New stellarator configurations with precise quasisymmetry and energetic particle confinement

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Remarkable progress in stellarator $\alpha$-particle confinement in the last year

Since 2021, This talk

Fraction of $\alpha$-particle energy lost before thermalization

All configurations scaled to same minor radius and $|B|$. See also Bader et al, Nuclear Fusion (2021), Sanchez et al (this session), Goodman et al (this meeting).
These new configurations with good alpha confinement use the principle of *quasisymmetry*.

\[
B = B(s, \theta - N \varphi)
\]

\[
\int (v_d \cdot \nabla s) dt = 0
\]
Goal: $B = B(s, \theta - N\varphi)$
• Minimal optimization recipe (low $\beta$)
• Self-consistent bootstrap current at high $\beta$
• Future directions
• Minimal optimization recipe (low $\beta$)

• Self-consistent bootstrap current at high $\beta$

• Future directions
Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:
  \[ f_{QS} = \int d^3x \left( \frac{1}{B^3} \left[ (N - t) \mathbf{B} \times \nabla \mathbf{B} \cdot \nabla \psi - \left( G + NI \right) \mathbf{B} \cdot \nabla \mathbf{B} \right] \right)^2 \]
  \[ f_{QH} = (A - A_*)^2 + f_{QS} \quad f_{QA} = (A - A_*)^2 + \left( t_0 - \int_0^1 t \, ds \right)^2 + f_{QS} \]

Goal: \( B = B(s, \theta - N \varphi) \).

For quasi-axisymmetry, \( N = 0 \).

For quasi-helical symmetry, \( N \) is the number of field periods,

\( e.g. \ N = 4 \) here
• 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.

• Objective functions:

\[
f_{QS} = \int d^3 x \left( \frac{1}{B^3} \left[ (N-t)\mathbf{B} \times \nabla \mathbf{B} \cdot \nabla \psi - (G+N)\mathbf{B} \cdot \nabla \mathbf{B} \right] \right)^2
\]

\[
f_{QH} = (A - A_*)^2 + f_{QS}
\]

\[
f_{QA} = (A - A_*)^2 + \left( l_* - \int_0^1 l ds \right)^2 + f_{QS}
\]

Boundary aspect ratio

• Parameter space: \( R_{m,n} \) & \( Z_{m,n} \) defining a toroidal boundary

\[
R(\theta,\phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \quad Z(\theta,\phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)
\]

• Codes used: SIMSOPT with VMEC

• Cold start: circular cross-section torus

• Vacuum fields (\( \nabla \times \mathbf{B} = 0 \)) at first, allowing precise checks

• Algorithm: default for least-squares in scipy (trust region reflective)

• 6 steps: increasing # of modes varied & equilibrium resolution

• Run many optimizations, pick the best
Straight $|B|$ contours are possible for quasi-axisymmetry

aspect = 6

$|B|$ on flux surfaces of the quasi-axisymmetric field

Straight $|B|$ contours are possible for quasi-helical symmetry.

$|B|$ on flux surfaces of the quasi-helically symmetric field.

Nearly as good quasisymmetry exists also at lower aspect ratio

Aspect ratio 3.3
Nearly as good quasisymmetry exists also at lower aspect ratio or different # of field periods.

Aspect ratio 3.3

3 field periods, aspect ratio 6
Good symmetry also exists with magnetic well

\[
\frac{d^2 (\text{flux surface volume})}{d (\text{toroidal flux})^2} < 0 \text{ everywhere}
\]

16-coil solutions have been found for the quasi-axisymmetric configurations.


With magnetic well

Without magnetic well

<\text{R}> / 10 between filament centers.
Symmetry-breaking modes can be made extremely small

New QA configuration

Fourier amplitudes $|B_{m,n}|$ [Tesla]

$m = 0, n = 0$ (Background)

$m \neq 0, n = 0$ (Quasiasysymmetric)

Geomagnetic field

$n \neq 0$ (Symmetry-breaking)

$s = $ Normalized toroidal flux

Symmetry-breaking $B_{m,n}$ [Tesla]

- NCSX
- IPP QA
- ARIES-CS
- IPP QH
- NYU
- HSX
- CFQS
- Wistell-A

Previous configurations

Dotted = with coils

New QH+well
New QH
New QA+well
New QA
$|B|$ in Boozer coordinates was verified by independent SPEC calculations.

By Elizabeth Paul

$max \ |B_{m,n}|$ [Tesla] with $n \neq 0$

$(N_{tor} = M_{pol}, L_{rad} = M_{pol} + 4)$
Quasisymmetry works: alpha particle confinement is significantly improved

- All configs scaled to minor radius and $|B|$ of ARIES-CS.
- Fusion alpha birth distribution.
- Same $n(s)$ and $T(s)$ profiles for alpha birth & collisions in each config.
- ANTS code, with collisions.
- Particle considered lost when $s > 1$.

### Fraction of alpha particle energy lost before thermalization

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>W7-X (high mirror, $\beta = 4%$)</td>
<td>0.30</td>
</tr>
<tr>
<td>CFQS</td>
<td>0.27</td>
</tr>
<tr>
<td>NCSX (ii383)</td>
<td>0.25</td>
</tr>
<tr>
<td>IPP QA (Henneberg)</td>
<td>0.24</td>
</tr>
<tr>
<td>HSX</td>
<td>0.23</td>
</tr>
<tr>
<td>LHD $R=3.75$</td>
<td>0.22</td>
</tr>
<tr>
<td>ARIES-CS</td>
<td>0.21</td>
</tr>
<tr>
<td>NYU (Garabedian)</td>
<td>0.20</td>
</tr>
<tr>
<td>IPP QH (Nuhrenberg)</td>
<td>0.19</td>
</tr>
<tr>
<td>Wistell-A</td>
<td>0.18</td>
</tr>
<tr>
<td>LHD $R=3.6$</td>
<td>0.17</td>
</tr>
<tr>
<td>W7-X (without coils, $\beta = 4%$)</td>
<td>0.16</td>
</tr>
<tr>
<td>Wistell-B</td>
<td>0.15</td>
</tr>
<tr>
<td>Giuliani QA</td>
<td>0.14</td>
</tr>
<tr>
<td>Near-axis expansion QH $nfp=3$ aspect=5</td>
<td>0.13</td>
</tr>
<tr>
<td>Wechsung QA</td>
<td>0.12</td>
</tr>
<tr>
<td>LandremanPaul QA</td>
<td>0.11</td>
</tr>
<tr>
<td>Wechsung QA+well</td>
<td>0.10</td>
</tr>
<tr>
<td>LandremanPaul QA+well</td>
<td>0.09</td>
</tr>
<tr>
<td>QA, $\beta = 3%$</td>
<td>0.08</td>
</tr>
<tr>
<td>Near-axis expansion QH $nfp=4$ aspect=5</td>
<td>0.07</td>
</tr>
<tr>
<td>LandremanPaul QH+well</td>
<td>0.06</td>
</tr>
<tr>
<td>QH, $\beta = 5%$, aspect=6.5</td>
<td>0.05</td>
</tr>
<tr>
<td>QH, $\beta = 0$, aspect=6.5</td>
<td>0.04</td>
</tr>
<tr>
<td>LandremanPaul QH</td>
<td>0.03</td>
</tr>
<tr>
<td>ITER tokamak, without coil ripple</td>
<td>0.02</td>
</tr>
</tbody>
</table>

0.00 | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30
Alpha confinement in quasi-helical stellarators can be better than in a tokamak due to thinner bananas.

Fraction of alpha particle energy lost before thermalization

See poster by Elizabeth Paul this afternoon

\[ \Delta s \propto \frac{1}{|t - N|} \]

Width of banana orbit \( \Delta s \propto 1/|t - N| \).

\( N = 0 \) for QA, \( N = \# \) of field periods for QH.
The symmetry also yields extremely low collisional transport for a thermal plasma.

Radial neoclassical transport coefficient

Previous configurations

New QH+well
New QA+well
New QH
New QA

Dotted = with coils

Wechsung et al, PNAS (2022)
Even better quasisymmetry and $\varepsilon_{\text{eff}}$ is achieved by refinement with combined plasma-and-coil optimization

### Objective:

$$f = \sum_{\text{surfaces}} \left( (\text{QS error})^2 + (\bar{\ell} - \bar{\ell}_*)^2 + \left( \begin{array}{c} \text{coil length, mean curvature,} \\ \text{max curvature, coil-coil distance terms} \end{array} \right) \right)$$

### Parameter space:
- coil shapes

### Derivatives:
- analytic + adjoint method

#### QA
- 16 coils
- $\iota > 0.4$

#### $\alpha$ losses
- < 2%

#### $\varepsilon_{\text{eff}}^{3/2}$
- $< 5 \times 10^{-9}$

#### $B_{m,n}^{\text{non-QS}}$
- $< 3 \times 10^{-5}$

### Collisional heat flux

*Giuliani et al, arXiv:2203.03753 (2022)*
• Minimal optimization recipe (low $\beta$)

• Self-consistent bootstrap current at high $\beta$

• Future directions
How can bootstrap current be included self-consistently in stellarator optimization?

- Need **self-consistency** between MHD equilibrium and drift-kinetic equation.
- Previous method: fixed-point iteration, only after an optimization.
  
  VMEC: given \( I_0(s) \), determine \( B_0 \).
  
  SFINCS: given \( B_0 \), determine \( I_1(s) \).
  
  VMEC: given \( I_1(s) \), determine \( B_1 \).
  
  SFINCS: given \( B_1 \), determine \( I_2(s) \).
  
  ...

- Accurate drift-kinetic bootstrap calculations in stellarators are computationally expensive. Preferably not in the optimization loop.

\[-\langle J \cdot B \rangle \text{ [MA T]} \text{ from VMEC}\]
New idea: exploit quasisymmetry & use analytic expressions for tokamaks


\[ t \rightarrow t - N \]

Bootstrap current (& other quantities) in quasisymmetry are the same as in axisymmetry, up to some substitutions:

Should be accurate for the new precisely quasisymmetric configurations.

A new set of analytical formulae for the computation of the bootstrap current and the neoclassical conductivity in tokamaks

Cite as: Phys. Plasmas 28, 022502 (2021); doi: 10.1063/5.0012664
Submitted: 6 May 2020 · Accepted: 11 December 2020 · Published Online: 2 February 2021

A. Redl, C. Angioni, E. Belli, O. Sauter, ASDEX Upgrade Team and EUROfusion MST1 Team
Before doing new optimizations: Redl formula is accurate in previous QA & QH stellarators

\[ n_e = (1 - s^5) \times 10^{20} \text{m}^{-3}, \quad T_e = T_i = (1 - s) \times 12 \text{ keV} \]

(Not self-consistent yet)
Optimization recipe

- Objective function: \( f = f_{qs} + f_{bootstrap} + (A - 6.5)^2 + (a - a_{ARIES-CS})^2 + (\langle B \rangle - \langle B \rangle_{ARIES-CS})^2 \)

  \[
  f_{qs} = \int d^3x \left( \frac{1}{B^3} \left[ (N - i) \mathbf{B} \times \nabla \mathbf{B} \cdot \nabla \psi - (G + NI) \mathbf{B} \cdot \nabla \mathbf{B} \right] \right)^2
  \]

  \[
  f_{bootstrap} = \frac{\int_0^1 ds \left[ \langle j \cdot \mathbf{B} \rangle_{vme} - \langle j \cdot \mathbf{B} \rangle_{Redl} \right]^2}{\int_0^1 ds \left[ \langle j \cdot \mathbf{B} \rangle_{vme} + \langle j \cdot \mathbf{B} \rangle_{Redl} \right]^2}
  \]

- Parameter space: \( \{R_{m,n}, Z_{m,n}, \text{toroidal flux, current spline values} \} \)
  or \( \{R_{m,n}, Z_{m,n}, \text{toroidal flux, iota spline values} \} \)

- Cold start

- Algorithm: default for least-squares in scipy (trust region reflective)

- Steps: increasing # of modes varied: m and \(|n/nfp| \) up to j in step j
Example of optimization with self-consistent bootstrap current

\[ n_{e0} = 2.2 \text{e}20 / \text{meters}^3 \]
\[ T_{e0} = T_{i0} = 10 \text{ keV} \]
\[ \beta = 2.5\%, \quad I_p = 1.2 \text{ MA} \]

All input/output files and optimization scripts online at doi.org/10.5281/zenodo.6520103
To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included.

Crossing iota=1, the worst resonance, is probably unacceptable.

\[ \beta = 0, \beta = 5\% \]

\[ n_e = 3 \times 10^{20}/\text{meters}^3, T_e = T_i = 15 \text{ keV} \]
To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included. Crossing iota=1, the worst resonance, is probably unacceptable.

Solution: Add barrier term in objective

\[ f^+ = \int_0^1 ds \left[ \min\left( |\iota(s)| - 1.03, 0 \right) \right]^2 \]

Rotational transform \( \iota \)

\( \beta = 0 \)

\( \beta = 5\% \) with \( \iota \) barrier

Quasisymmetry & bootstrap consistency remain good:
If you want perfectly self-consistent current, you can do a few fixed-point iterations at the end.

No significant degradation in quasisymmetry:

Optimization with Redl current:

After SFINCS fixed-point iterations:

\[ \langle \beta \rangle = 5\%, \quad \epsilon_{\text{eff}}^{3/2} < 6 \times 10^{-5} \]

\( \alpha \)-particle losses < 0.3%
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry.

Bootstrap current profile

Symmetry is not as good as for vacuum, but sufficient for excellent confinement.

\[ \langle \beta \rangle = 2.5\%, \quad \epsilon_{eff}^{3/2} < 2 \times 10^{-5} \]

\( \alpha \)-particle losses < 1.5%

Possible islands where \( \iota = 1/4, 2/5, 1/3 \)?
Redl formula is more accurate than long-mean-free-path stellarator bootstrap formula, & free of resonances

Stellarator bootstrap formulae for long-mean-free-path (low collisionality):

*Shaing & Callen (1983)*,
*Shaing et al (1989)*,
*Helander, Parra & Newton (2017)*

**BOOTSJ ad-hoc smoothing:**

\[
\frac{1}{m - n/\nu} \rightarrow \frac{m - n/\nu}{(m - n/\nu)^2 + m^2d^2}
\]
• Minimal optimization recipe (low $\beta$)

• Self-consistent bootstrap current at high $\beta$

• Future directions
Future directions

• For the high $\beta$ configurations, check surface quality, & eliminate any islands.

• Coils & MHD stability for the high $\beta$ configurations.

• Check robustness to uncertainty in the pressure profile.

• Similar recipes for quasi-poloidal symmetry or quasi-isodynamic?
It is now possible to design stellarators with alpha confinement close to or better than a tokamak.
Extra slides
The new configurations have small magnetic shear

New QH
New QH+well

New QA
New QA+well
Good flux surface exist with coils

New QA

New QA+well
SPEC confirms the new QA/QH configurations have good surfaces.

- New QA
- New QH
- New QA+well
- New QH+well
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry.

$\langle \beta \rangle = 3\%$, $\varepsilon_{\text{eff}}^{3/2} < 7 \times 10^{-6}$

$\alpha$-particle losses < 1%

Symmetry is not as good as for vacuum, but sufficient for excellent confinement.

Possible islands where $\iota = 2/3$, $\iota = 4/7 = 0.57$?
Why do the configurations with best quasisymmetry not have the best trajectory confinement?

Lost trajectories in the new QA look like this:

Fraction of alpha energy lost

Time [sec]

Lost trajectories in the new QA look like this:

Width of banana orbit \( \Delta s \approx \left| \frac{mvR\sqrt{2r\bar{\eta}}}{(t-N)\psi_{edge}Ze} \right| \propto \frac{1}{t-N} \)

For fixed minor radius, \( \frac{\Delta s_{QA}}{\Delta s_{QH}} \sim 4 \)

\( \Delta s_{QH} \)
2 types of quasisymmetry

Quasi-axisymmetry (QA): $B = B(r, \theta)$

Quasi-helical symmetry (QH): $B = B(r, \theta - N\varphi)$

General stellarator (not symmetric)

Contours of $B = |B|$: $B_{\text{min}}$ to $B_{\text{max}}$
Previous quasisymmetric configurations

We want
\[ B = B(r, \theta - N \varphi) \]

(a) Zarnstorff et al (2001)
(b) Najambadi et al (2008)
(c) Garabedian (2008)
(d) Liu et al (2018)
(e) Henneberg et al (2019)
(f) Nuhrenberg & Zille (1988)
(g) Anderson et al (1995)
(h) Bader et al (2020)

Is there an optimization recipe that can give consistently straight |B| contours?
New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Redl (2021)

ASDEX Upgrade #33173, time = 4.75 sec

Geometry enters through

\[ f_t = 1 - f_c = 1 - \frac{3}{4} \langle B^2 \rangle \int_0^{1/B_{\text{max}}} \frac{\lambda d\lambda}{\sqrt{1 - \lambda B}} \]

\[ \nu_{e*} = 6.921 \cdot 10^{-18} \frac{q R n_e Z \ln \Lambda_e}{T_e^{2/3} \epsilon^{3/2}} , \]

\[ \nu_{i*} = 4.90 \cdot 10^{-18} \frac{q R n_i Z^4 \ln \Lambda_{ii}}{T_i^{2/3} \epsilon^{3/2}} , \]
The symmetry yields extremely good confinement of collisionless trajectories.

All configurations scaled to ARIES-CS minor radius (1.7 m) and $|B|$ (5.7 T).

5000 alpha particles initialized isotropically at $s=0.3$.

*SIMPLE code: Albert et al, JCP (2020).*
Previous quasisymmetric configurations \((s=0.5)\)
Previous quasisymmetric configurations \((s=1)\)
$|B|$ along a field line for new QA

- $s=0.01$
- $s=0.02$
- $s=0.04$
- $s=0.25$
- $s=0.50$
- $s=1.00$
$|B|$ along a field line for new QH

For different values of $s$: 0.01, 0.02, 0.04, 0.25, 0.50, 1.00.
Along a field line for new QA with magnetic well.