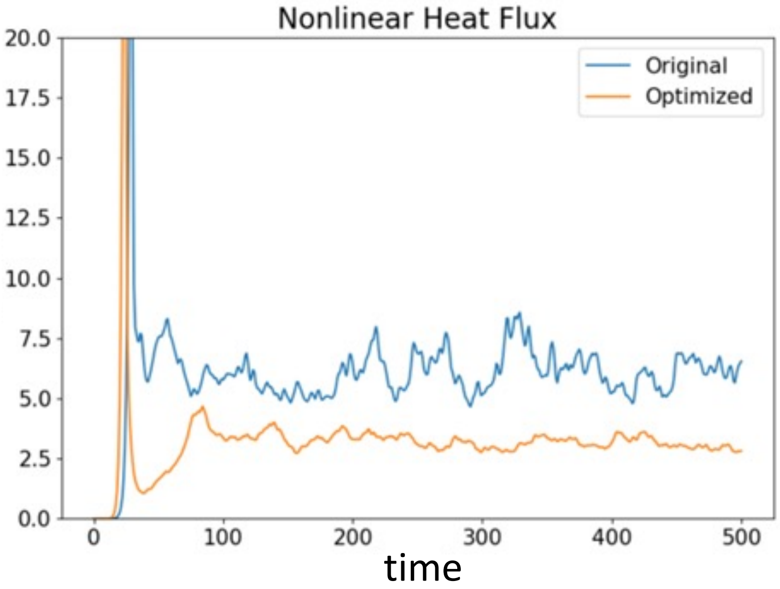
But linear growth rate is not a perfect predictor of the nonlinear heat flux…

First optimizations with nonlinear turbulence calculations in the objective

By Patrick Kim (UMD undergraduate!)

Using stochastic gradient descent with momentum





Bayesian optimization with 20-100 design variables would be nice…

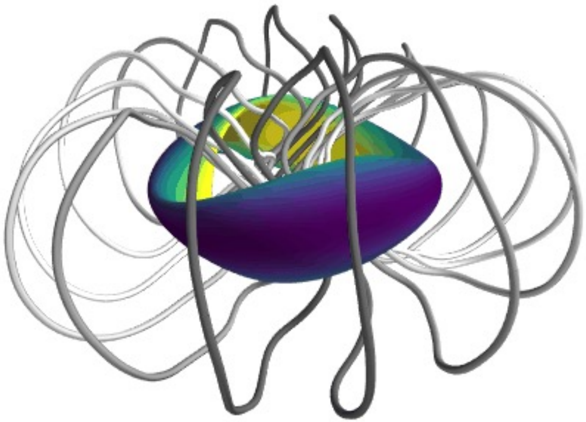
Outline

- Optimizing plasma shape, directly targeting particle confinement.
- Optimizing plasma shape using a surrogate (“quasisymmetry”)
- Optimizing plasma shape to reduce turbulence
- Topology optimization for magnets

An established method for electromagnetic coil design is available, but has some limitations

Assume plasma shape has already been optimized, so target **B** field is known.

Coils represented as space curves.
Design variables: Fourier modes of Cartesian components.



$$x(t) = x_{c,0} + \sum_{n=1}^{N_F} \left[x_{c,n} \cos(nt) + x_{s,n} \sin(nt) \right]$$

Objective:

$$f = \underbrace{\int_{\substack{plasma\\surf}} (\boldsymbol{B} \cdot \boldsymbol{n})^2}_{\text{Match target B}} + \underbrace{\lambda (length - target)^2 + \ldots}_{\text{Regularization}}$$

Issues:

- Non-convex, so there are multiple local minima. Need good initial guess.
- Coil topology & number of coils are fixed by initial guess.

New topology optimization method for electromagnetic coils

Pre-define grid of voxels where current might flow.

Current density $\boldsymbol{J}$ in each voxel represented by basis of 11 divergence-free functions. Amplitudes are the design variables.

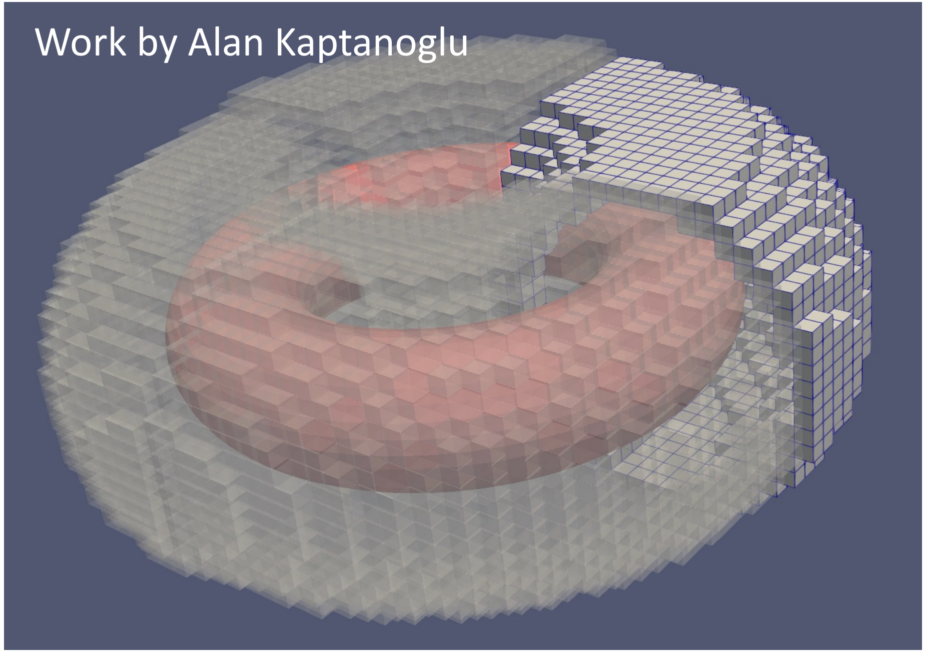
Current conservation at each cell face gives linear equality constraints.

Given basis functions, $\boldsymbol{B}$ can be computed by Biot-Savart Law:

$$\boldsymbol{B}(r) = \frac{\mu_0}{4\pi}\int d^3r' \frac{\boldsymbol{J}(r')\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}'|^3}$$

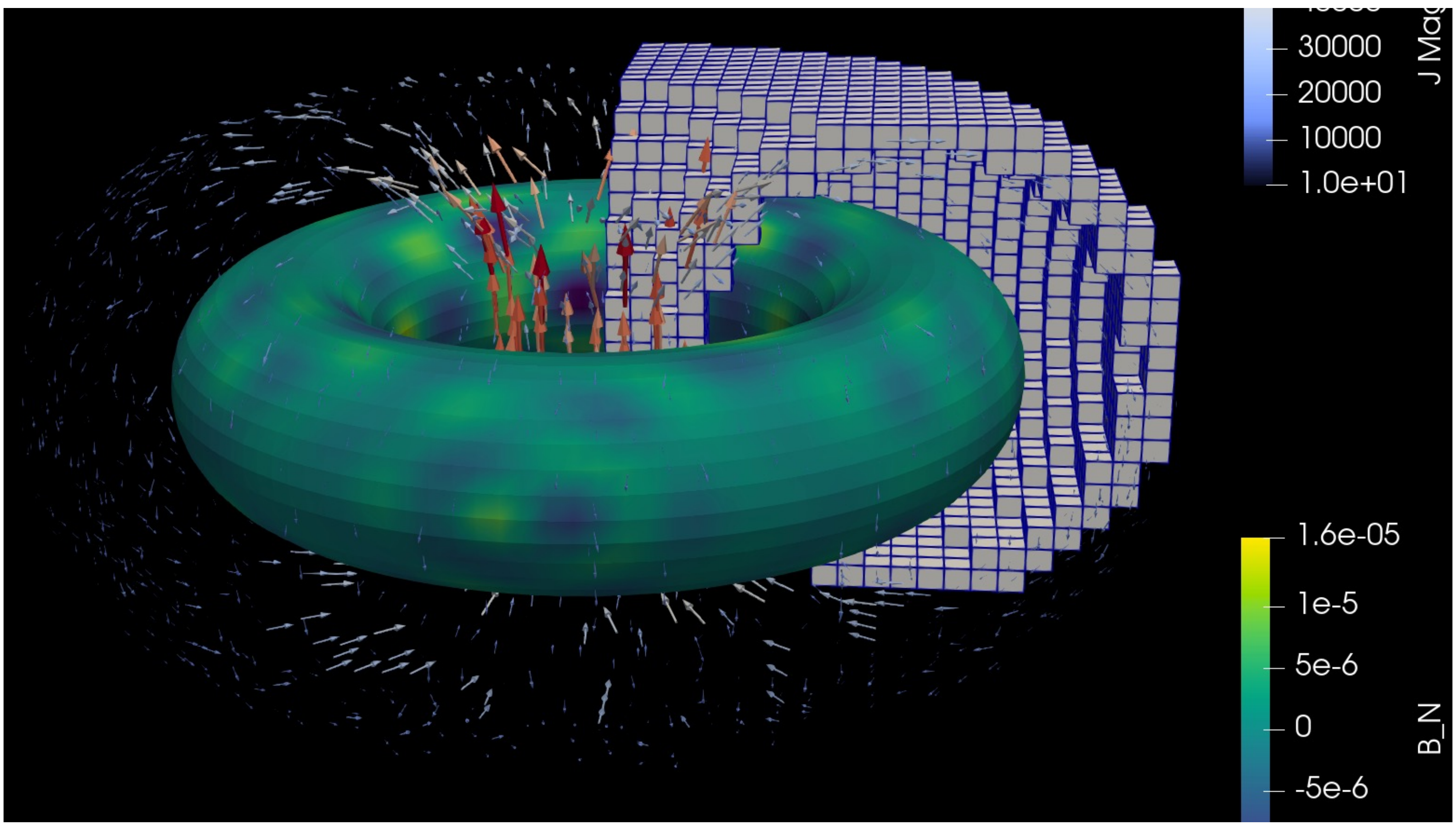
Objective:

$$f = \underbrace{\int_{\substack{plasma\\ surf}} (\boldsymbol{B}\cdot\boldsymbol{n})^2}_{\text{Match target B}} + \lambda \underbrace{\int_{voxels} \|\boldsymbol{J}\|_2^2 + v\|\boldsymbol{J}\|_1 + \eta\|\boldsymbol{J}\|_0}_{\text{Regularization}}$$

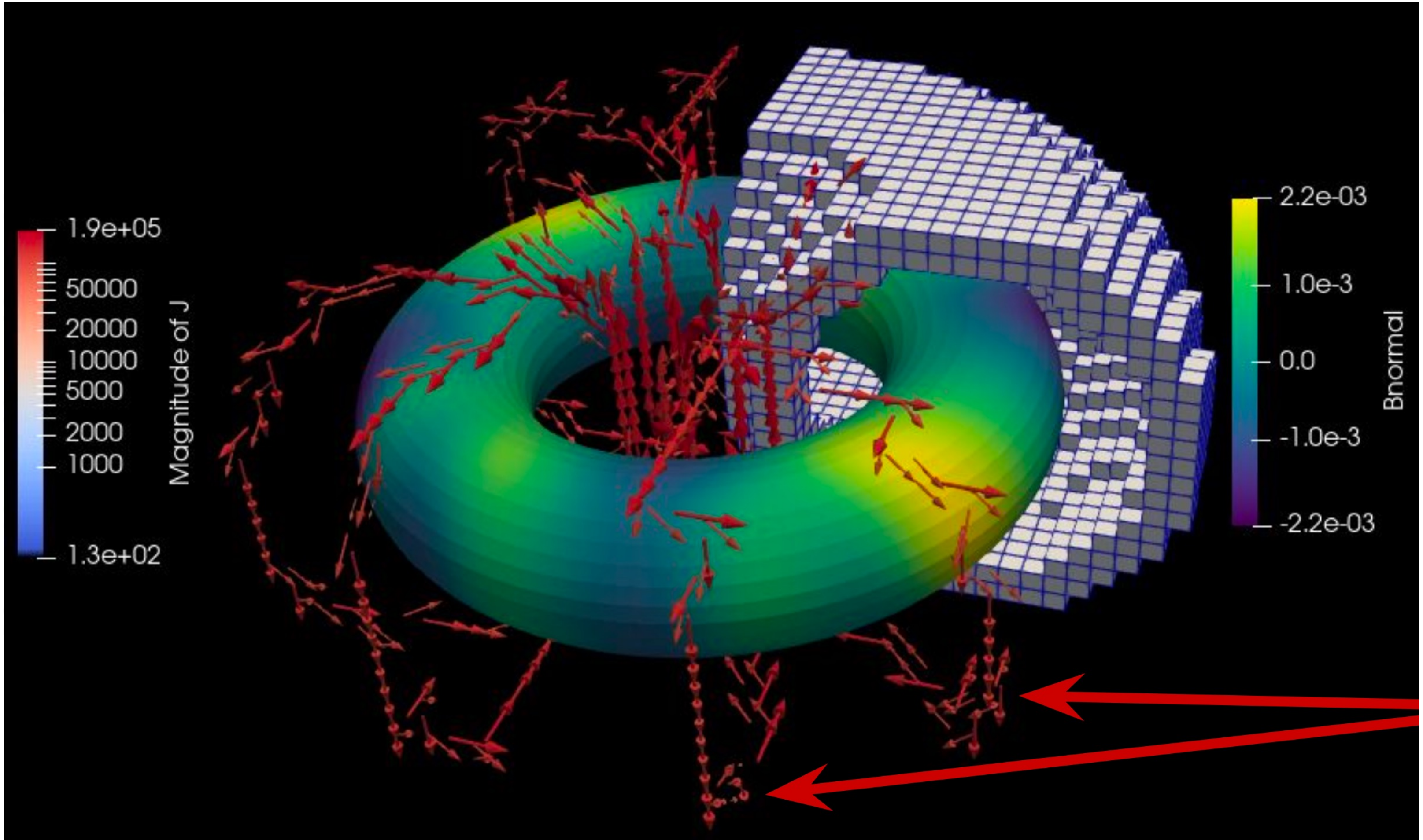


- Without sparsity-promoting terms, problem is linear.
- With l1 sparsity, problem remains convex.
- With l0 sparsity, good algorithms exist.

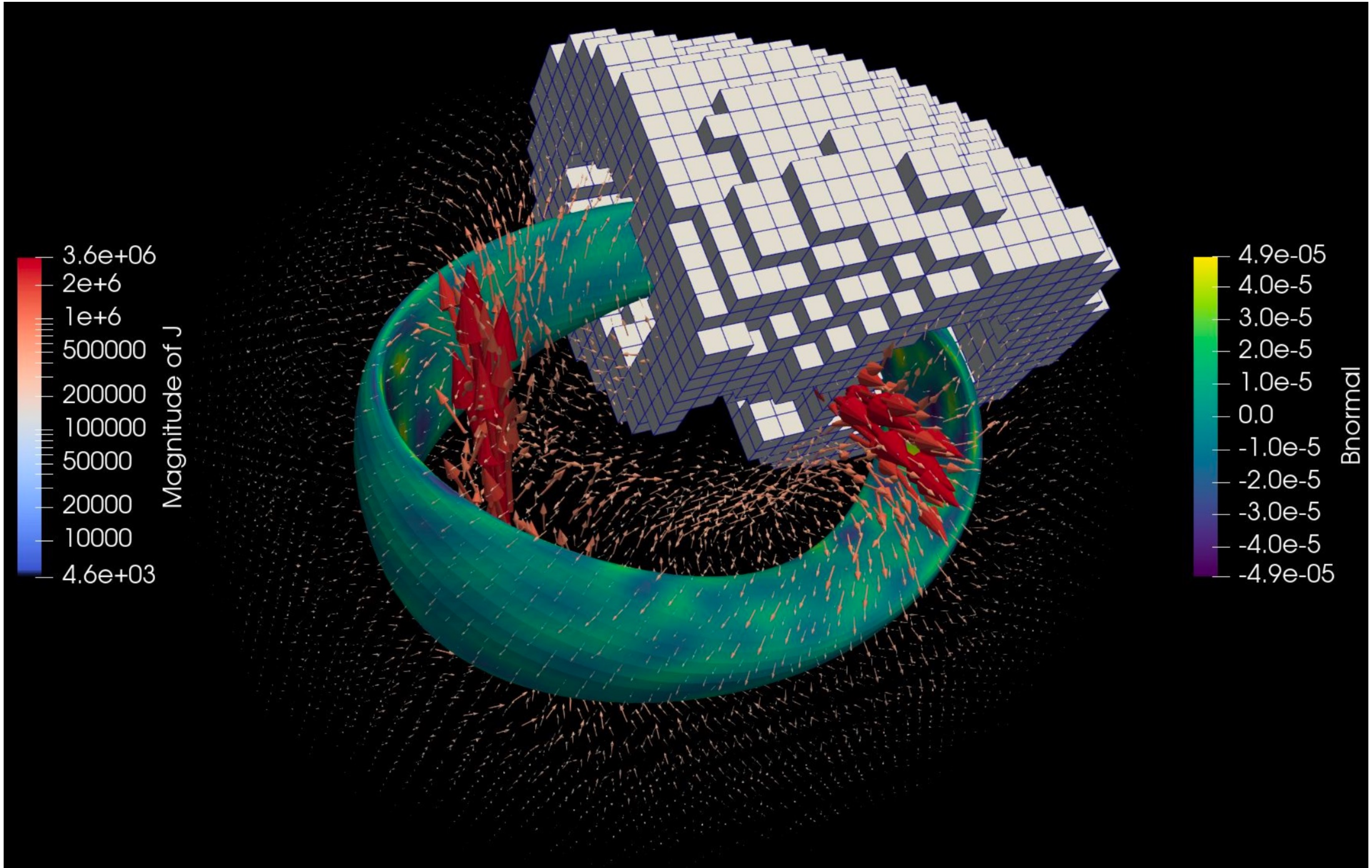
Preliminary results from electromagnet topology optimization: In axisymmetry, expected currents are recovered



Preliminary results from electromagnet topology optimization: With sparsity objective, currents coalesce into discrete coils



Preliminary results from electromagnet topology optimization: Starting to look at non-axisymmetric geometries



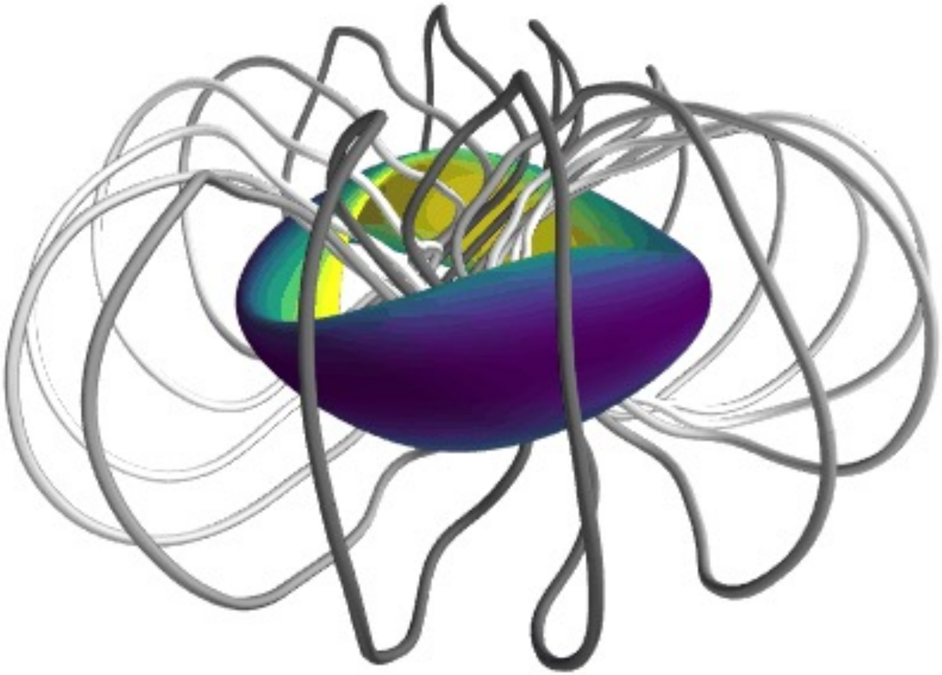
Conclusions: stellarator optimization is a subject with many challenging numerical problems

This talk:

- Shape optimization for confining particle trajectories, with PDE & possibly ODE solves.
- Shape optimization with linear stability or nonlinear turbulence simulations in the loop.
- Topology optimization for electromagnets.

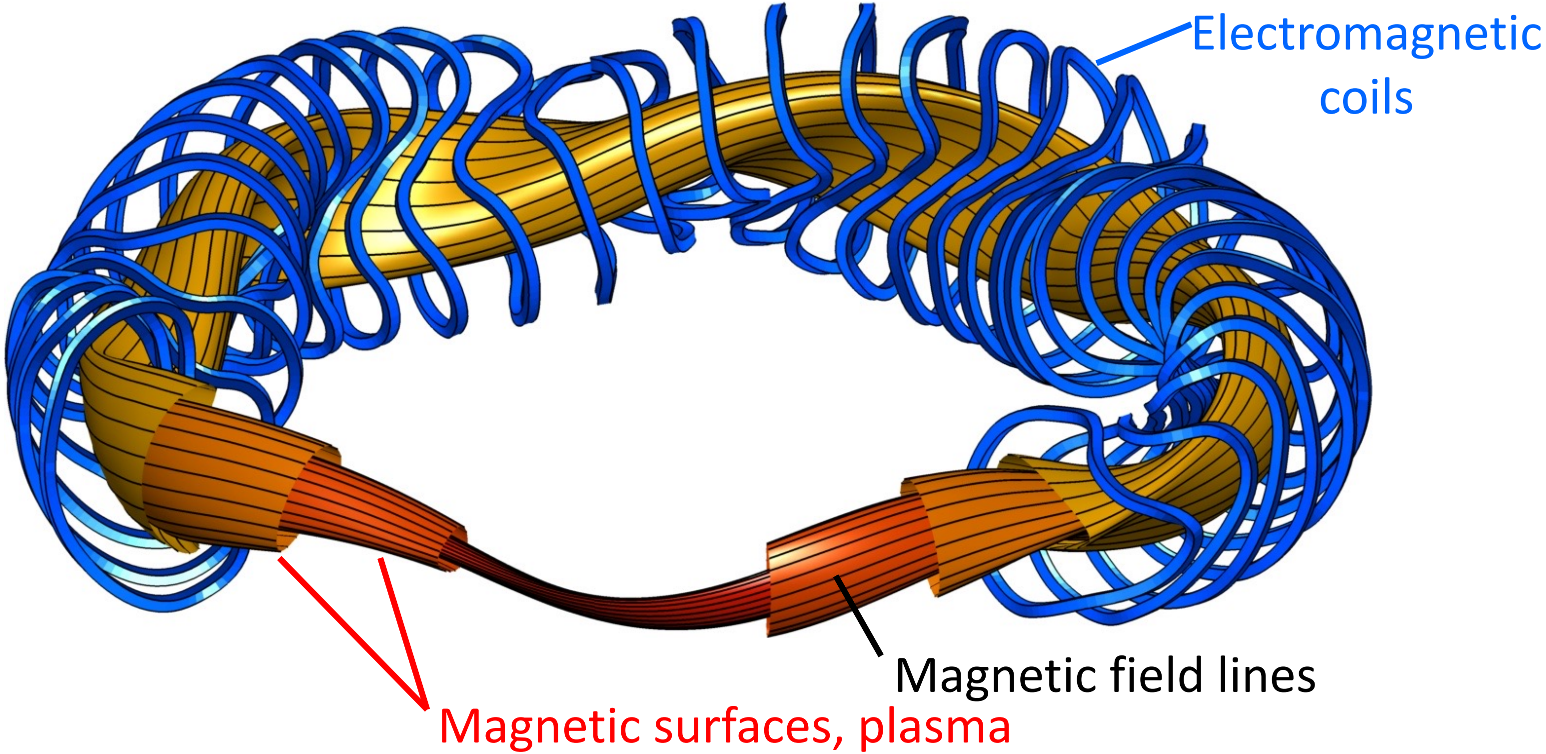
Many other directions:

- Other surface & curve discretizations
- Multi-fidelity
- Surrogates for confinement or turbulence
- ≥ 4D advection-dominated PDEs
- Uncertainty quantification
- …



Extra slides

A *stellarator* is a configuration of magnets for confining plasma without continuous rotation symmetry

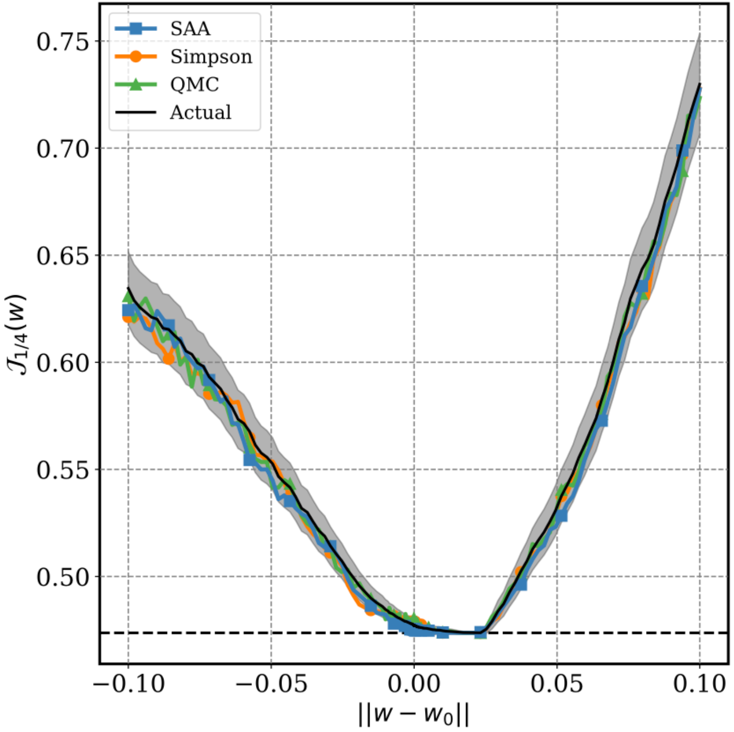


- Applications: fusion energy, or electron-positron plasmas (astrophysics)
- Shape of the plasma and of the coils is chosen by numerical optimization.

Direct optimization of particle confinement is challenging due to computational cost & noise

Objective function to measure energy loss:

$$\mathcal{J}(\mathbf{w}) := \int_{\mathbf{x}} \int_{v_{\|}} \underbrace{3.5e^{-2\mathcal{T}(\mathbf{x},v_{\|},\mathbf{w})/t_{\max}}}_{\text{Energy at loss time}} \underbrace{f(\mathbf{x}, v_{\|})}_{\text{1 if particle is lost, else 0}} \underbrace{dv_{\|}\, d\mathbf{x}}_{\text{Initial condition}}.$$



Divergence-free basis functions for current density J

$$\phi_i^{(k)} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} Y_k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Z_k \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ X_k \end{bmatrix},$$
$$\begin{bmatrix} 0 \\ 0 \\ Y_k \end{bmatrix}, \begin{bmatrix} 0 \\ Z_k \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ X_k \\ 0 \end{bmatrix}, \frac{\sqrt{3}}{2} \begin{bmatrix} X_k \\ -Y_k \\ 0 \end{bmatrix}, \frac{\sqrt{3}}{2} \begin{bmatrix} X_k \\ 0 \\ -Z_k \end{bmatrix}$$

where

$$X_k \equiv \frac{x - x_k}{\Delta x_k}, \quad Y_k \equiv \frac{y - y_k}{\Delta y_k}, \quad Z_k \equiv \frac{z - z_k}{\Delta z_k},$$