Physics of Plasmas ARTICLE

Optimization of quasi-symmetric stellarators with self-consistent bootstrap current and energetic particle confinement

Published Online: 2 August 2022

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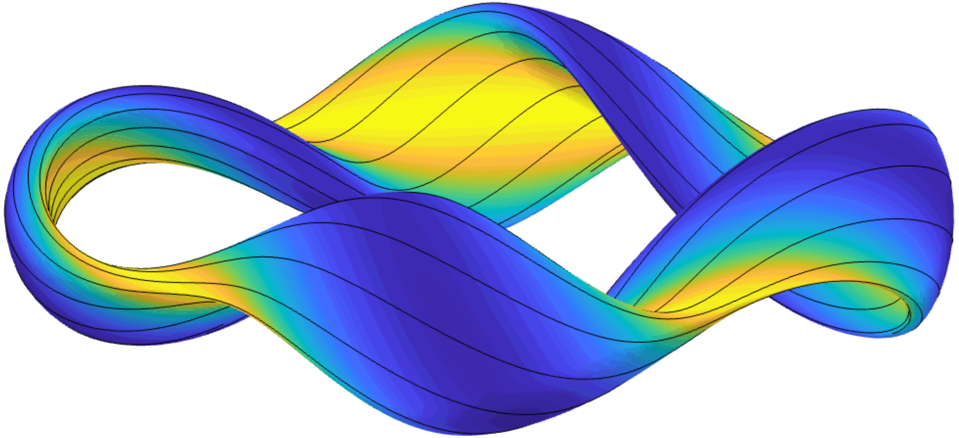
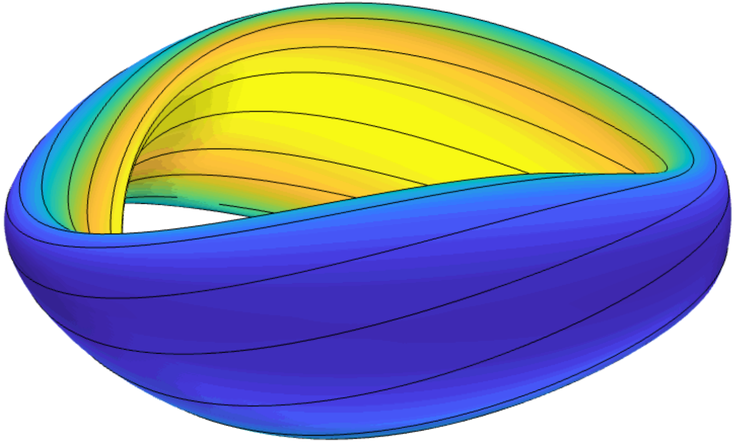
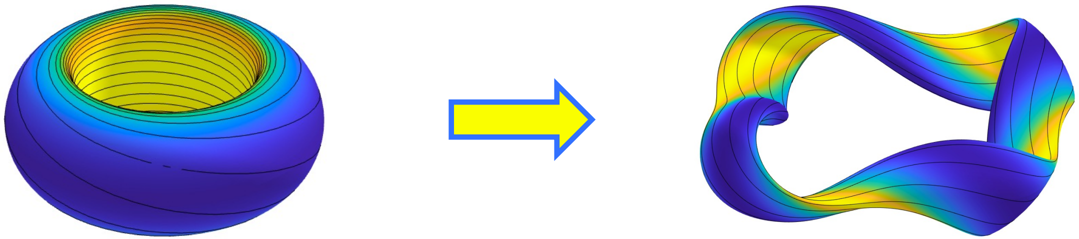


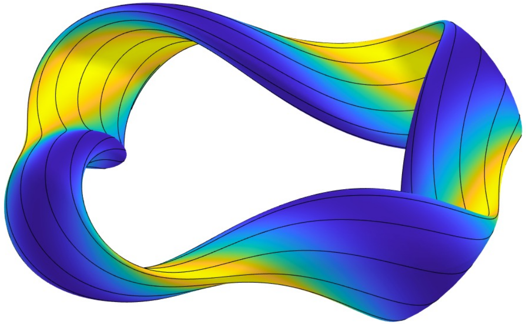


Main ideas

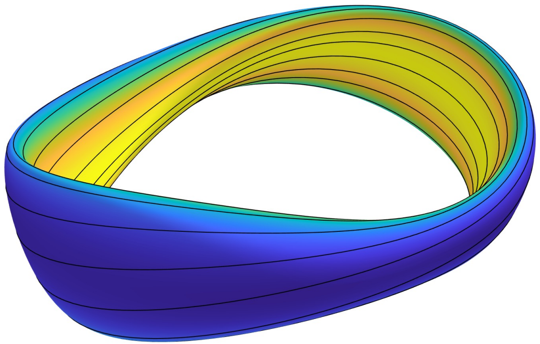
1. A recent advance in computing the “bootstrap current” in tokamaks immediately gives an efficient new way to compute the bootstrap current in many stellarators.



2. Great progress has been made in the last year on a long-standing challenge for stellarators: confining energetic particles.



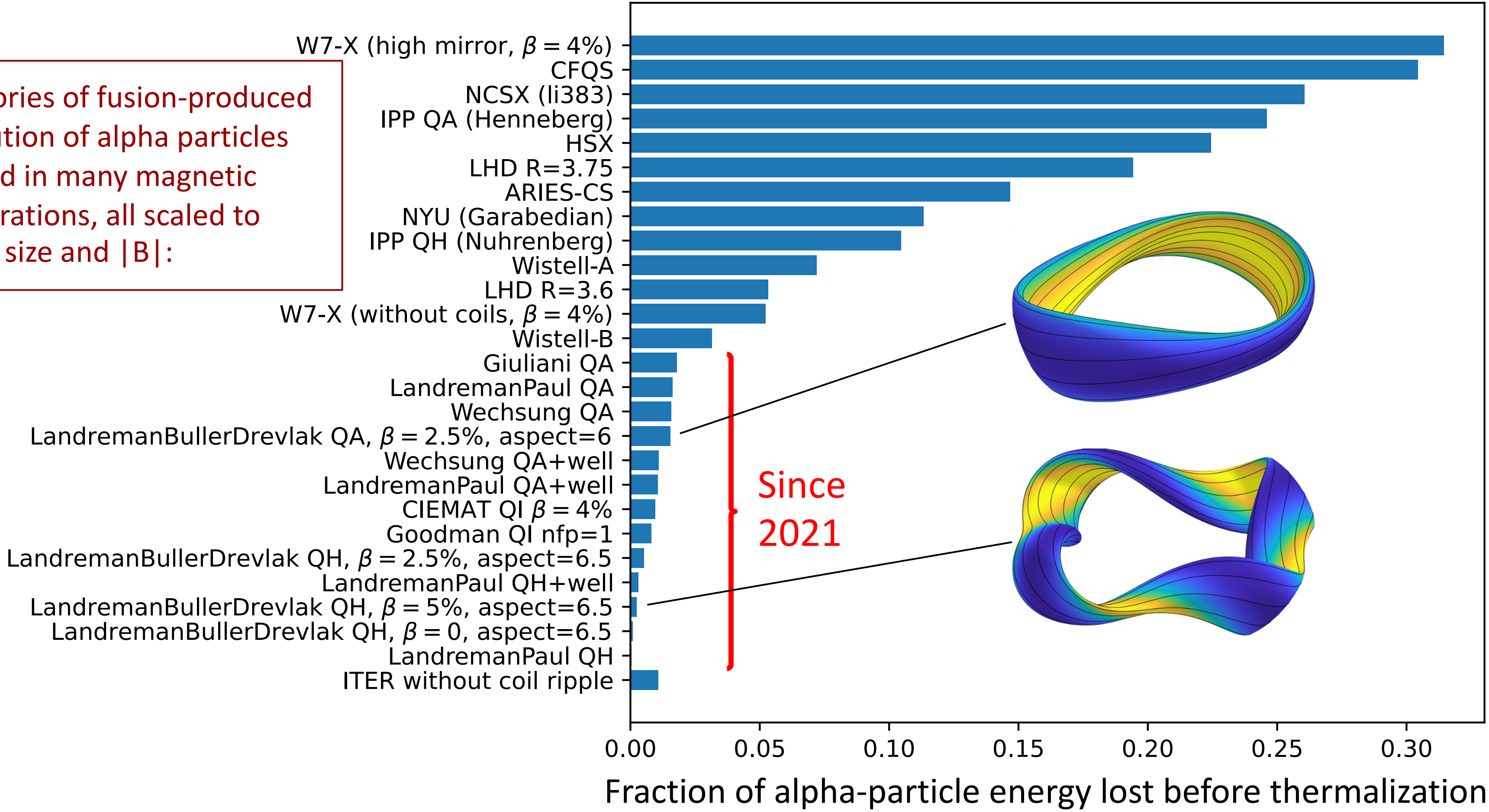
α-particle energy losses < 0.3%



< 1.5%

Stellarators can now be designed with comparable or lower α-particle losses to tokamaks

Trajectories of fusion-produced distribution of alpha particles followed in many magnetic configurations, all scaled to reactor size and |B|:



Outline

- Introduction
  - Quasisymmetry
  - Bootstrap current
- Isomorphism: Applying tokamak bootstrap formula to stellarators
- Putting it all together in optimization

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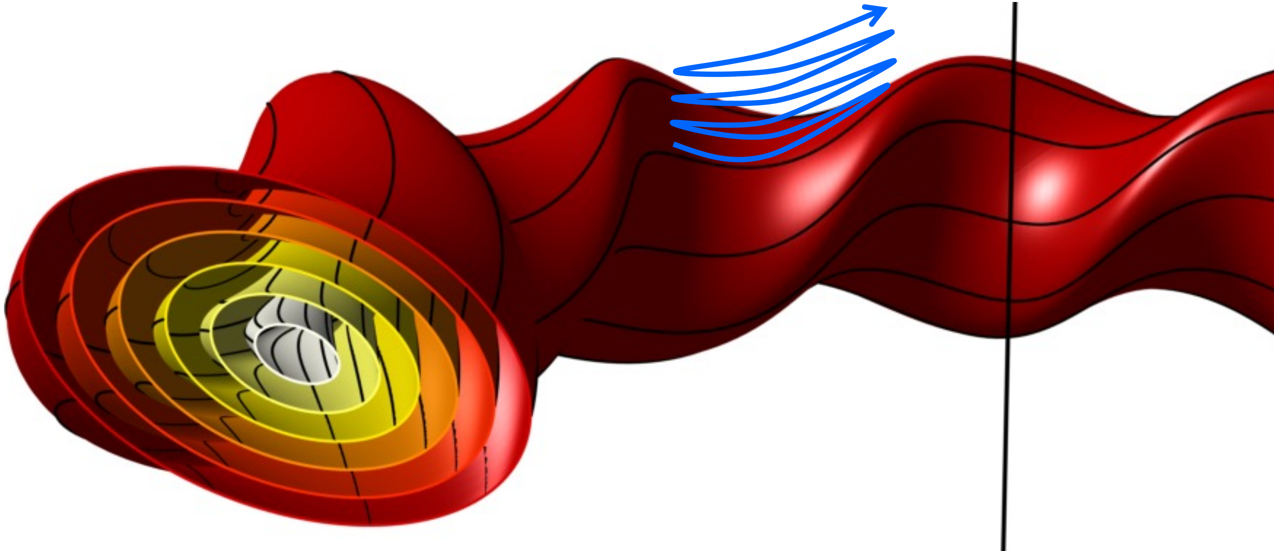
- But, alpha-particle losses & neoclassical transport would be too large unless you carefully choose the geometry.



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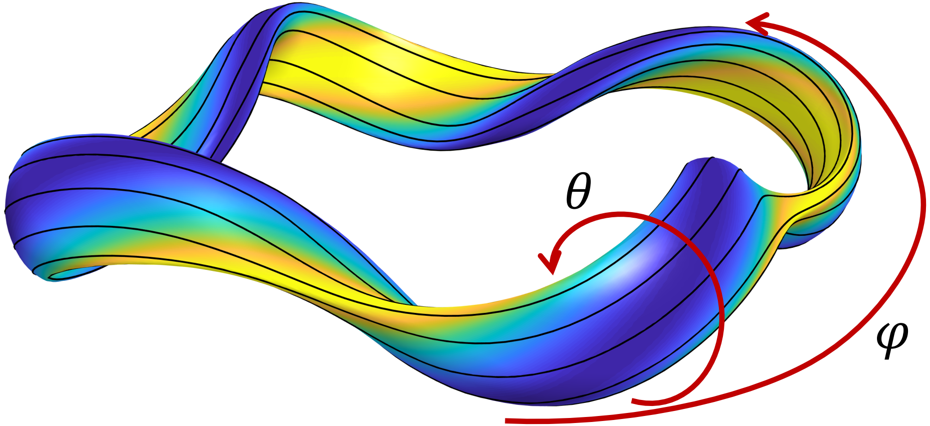
- But, alpha-particle losses & neoclassical transport would be too large unless you carefully choose the geometry.
- Quasisymmetry is a solution:



$$B = |\boldsymbol{B}| = B(s, \theta - N\varphi)$$

Flux surface label

Boozer angles

To understand why quasisymmetry works, let’s recall why confinement is so good in axisymmetry.

Complicated particle trajectories can be confined in axisymmetry due to Noether's theorem

Lagrangian for a particle in a **B** field: $\mathcal{L} = \frac{m}{2}|\dot{\boldsymbol{x}}|^2 + q\boldsymbol{A}\cdot\boldsymbol{x}$

Vector potential: $\boldsymbol{B} = \nabla\times\boldsymbol{A}$

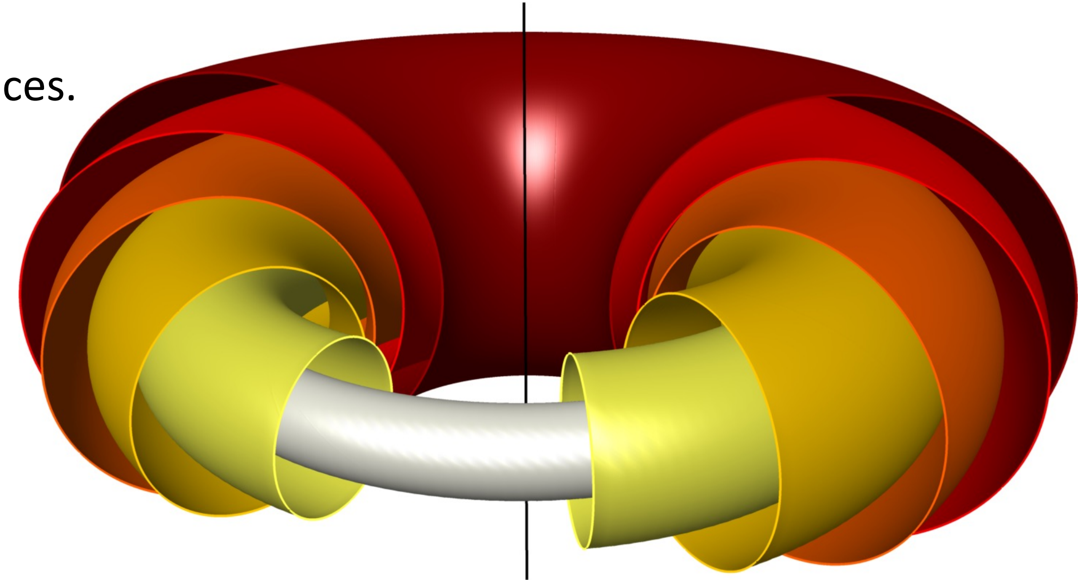
Continuous rotational symmetry

$\Rightarrow$ Canonical angular momentum is conserved: $\frac{\partial\mathcal{L}}{\partial\dot{\phi}} = mv_\phi R + qA_\phi R = \text{constant}$

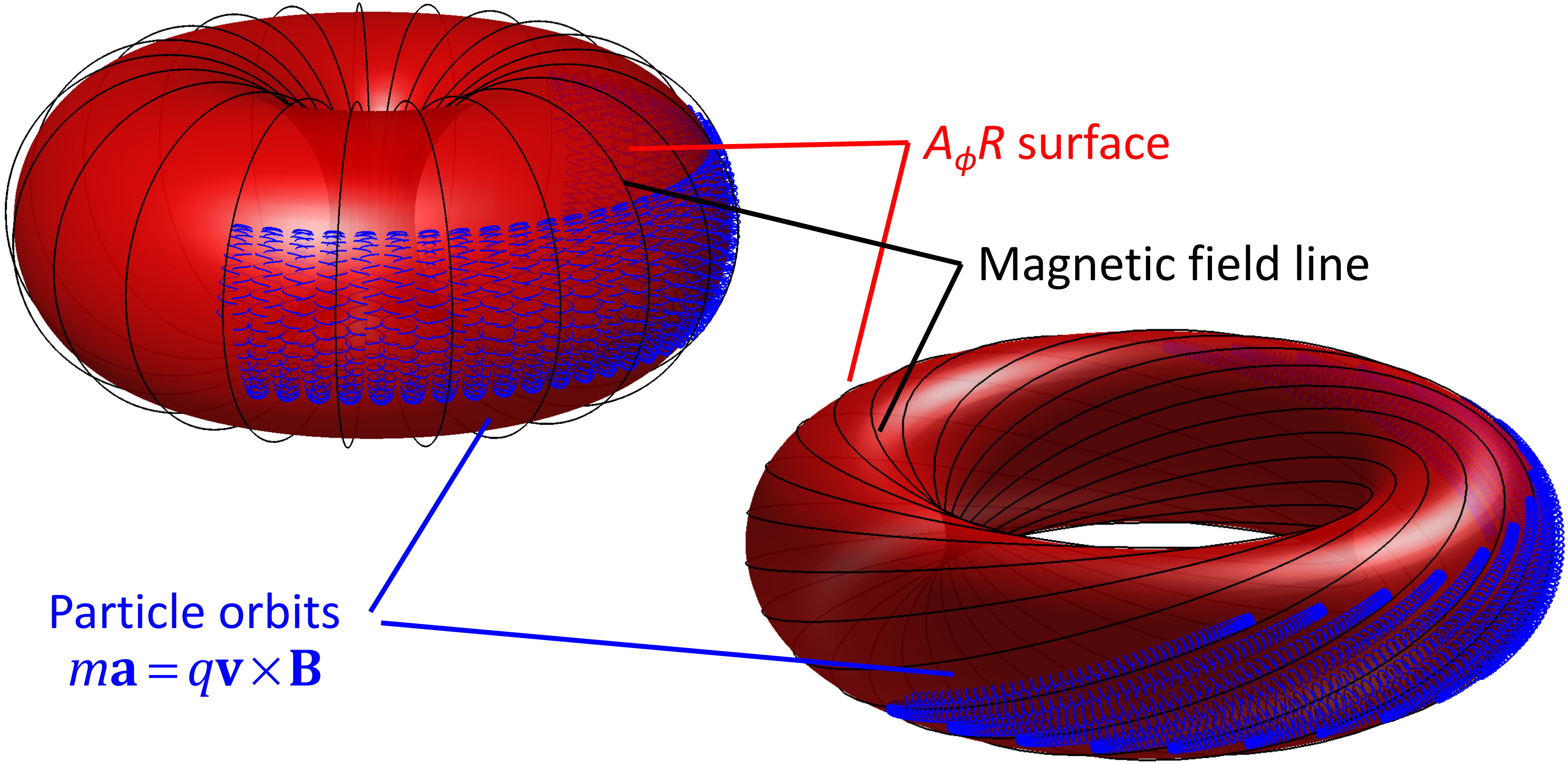
Strong **B** limit $\Rightarrow$ $|mv_\phi| \ll |qA_\phi|$

$\Rightarrow$ Particles stuck to constant-$A_\phi R$ surfaces.

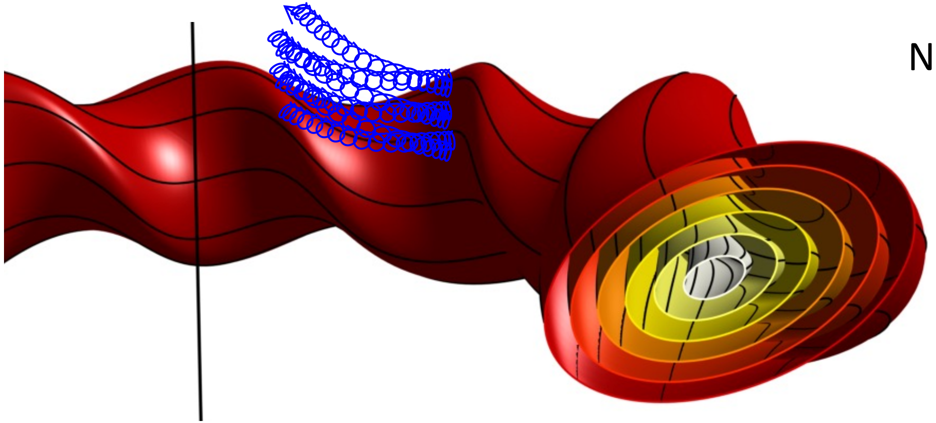
If $A_\phi R$ surfaces are bounded like this, then particles will be confined:



In axisymmetry, particles are confined (close) to $A_\phi R$ surfaces, despite complicated orbits.



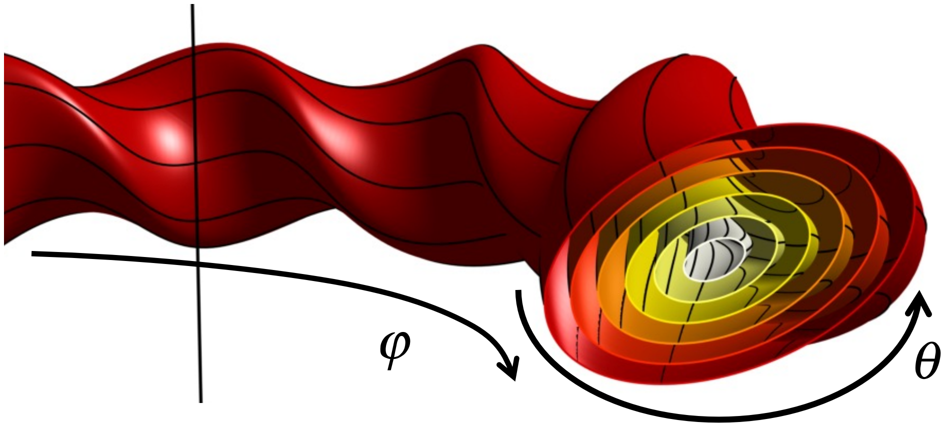
Without axisymmetry, confinement is not as good in general



No reason for particles to stay close to a flux surface.

$\Longrightarrow$ Large neoclassical transport & losses of energetic particles.

Quasisymmetry is a condition that preserves good confinement without requiring axisymmetry



Lagrangian for particle in magnetic field:

$$\mathcal{L} = \frac{m}{2}|\dot{\boldsymbol{x}}|^2 + q\boldsymbol{A}\cdot\boldsymbol{x}$$

Average over fast gyration, use Boozer angles:

Independent of θ and φ

$$\mathcal{L} = \frac{mG^2\dot{\varphi}^2}{2B^2} - \mu B + q\psi\dot{\theta} - q\chi\dot{\varphi}$$

Only depends on θ and φ through $B = |\boldsymbol{B}|$!

If $\partial B/\partial\varphi = 0$, then canonical angular momentum $\partial\mathcal{L}/\partial\varphi$ is conserved $\implies$ Good confinement.

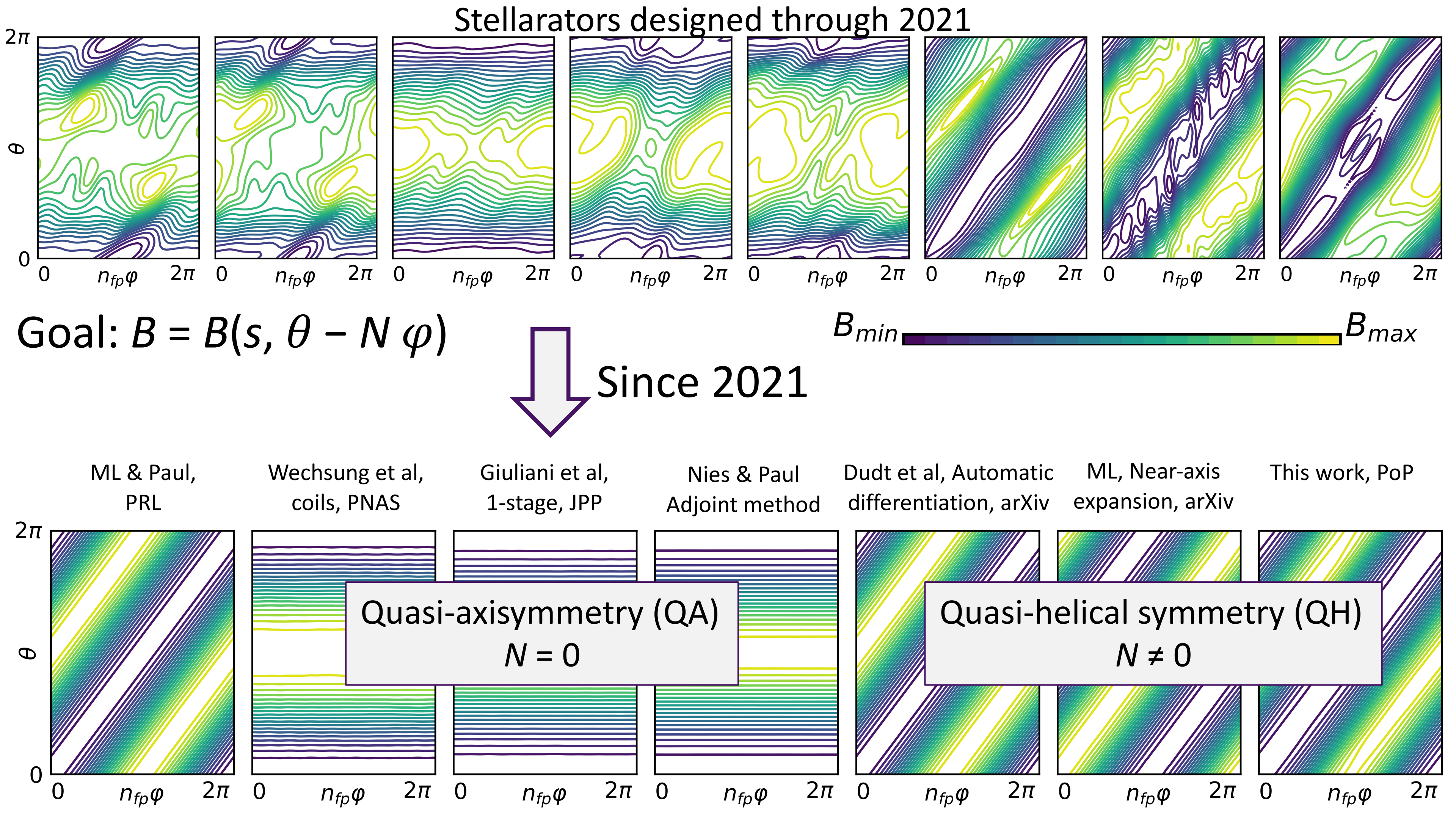
Quasisymmetry: $B = B(s, \theta - N\varphi)$

Flux surface label

Any integer

In quasisymmetry, guiding-center trajectories are **isomorphic** to trajectories in axisymmetry.

Boozer (1980), Nuhrenberg & Zille (1988), Rodriguez et al (2020)


Stellarators designed through 2021
2π
θ
0
$n_{fp}\varphi$
2π
Goal: $B = B(s, \theta - N\,\varphi)$
B_{min}
B_{max}
Since 2021
ML & Paul, PRL
Wechsung et al, coils, PNAS
Giuliani et al, 1-stage, JPP
Nies & Paul Adjoint method
Dudt et al, Automatic differentiation, arXiv
ML, Near-axis expansion, arXiv
This work, PoP
Quasi-axisymmetry (QA)
$N = 0$
Quasi-helical symmetry (QH)
$N \neq 0$

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The bootstrap current arises in tokamaks & stellarators when the density & temperature become significant

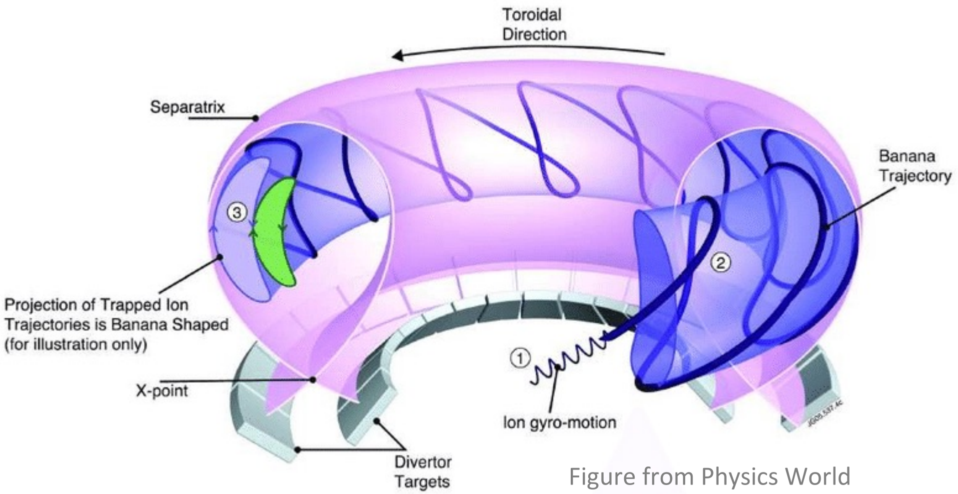
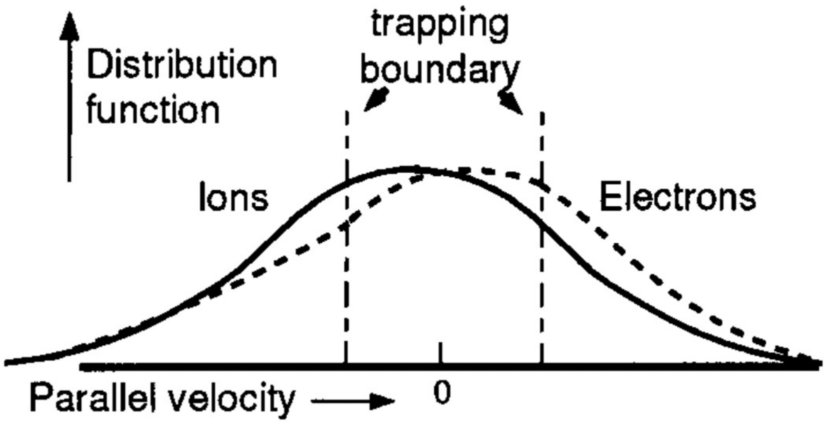


Figure from Physics World



Peeters, PPCF (2000)

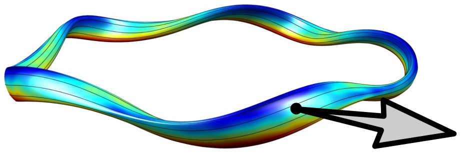
- Ions and electrons have different trajectories. Different mean flows = electric current.
- Current depends on geometry, density, & temperature.
- For $\beta > 0$, we don't know **B** until we include this effect.
- $J_{\text{bootstrap}}$ may be undesirable in a stellarator: increases sensitivity to pressure profile.
- How can a self-consistent $J_{\text{bootstrap}}$ calculation be integrated with stellarator optimization?

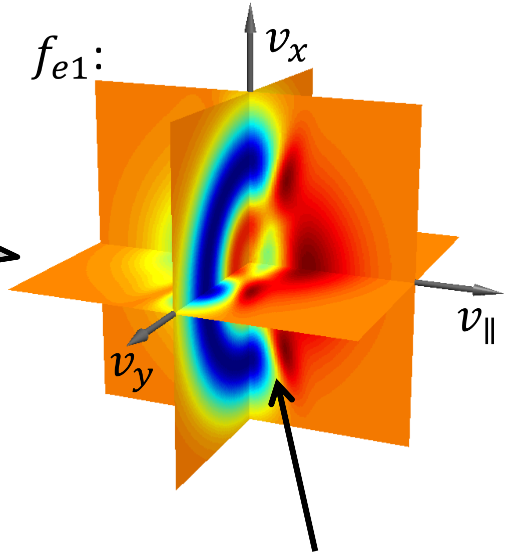
The bootstrap current can be calculated in stellarators, but it is numerically challenging

Must solve the drift-kinetic equation for $f_{i1},\ f_{e1}$:

$$v_\parallel \nabla_\parallel f_{i1} + \boldsymbol{v}_d \cdot \nabla s \frac{\partial f_{i0}}{\partial s} = C_{ii} + C_{ie}$$

$$v_\parallel \nabla_\parallel f_{e1} + \boldsymbol{v}_d \cdot \nabla s \frac{\partial f_{e0}}{\partial s} = C_{ei} + C_{ee}$$





$$\Rightarrow \quad j_\parallel = e \int d^3v\, v_\parallel f_{i1} - e \int d^3v\, v_\parallel f_{e1}$$

- Integro-differential equations.
- Steady advection-diffusion equations, advection-dominated.
- 5 coupled dimensions: $\theta, \varphi, v_\parallel, v$, species.
- Solutions (distribution functions) have internal boundary layers $\Longrightarrow$ need high resolution.

Solved by SFINCS code: ML et al, *Phys. Plasmas* (2014)

Need self-consistency between MHD equilibrium and drift-kinetic equation.

- Previous method: fixed-point iteration, only after an optimization.

MHD equilibrium code

Drift-kinetic code

VMEC: given $I_0(s)$, determine $\mathbf{B}_0$.

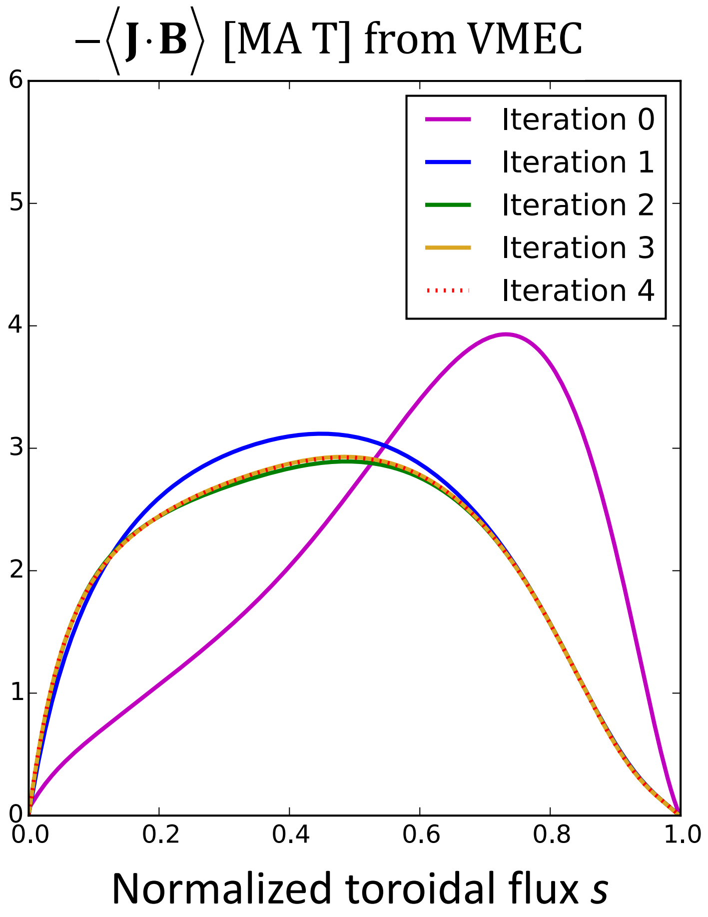
SFINCS: given $\mathbf{B}_0$, determine $I_1(s)$.

VMEC: given $I_1(s)$, determine $\mathbf{B}_1$.

SFINCS: given $\mathbf{B}_1$, determine $I_2(s)$.

…

- Accurate drift-kinetic bootstrap calculations in stellarators are computationally expensive. Preferably not in the optimization loop.



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Tokamak calculations of bootstrap current should apply also to quasisymmetric stellarators

In quasisymmetry, with $B = B(s, \theta - N\varphi)$,

- Guiding-center trajectories are **isomorphic** to trajectories in axisymmetry with the same 2D $B(s,\theta)$.
- Like the Lagrangian, $v_\| \nabla_\|$ & $\int d^3v\, v_\|$ only depend on θ and φ through B.
- Therefore, solutions of the drift-kinetic equation & their moments are isomorphic to those in axisymmetry as well.

$$v_\| \nabla_\| f_{i1} + \boldsymbol{v}_d \cdot \nabla s \frac{\partial f_{i0}}{\partial s} = C_{ii} + C_{ie}$$

$$j_\| = e \int d^3v\, v_\| f_{i1} - e \int d^3v\, v_\| f_{e1}$$

Pytte & Boozer (1981), Boozer (1983)

Need to substitute

$$\iota \to \iota - N$$

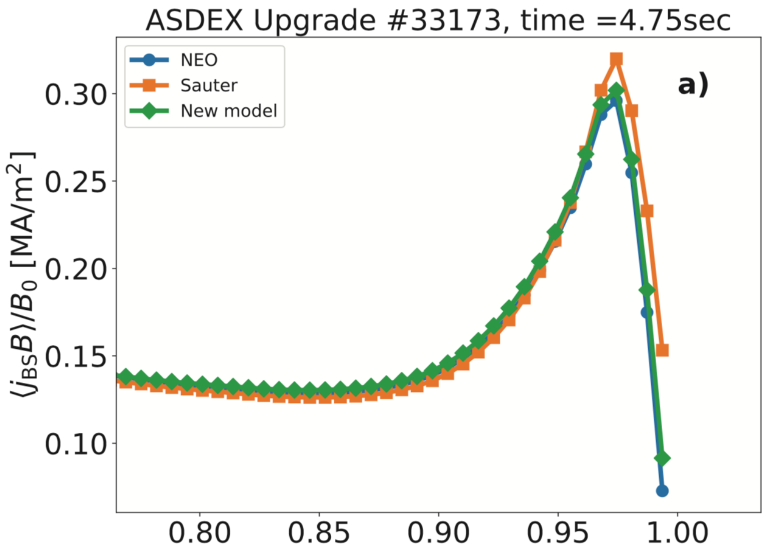
We'll exploit a recent advance in computing the bootstrap current in tokamaks

A new set of analytical formulae for the computation of the bootstrap current and the neoclassical conductivity in tokamaks

Cite as: Phys. Plasmas **28**, 022502 (2021); doi: 10.1063/5.0012664

A. Redl,[1,2,a)] C. Angioni,[1] E. Belli,[3] O. Sauter,[4] ASDEX Upgrade Team[b)] and EUROfusion MST1 Team[c)]

Bootstrap current



Flux surface label (minor radius)

Geometry enters through

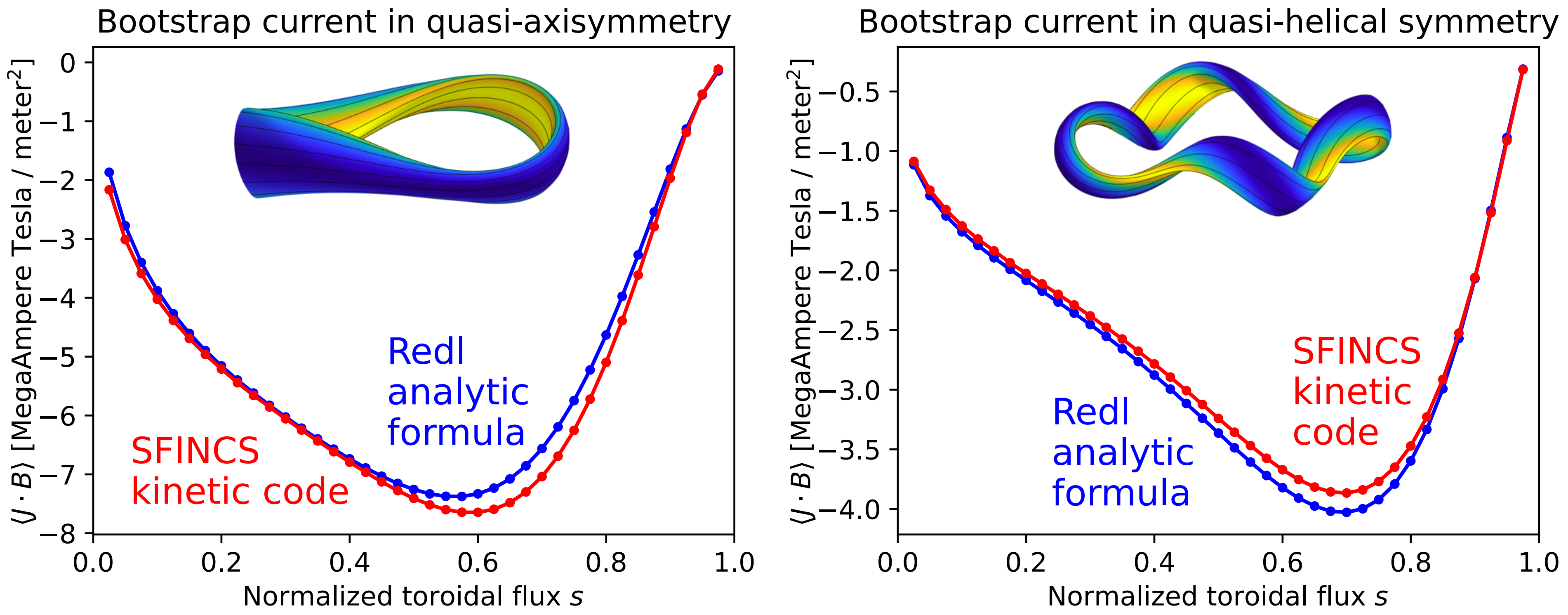
$$f_t = 1 - f_c = 1 - \frac{3}{4}\langle B^2\rangle \int_0^{1/B_{\max}} \frac{\lambda d\lambda}{\langle \sqrt{1-\lambda B}\rangle}.$$

$$\nu_{e*} = 6.921 \cdot 10^{-18} \frac{qRn_e Z \ln \Lambda_e}{T_e^2 \epsilon^{3/2}},$$

$$\nu_{i*} = 4.90 \cdot 10^{-18} \frac{qRn_i Z^4 \ln \Lambda_{ii}}{T_i^2 \epsilon^{3/2}},$$

Redl formula is accurate in previous quasisymmetric stellarators!

$n_e = (1 - s^5)\ 4\text{x}10^{20}\ \text{m}^{-3}$, $T_e = T_i = (1 - s)\ 12\ \text{keV}$



(Not self-consistent yet)

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Start with an optimization problem for β = 0

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

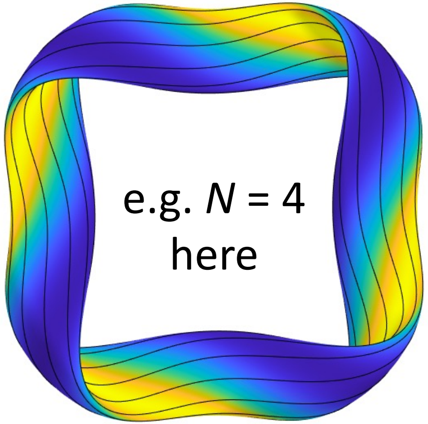
$$f_{QH} = \left(A - A_*\right)^2 + f_{QS} \qquad\qquad f_{QA} = \left(A - A_*\right)^2 + \left(\iota_* - \int_0^1 \iota\, ds\right)^2 + f_{QS}$$

Boundary aspect ratio

Goal: $B = B(s, \theta - N\varphi)$.

For quasi-axisymmetry, $N = 0$.

For quasi-helical symmetry, N is the number of field periods,



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$$f_{QH} = \left(A - A_* \right)^2 + f_{QS} \qquad f_{QA} = \left(A - A_* \right)^2 + \left(\iota_* - \int_0^1 \iota \, ds \right)^2 + f_{QS}$$

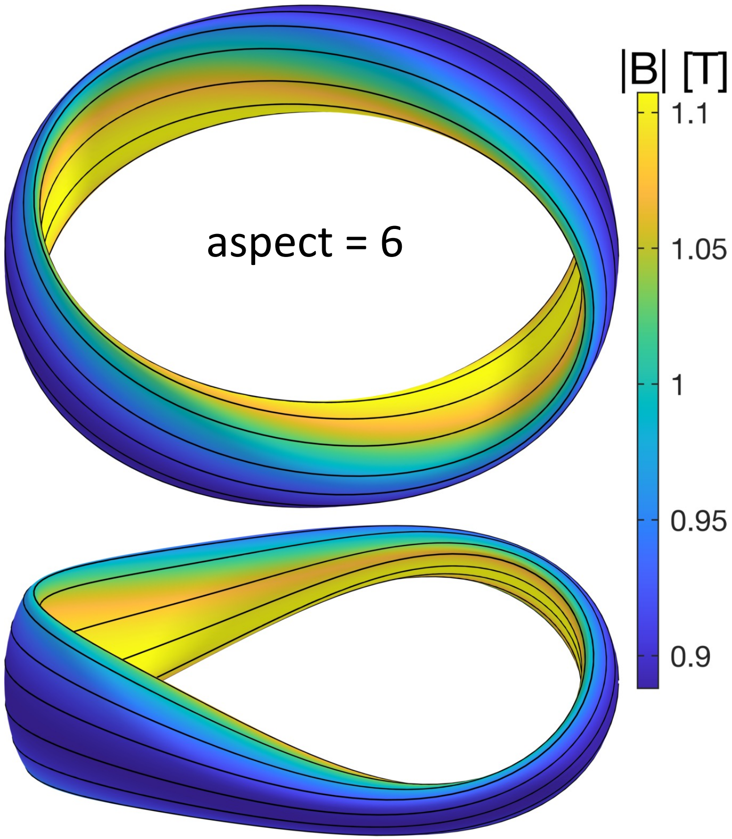
Boundary aspect ratio

- Parameter space: $R_{m,n}$ & $Z_{m,n}$ defining a toroidal boundary

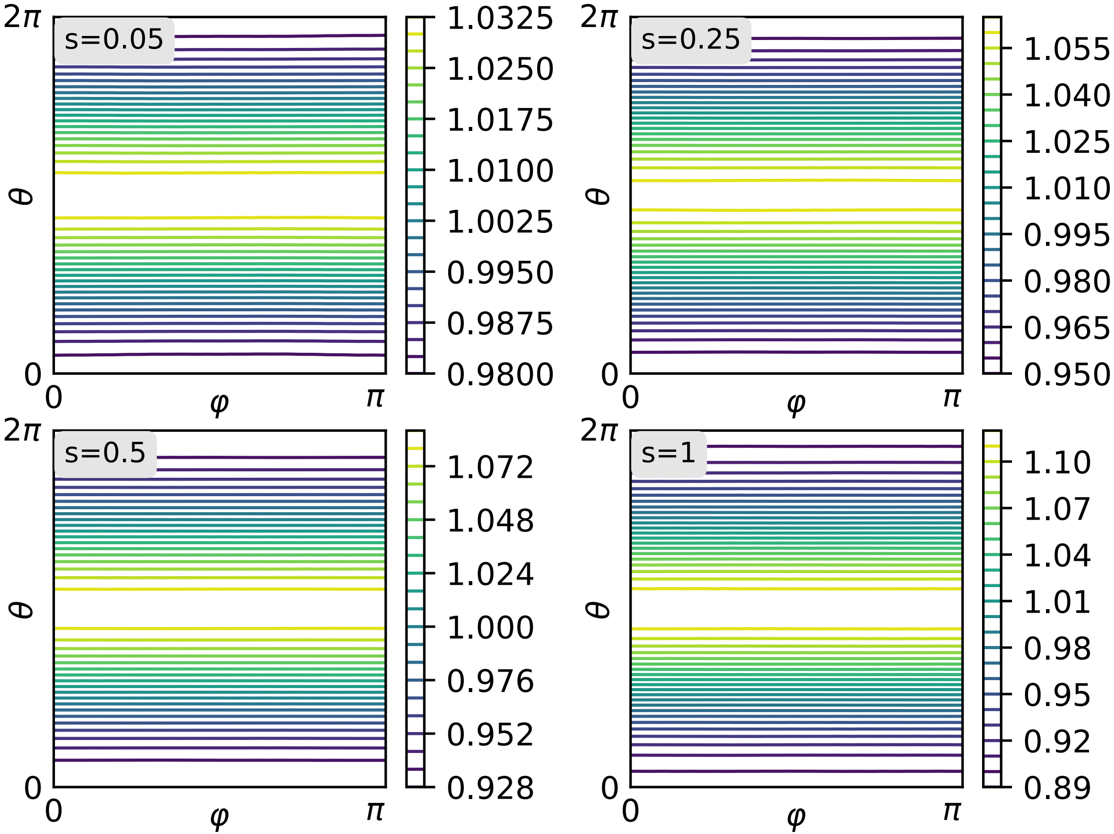
$$R(\theta,\phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \quad Z(\theta,\phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)$$

- Cold start: circular cross-section torus
- Algorithm: default for least-squares in scipy (trust region reflective)
- 6 steps: increasing # of modes varied & equilibrium resolution

Example of vacuum quasi-axisymmetry optimization

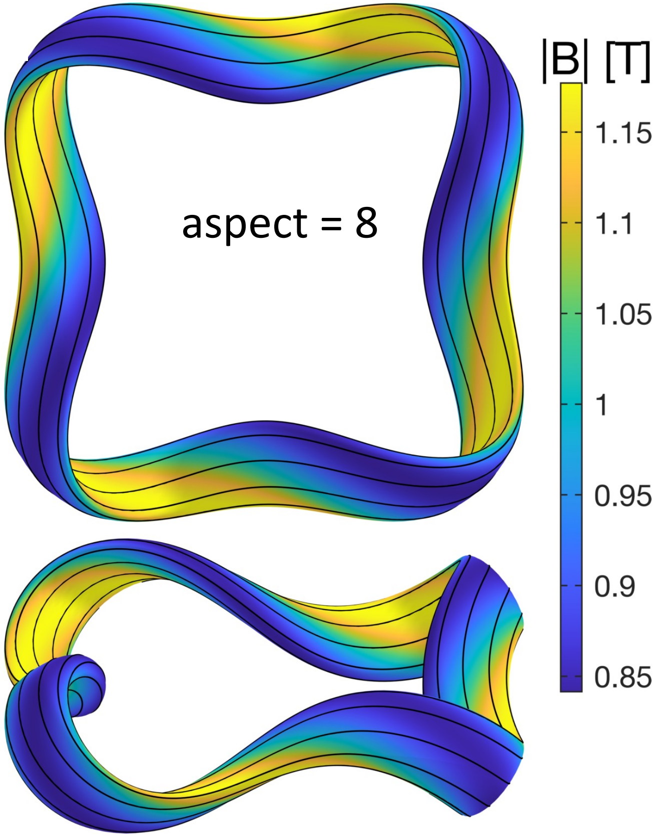


|B| on flux surfaces of the quasi-axisymmetric field

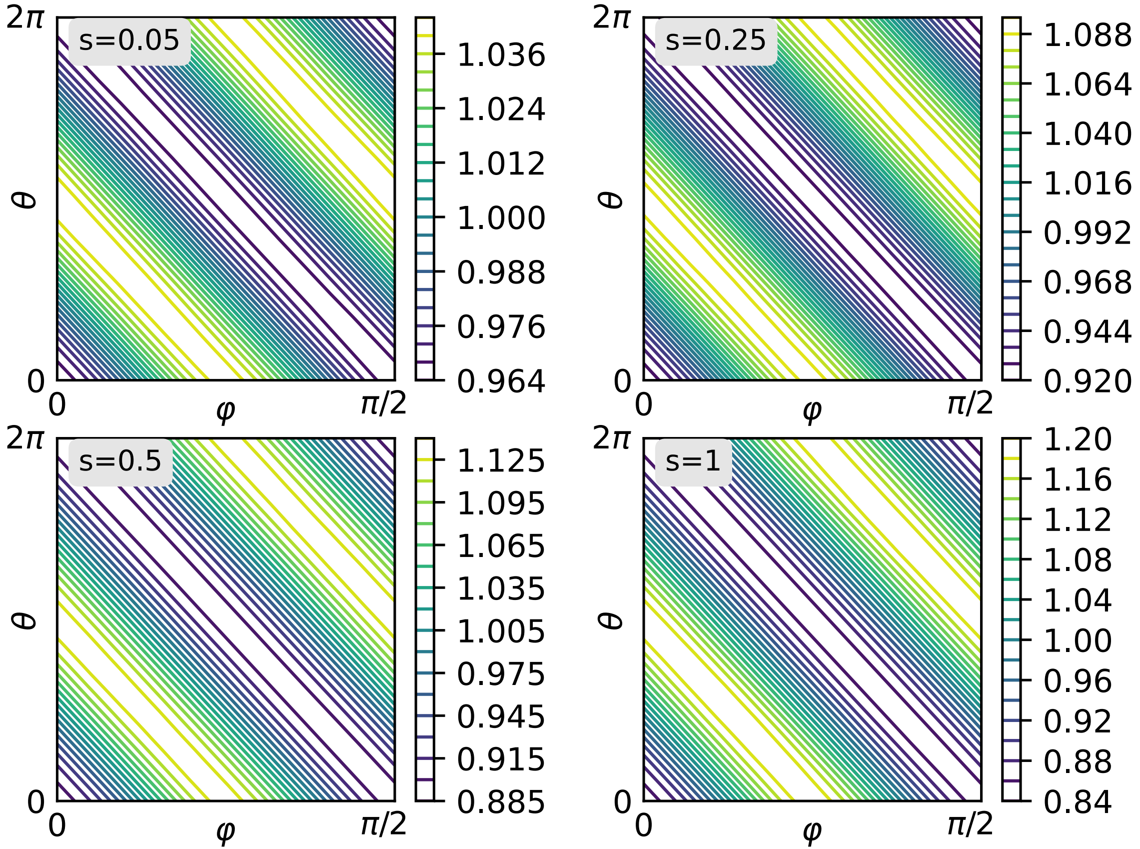


ML & Paul, PRL (2022).

Example of vacuum quasi-helical symmetry optimization



|B| on flux surfaces of the quasi-helically symmetric field



ML & Paul, PRL (2022).

Now add boostrap self-consistency to the optimization recipe

- Objective function: $$f = f_{QS} + f_{bootstrap} + \left(A - 6.5\right)^2 + \left(a - a_{ARIES-CS}\right)^2 + \left(\langle B \rangle - \langle B \rangle_{ARIES-CS}\right)^2$$

Boundary aspect ratio

Minor radius

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

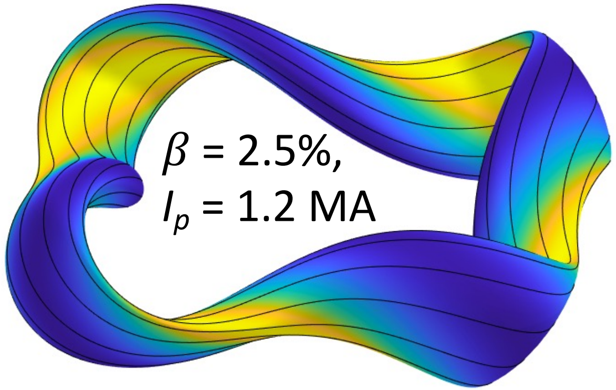
$$f_{bootstrap} = \frac{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{vmec}} - \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Redl}} \right]^2}{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{vmec}} + \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\mathrm{Redl}} \right]^2}$$

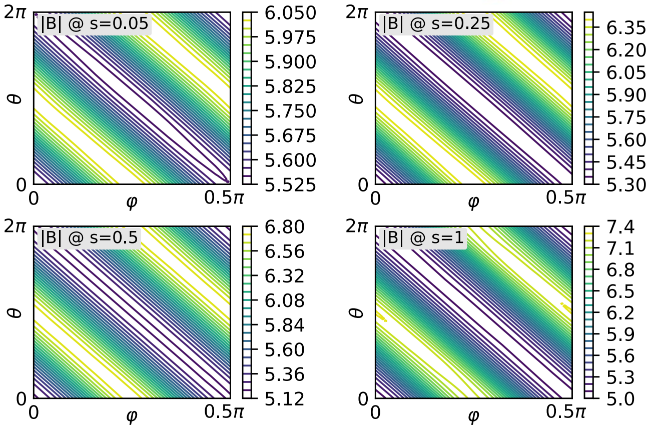
- Parameter space: {$R_{m,n}$, $Z_{m,n}$, toroidal flux, current spline values}

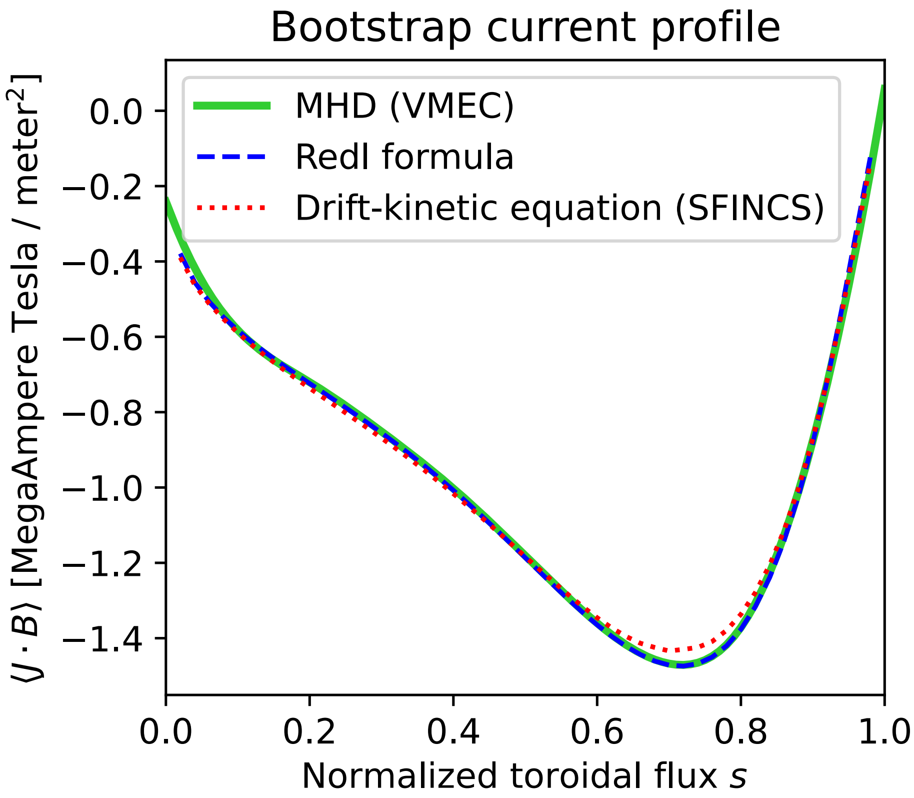
  or {$R_{m,n}$, $Z_{m,n}$, toroidal flux, iota spline values}

Example of optimization with self-consistent bootstrap current

n_{e0} = 2.2e20/meters^3
$T_{e0} = T_{i0}$ = 10 keV



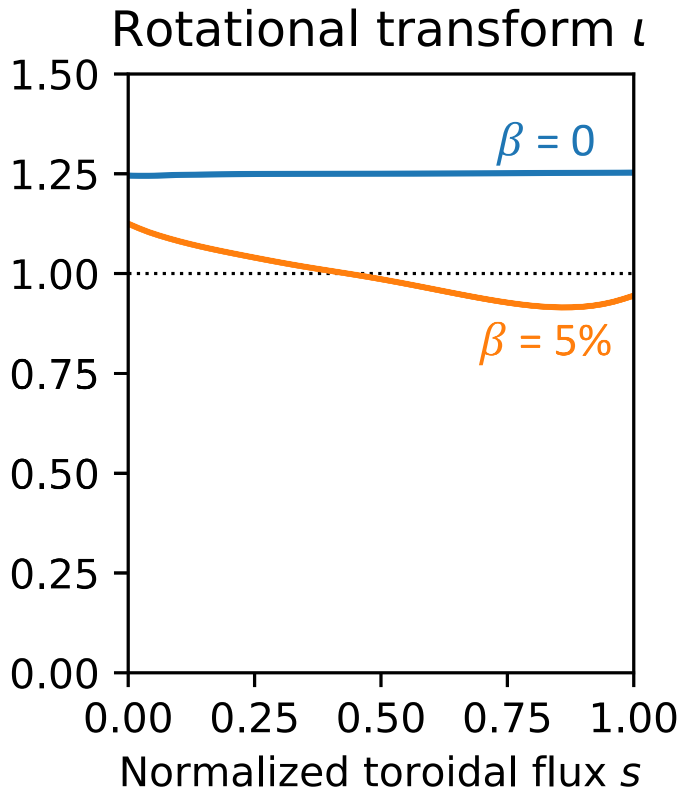




All input/output files and optimization scripts online at
doi.org/10.5281/zenodo.6520103

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

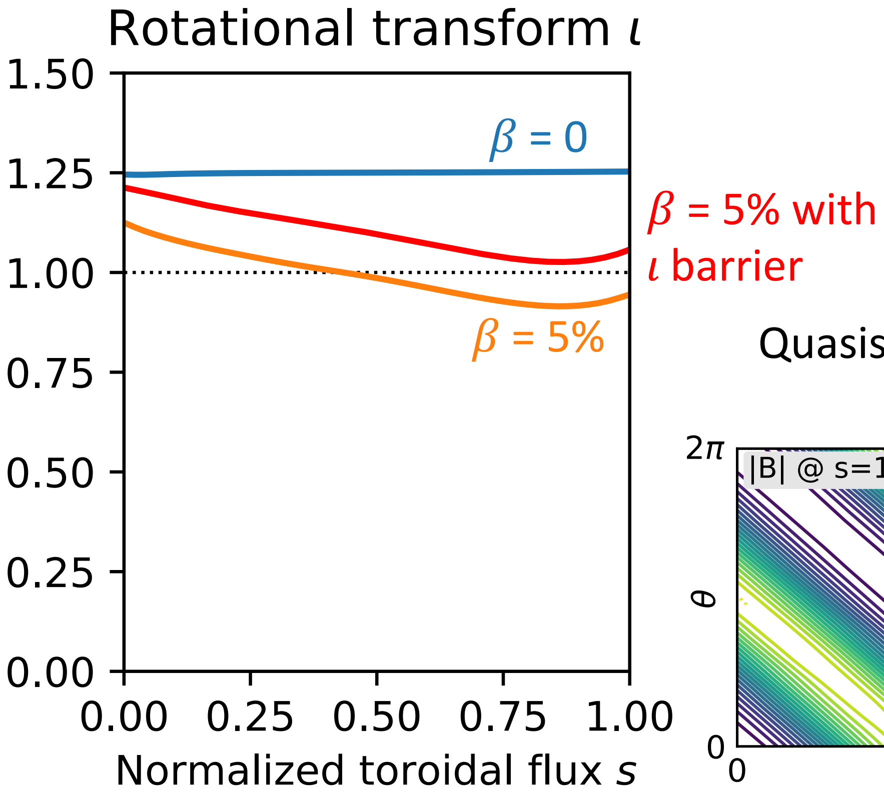
Crossing iota=1, the worst resonance, is probably unacceptable.



n_{e0} = 3e20/meters^3, T_{e0} = T_{i0} = 15 keV

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

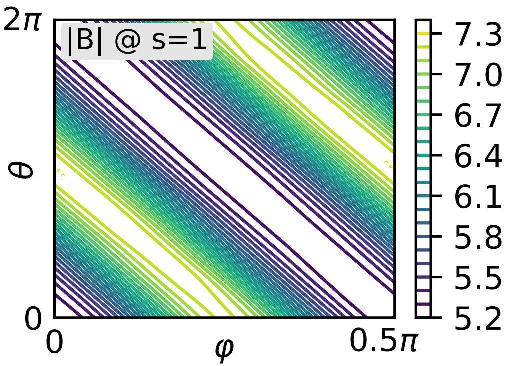
Crossing iota=1, the worst resonance, is probably unacceptable.

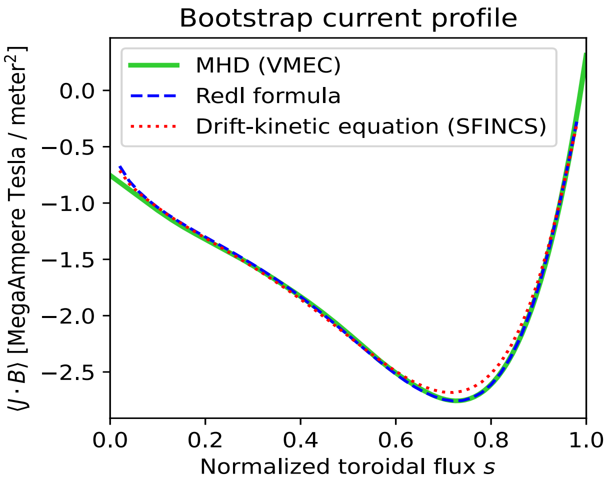


Solution: Add barrier term in objective

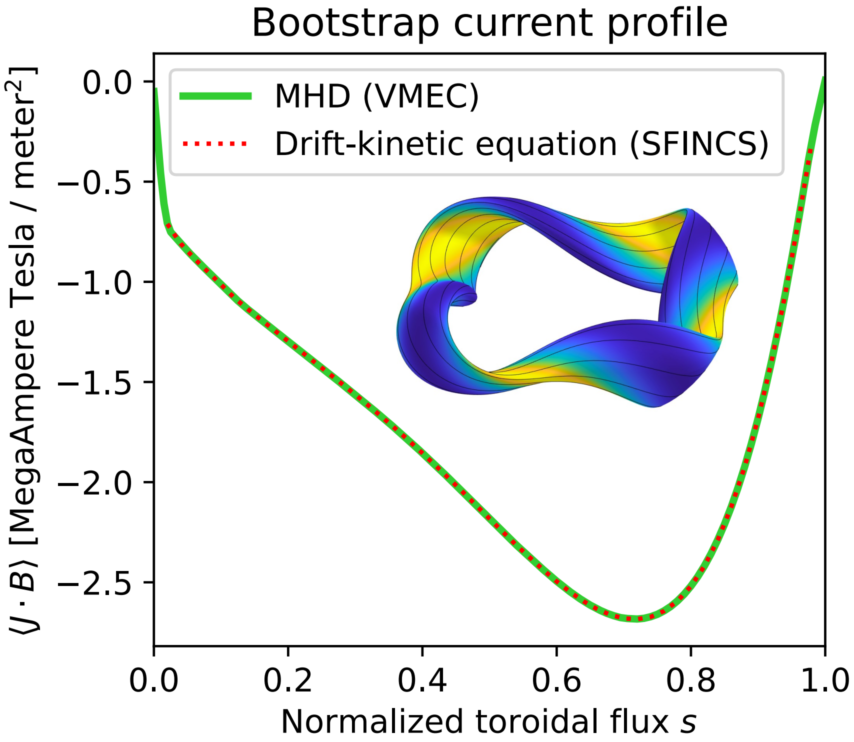
$$f \mathrel{+}= \int_0^1 ds\left[\min\left(\left|\iota(s)\right|-1.03,\ 0\right)\right]^2$$

Quasisymmetry & bootstrap consistency remain good:





If you want *perfectly* self-consistent current, you can do a few fixed-point iterations at the end

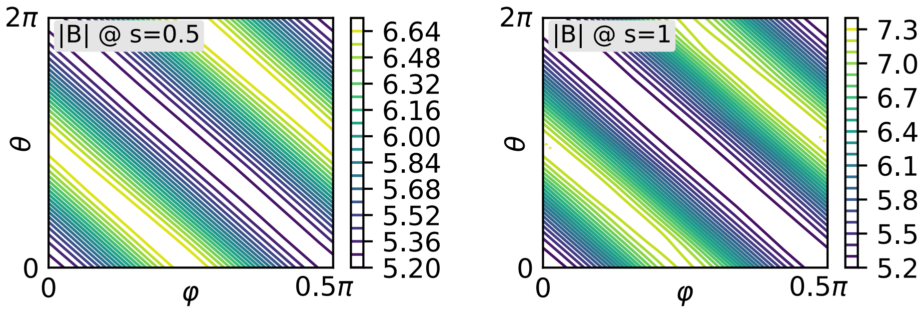


$$\langle \beta \rangle = 5\%, \qquad \varepsilon_{eff}^{3/2} < 6\times 10^{-5}$$

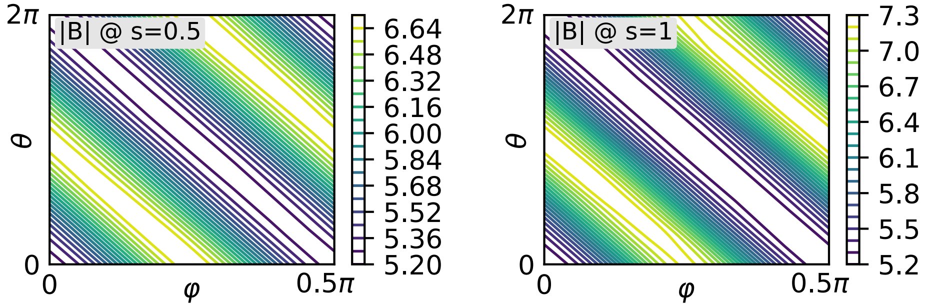
α-particle energy losses < 0.3%

No significant degradation in quasisymmetry:

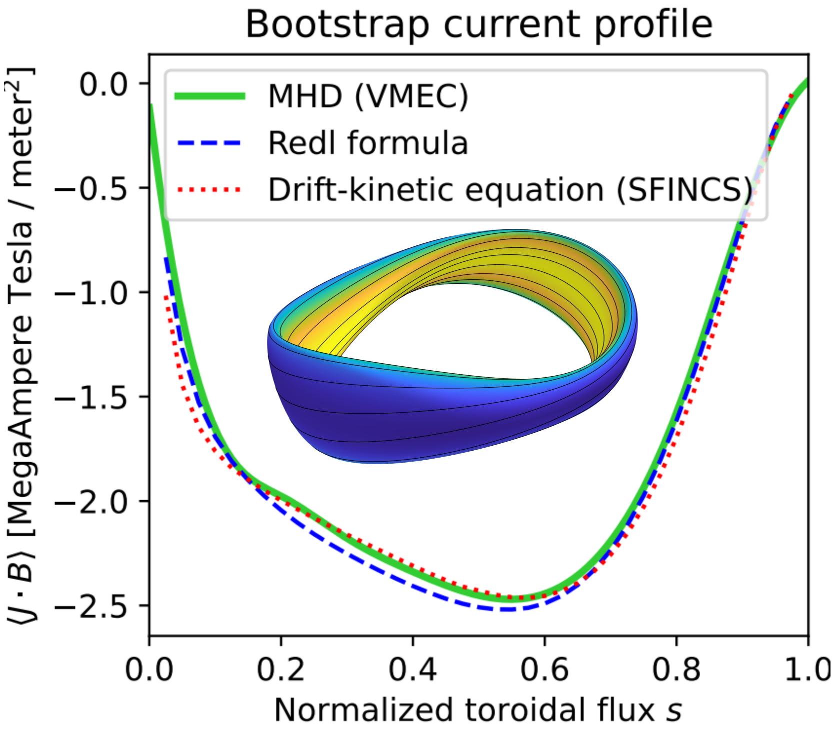
Optimization with Redl current



After SFINCS fixed-point iterations



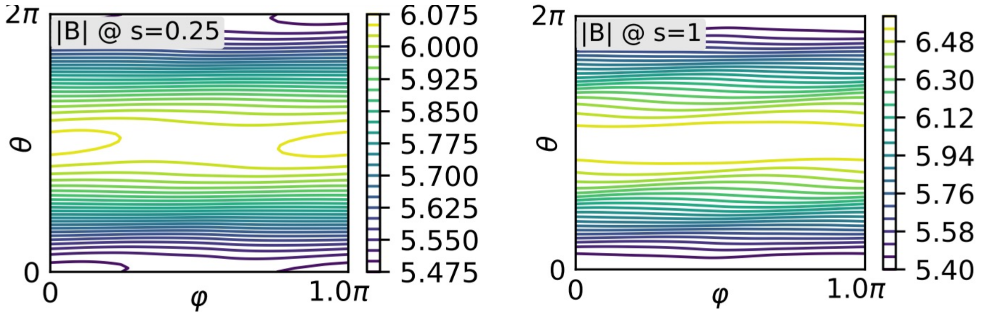
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry

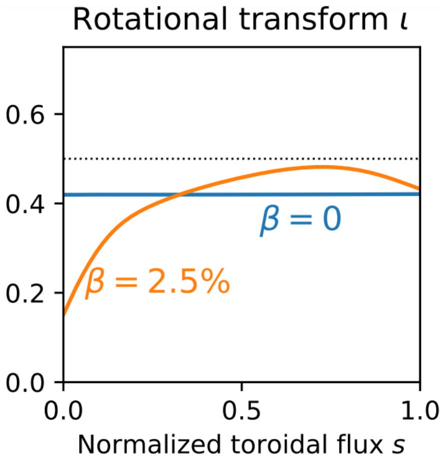


$$\langle\beta\rangle = 2.5\%, \qquad \epsilon_{eff}^{3/2} < 2 \times 10^{-5}$$

α-particle energy losses < 1.5%

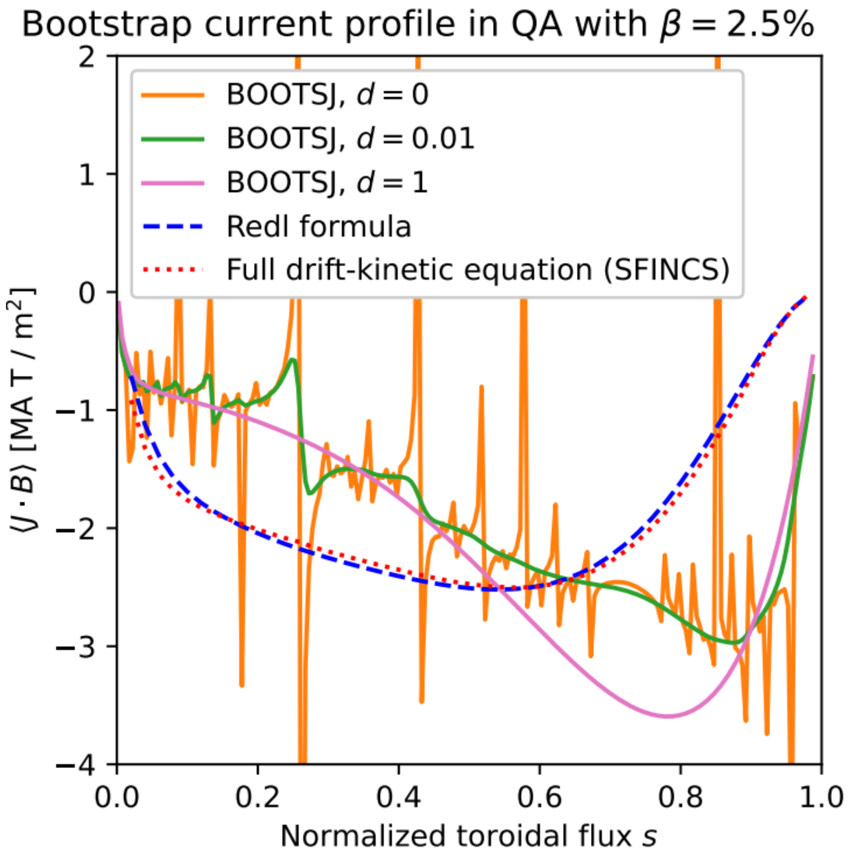
Symmetry is not as good as for vacuum, but sufficient for excellent confinement





Possible islands where ι = 1/4, 2/5, 1/3?

Redl formula is more accurate than long-mean-free-path stellarator bootstrap formula, & free of resonances



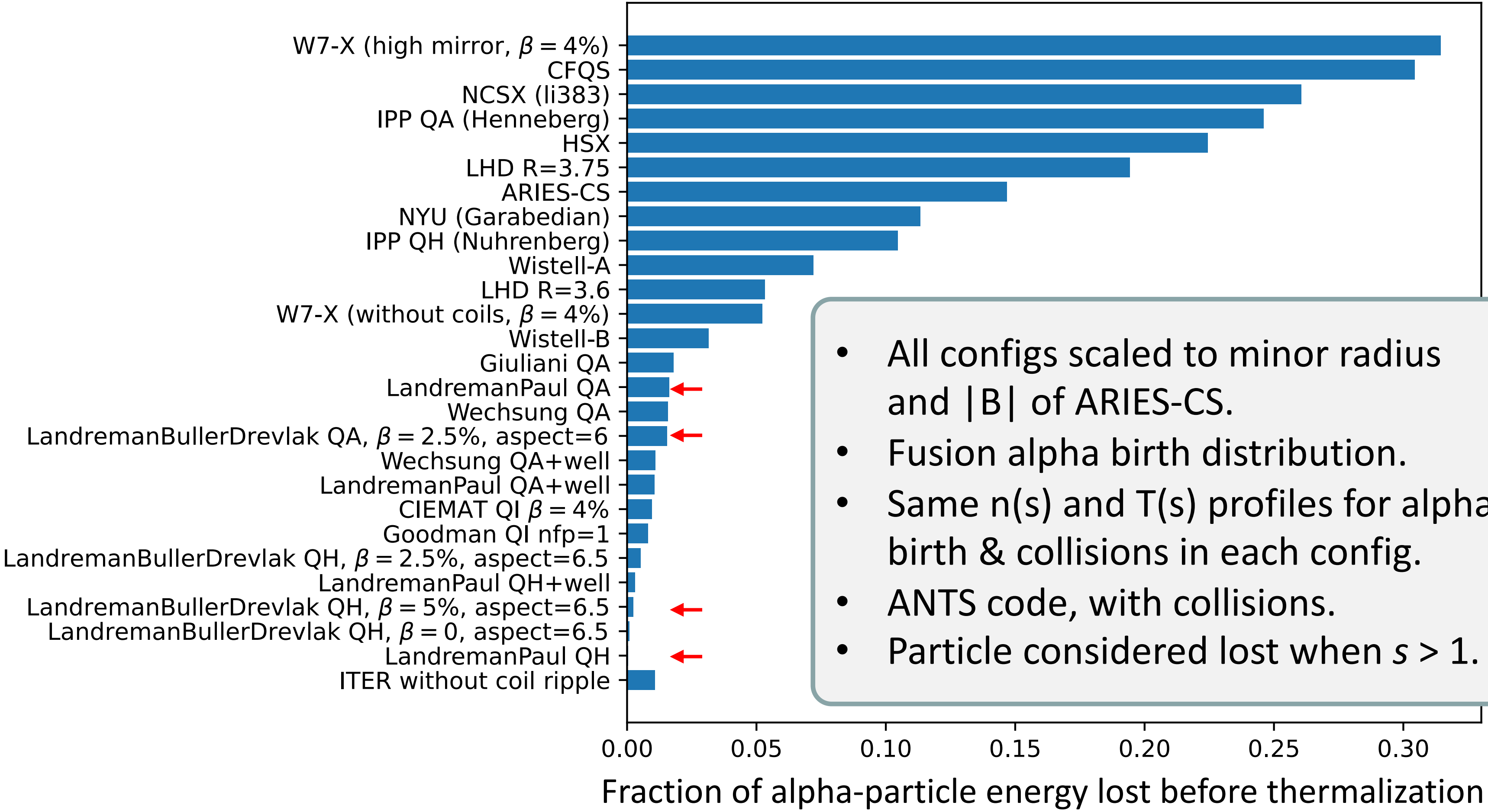
Stellarator bootstrap formulae for long-mean-free-path (low collisionality):

Shaing & Callen (1983),
Shaing et al (1989),
Helander, Parra & Newton (2017)

BOOTSJ ad-hoc smoothing:

$$\frac{1}{m - n/\iota} \rightarrow \frac{m - n/\iota}{(m - n/\iota)^2 + m^2 d^2}$$

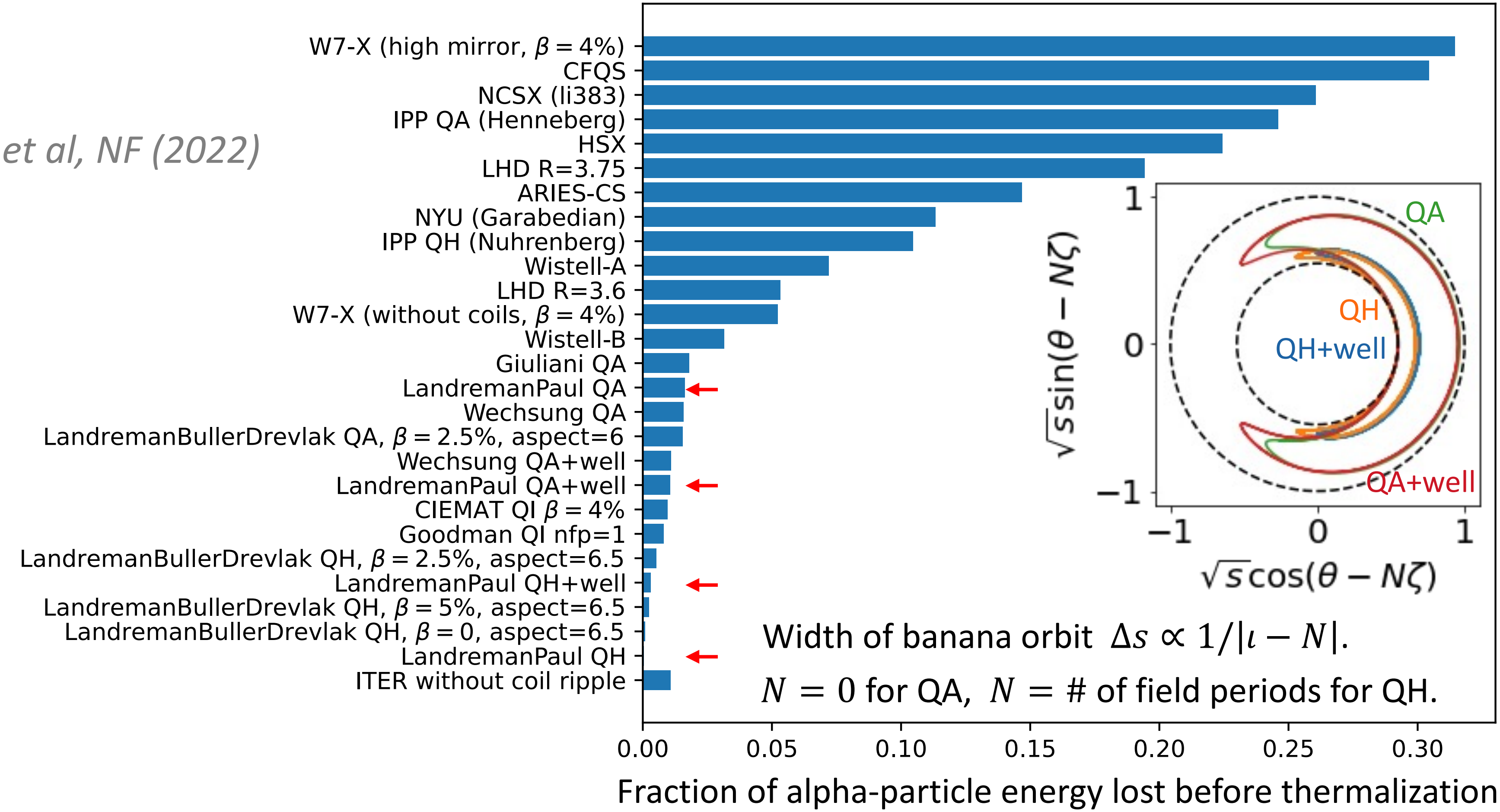
Quasisymmetry works: alpha particle confinement is significantly improved



- All configs scaled to minor radius and |B| of ARIES-CS.
- Fusion alpha birth distribution.
- Same n(s) and T(s) profiles for alpha birth & collisions in each config.
- ANTS code, with collisions.
- Particle considered lost when $s > 1$.

Alpha confinement in quasi-helical stellarators can be better than in a tokamak due to thinner bananas

Paul et al, NF (2022)

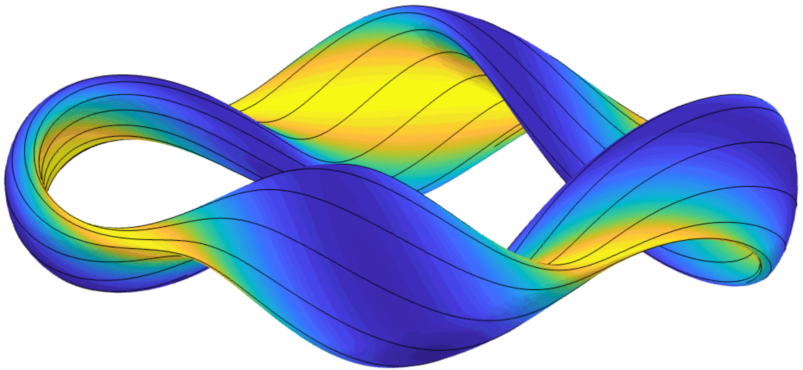


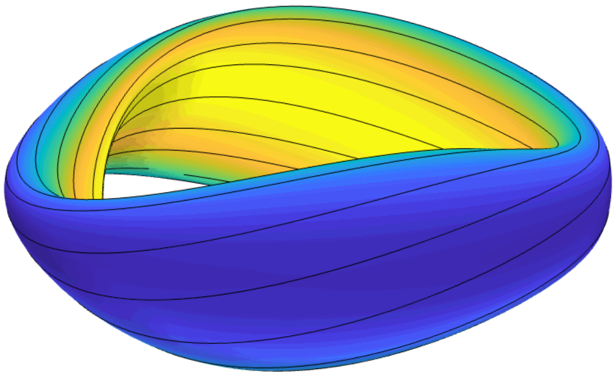
Width of banana orbit $\Delta s \propto 1/|\iota - N|$.

$N = 0$ for QA, $N = \#$ of field periods for QH.

Summary:

- Synergy with tokamaks: A new accurate formula is available for the bootstrap current in an important class of stellarators.
- It is now possible to design stellarators with α-particle confinement close to or better than a tokamak.





Future work:

- Include MHD stability
- Find coils
- Check flux surface quality, & eliminate any islands.
- Check robustness to uncertainty in the pressure profile.