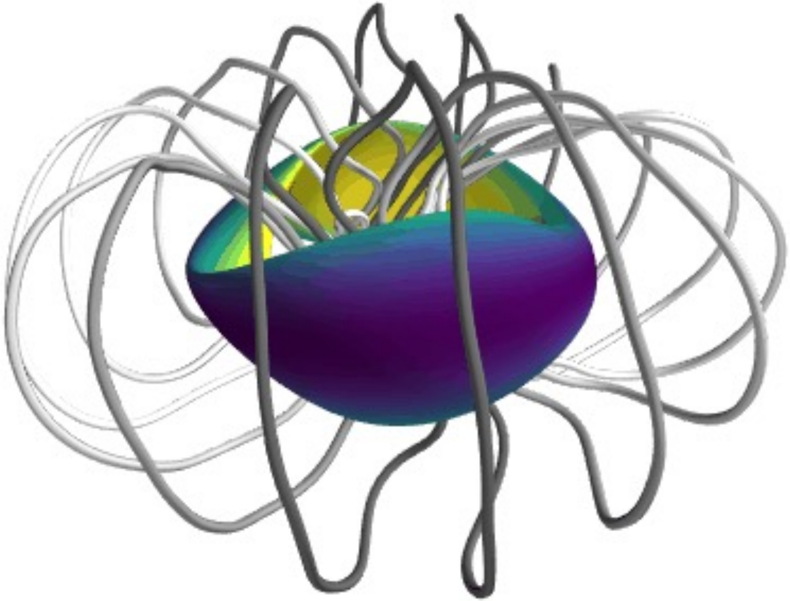
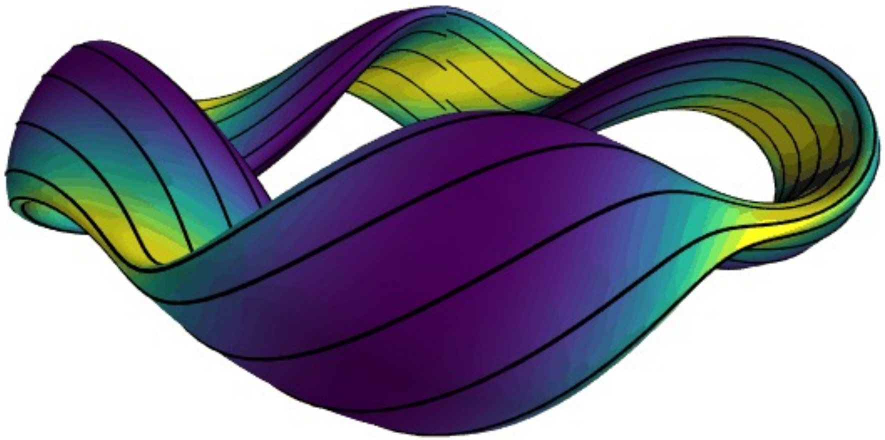
New stellarator configurations with precise quasisymmetry and energetic particle confinement



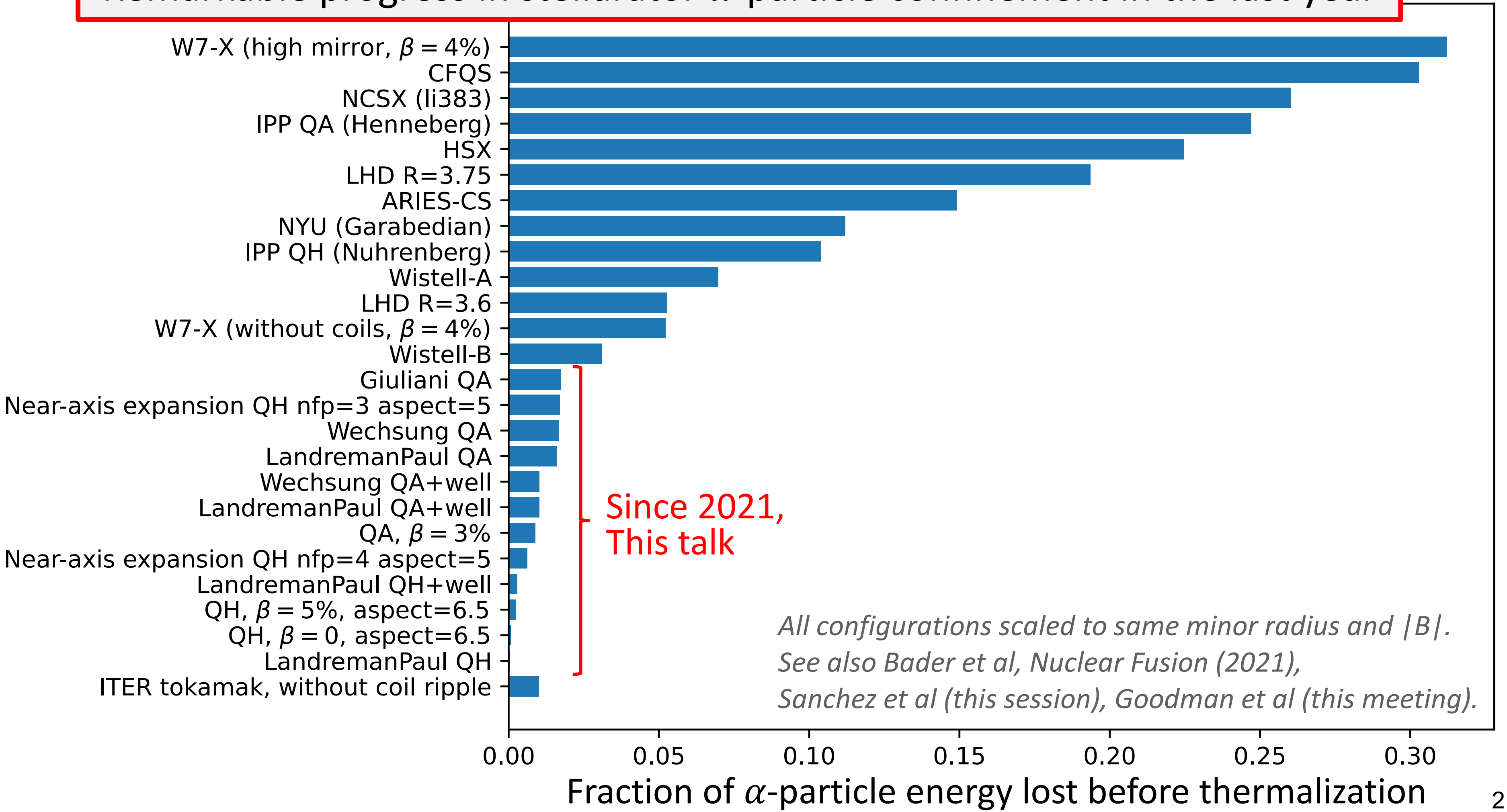


M Landreman[a], S Buller[a], A Cerfon[b], M Drevlak[c], A Giuliani[b], B Medasani[d],
M Padidar[e], E J Paul[d], G Stadler[b], F Wechsung[b], C Zhu[e]

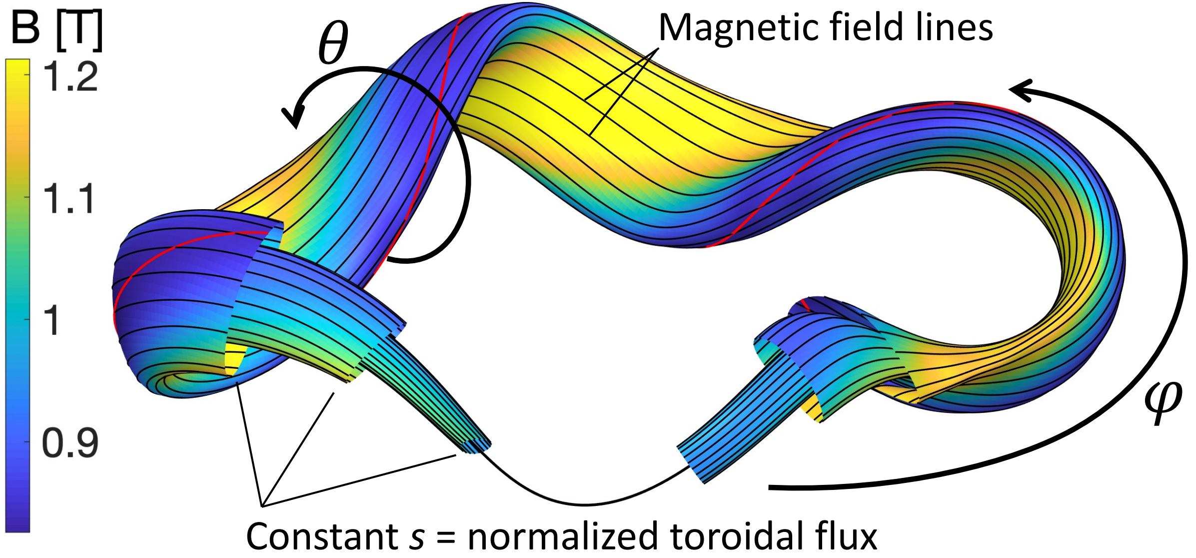
[a] U of Maryland, [b] New York U, [c] Max Planck Institute, [d] PPPL, [e] U of Science & Technology of China, [e] Cornell

ML & Paul, PRL (2022), Wechsung et al, PNAS (2022), Giuliani et al arXiv (2022), ML, Buller, & Drevlak arXiv (2022)

Remarkable progress in stellarator α-particle confinement in the last year



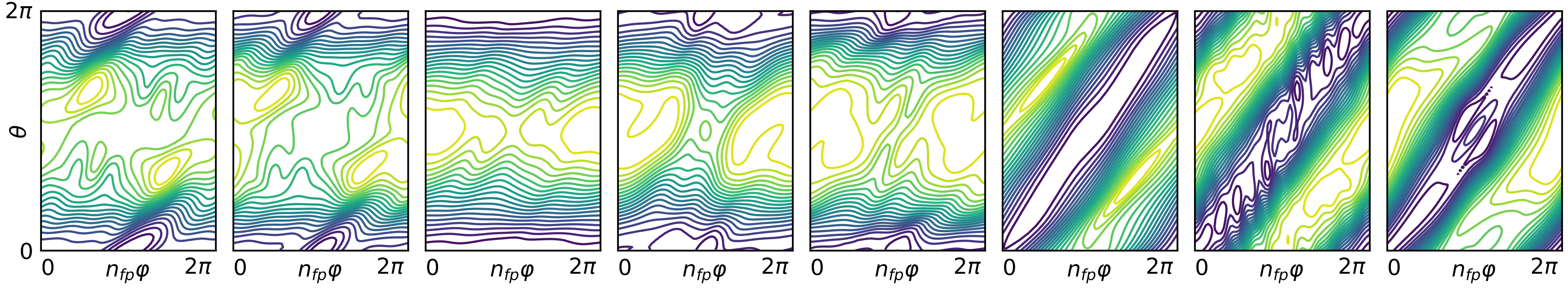
These new configurations with good alpha confinement use the principle of *quasisymmetry.*



$$B = B(s, \theta - N\,\varphi)$$

Boozer angles

$$\Rightarrow \quad \oint \left(\mathbf{v}_d \cdot \nabla s \right) dt = 0$$



Goal: $B = B(s, \theta - N\,\varphi)$

B_{min} — B_{max}

Since 2021

ML & Paul, Phys Rev Lett (2022)

Wechsung et al, PNAS (2022)

Giuliani et al, 1-stage, arXiv (2022)

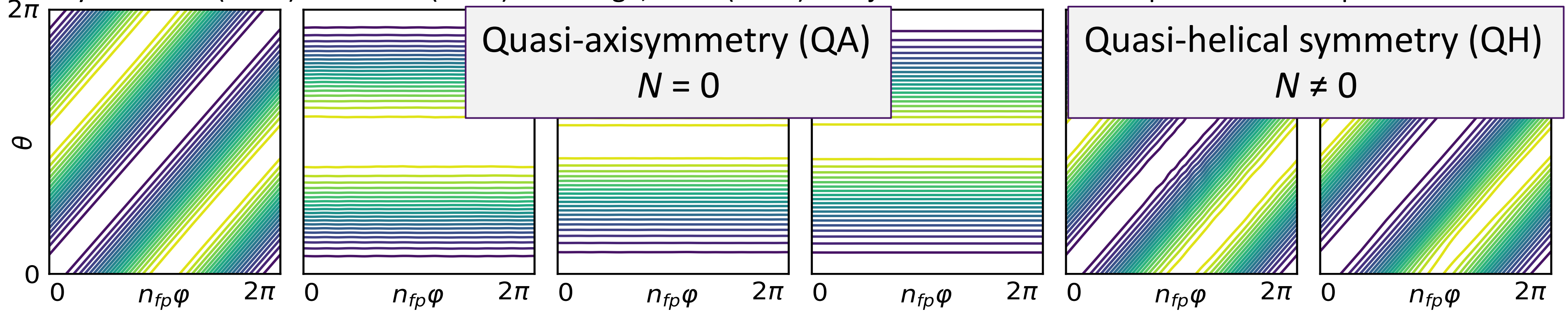
Nies & Paul Adjoint method

Near-axis expansion

5% β, Self-consistent plasma current

Quasi-axisymmetry (QA)
$N = 0$

Quasi-helical symmetry (QH)
$N \neq 0$



Also Dudt et al (this afternoon), Zarnstorff et al (yesterday)

- Minimal optimization recipe (low β)
- Self-consistent bootstrap current at high β
- Future directions

- Minimal optimization recipe (low β)
- Self-consistent bootstrap current at high β
- Future directions

Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[(N - \iota) \mathbf{B} \times \nabla B \cdot \nabla \psi - (G + NI) \mathbf{B} \cdot \nabla B \right] \right)^2$$

$$f_{QH} = (A - A_*)^2 + f_{QS}$$

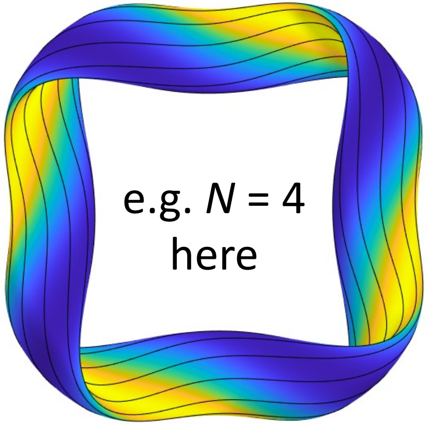
Boundary aspect ratio

$$f_{QA} = (A - A_*)^2 + \left(\iota_* - \int_0^1 \iota\, ds \right)^2 + f_{QS}$$

Goal: $B = B(s, \theta - N\varphi)$.

For quasi-axisymmetry, $N = 0$.

For quasi-helical symmetry, N is the number of field periods,



Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

$$f_{QH} = \left(A - A_*\right)^2 + f_{QS} \qquad f_{QA} = \left(A - A_*\right)^2 + \left(\iota_* - \int_0^1 \iota \, ds\right)^2 + f_{QS}$$

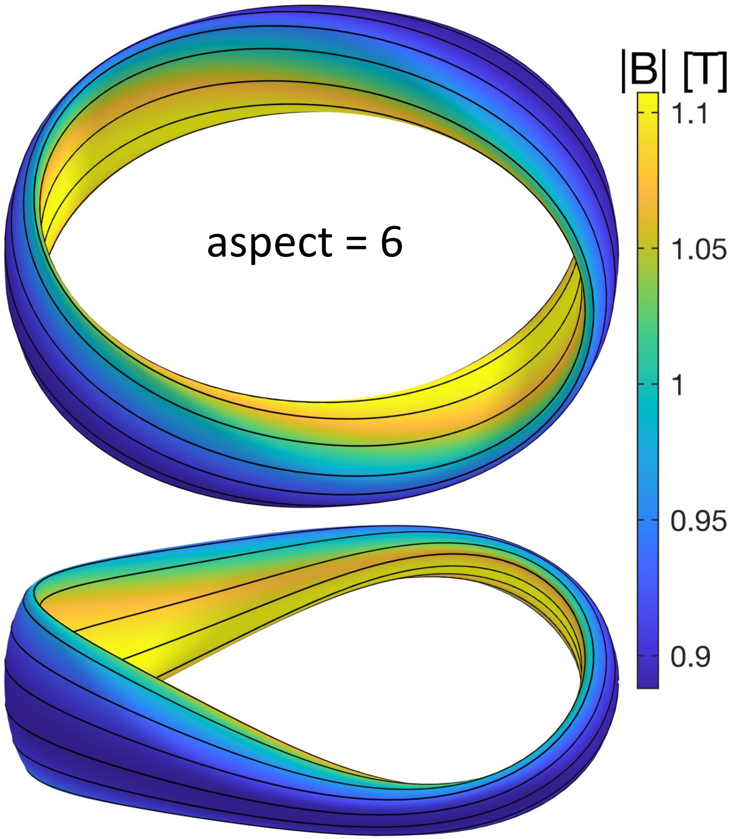
Boundary aspect ratio

- Parameter space: $R_{m,n}$ & $Z_{m,n}$ defining a toroidal boundary

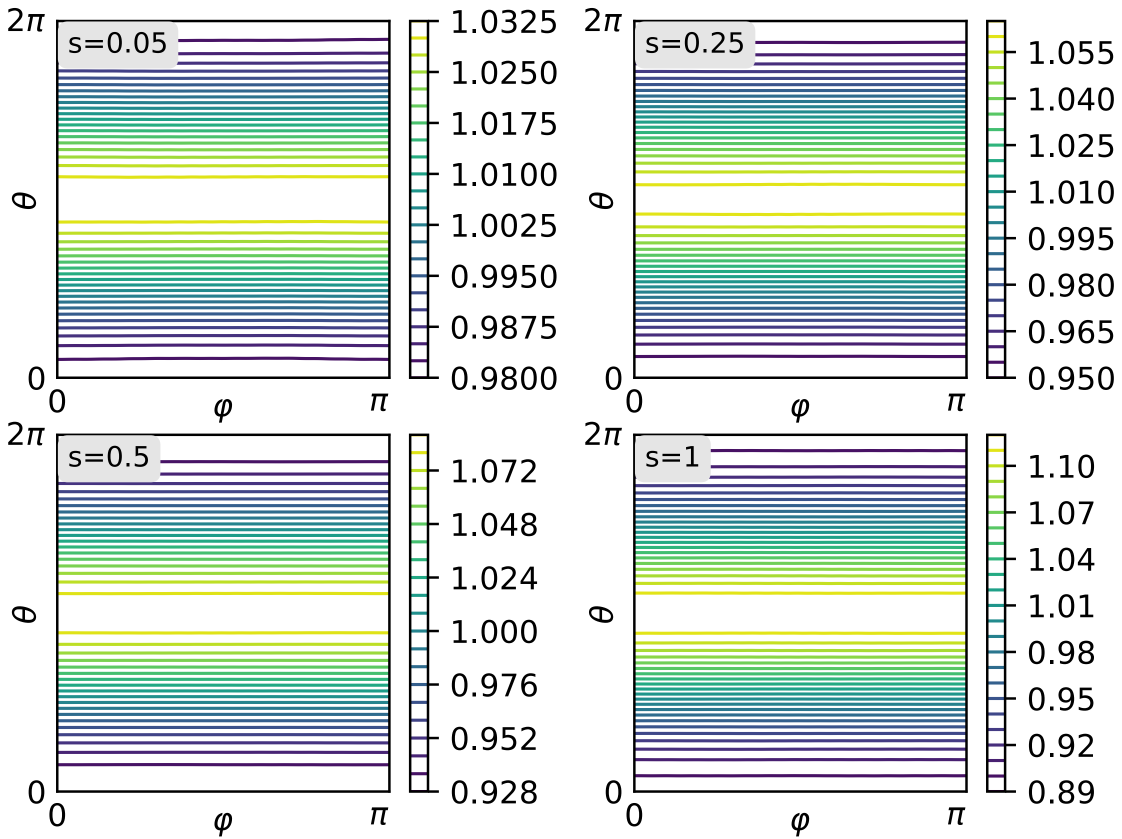
$$R(\theta,\phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \qquad Z(\theta,\phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)$$

- Codes used: SIMSOPT with VMEC
- Cold start: circular cross-section torus
- Vacuum fields ($\nabla \times \mathbf{B} = 0$) at first, allowing precise checks
- Algorithm: default for least-squares in scipy (trust region reflective)
- 6 steps: increasing # of modes varied & equilibrium resolution
- Run many optimizations, pick the best

Straight |B| contours are possible for quasi-axisymmetry

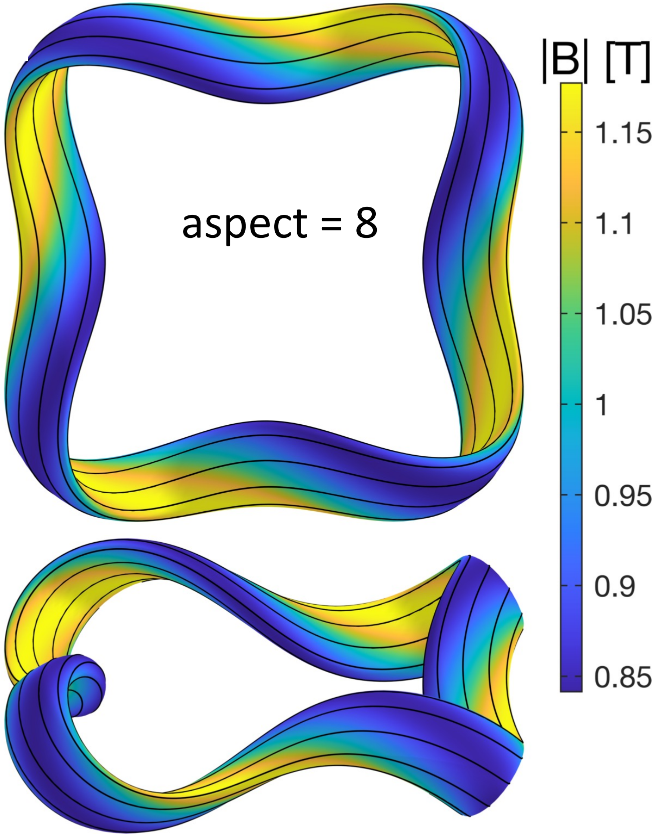


|B| on flux surfaces of the quasi-axisymmetric field

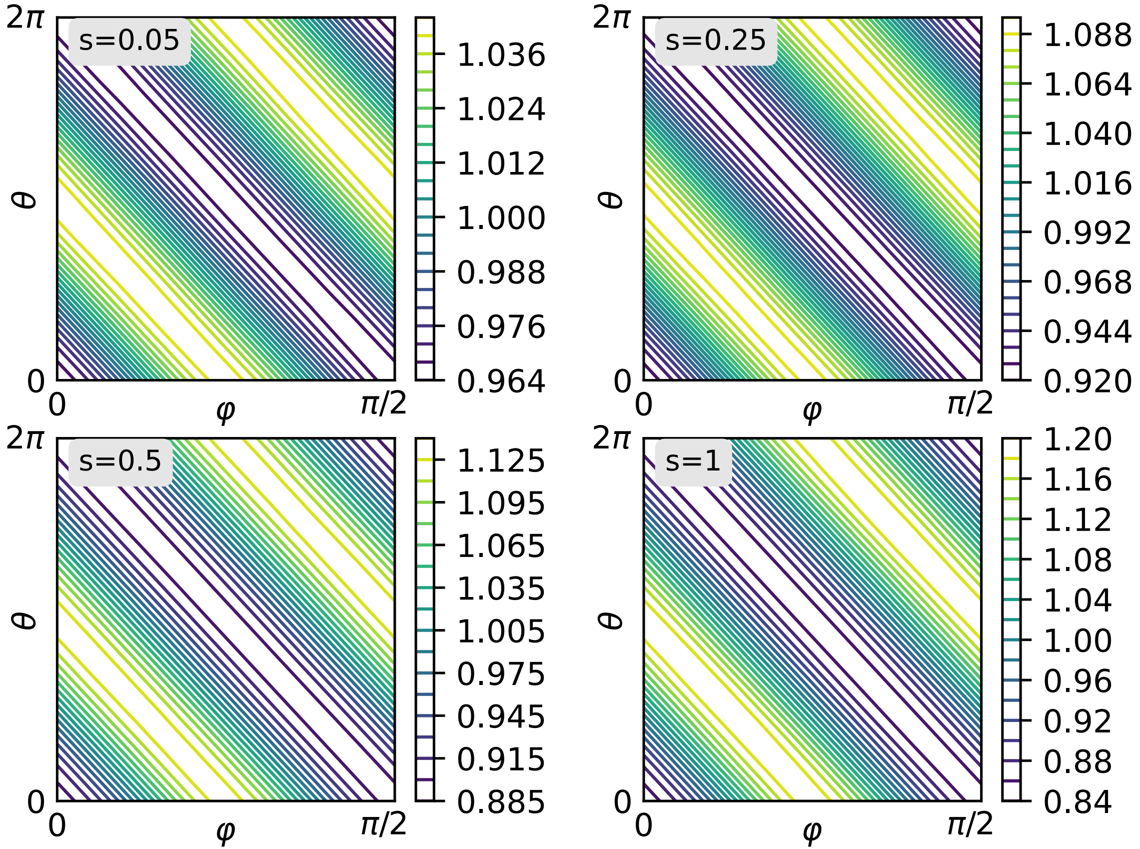


ML & Paul, PRL (2022). All input/output files and optimization scripts online at doi.org/10.5281/zenodo.5645412

Straight |B| contours are possible for quasi-helical symmetry



|B| on flux surfaces of the quasi-helically symmetric field

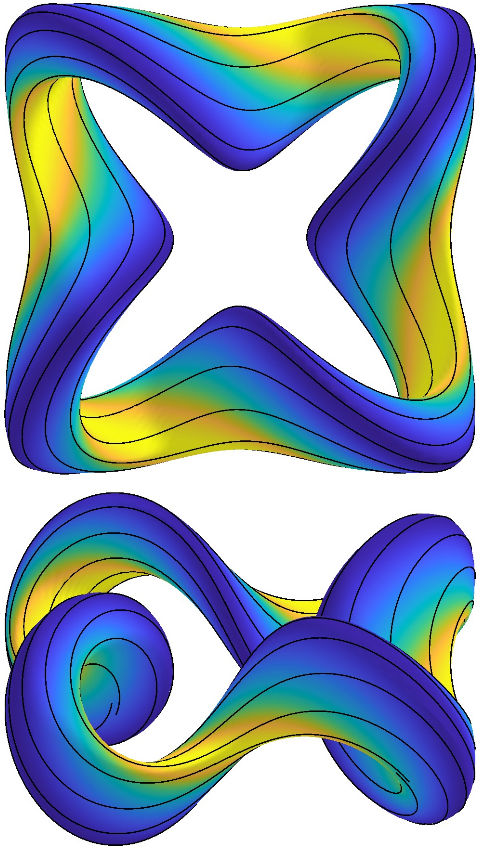


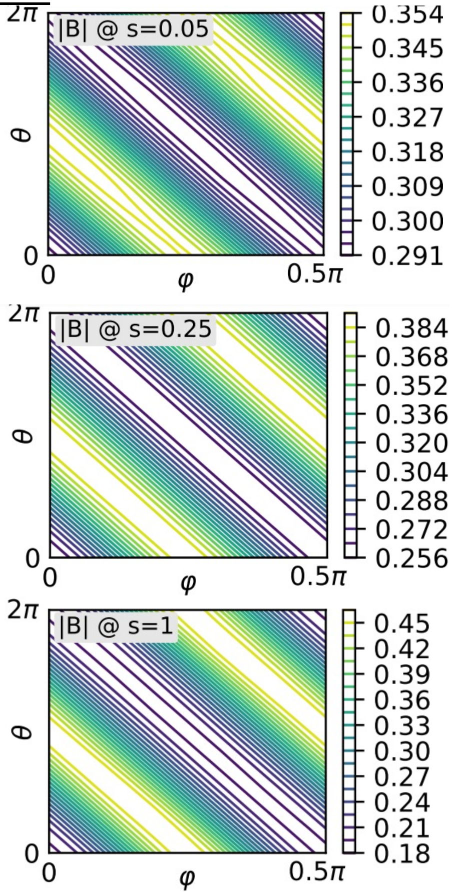
ML & Paul, PRL (2022).

All input/output files and optimization scripts online at doi.org/10.5281/zenodo.5645412

Nearly as good quasisymmetry exists also at lower aspect ratio

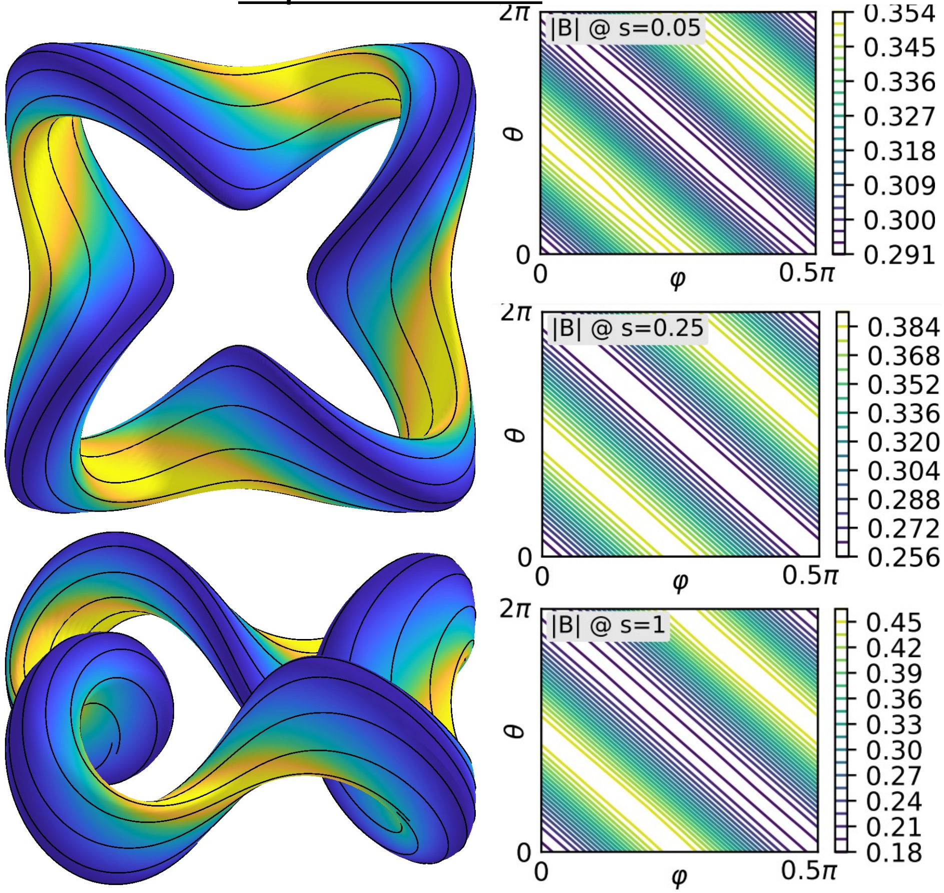
Aspect ratio 3.3



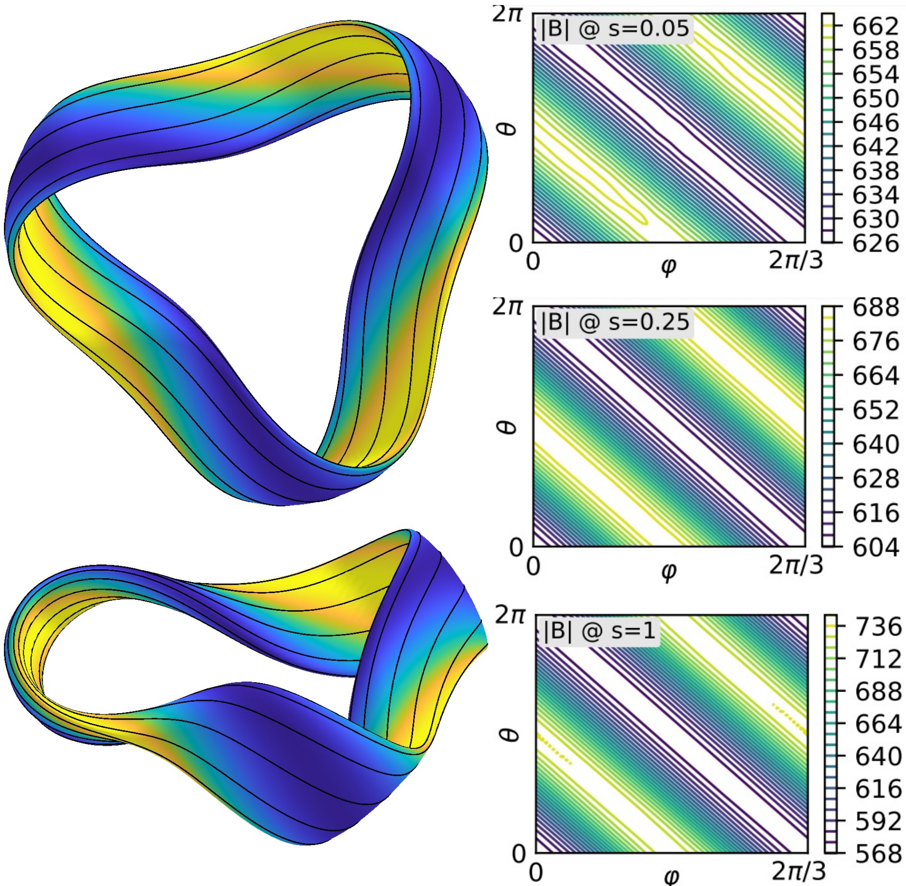


Nearly as good quasisymmetry exists also at lower aspect ratio or different # of field periods

Aspect ratio 3.3

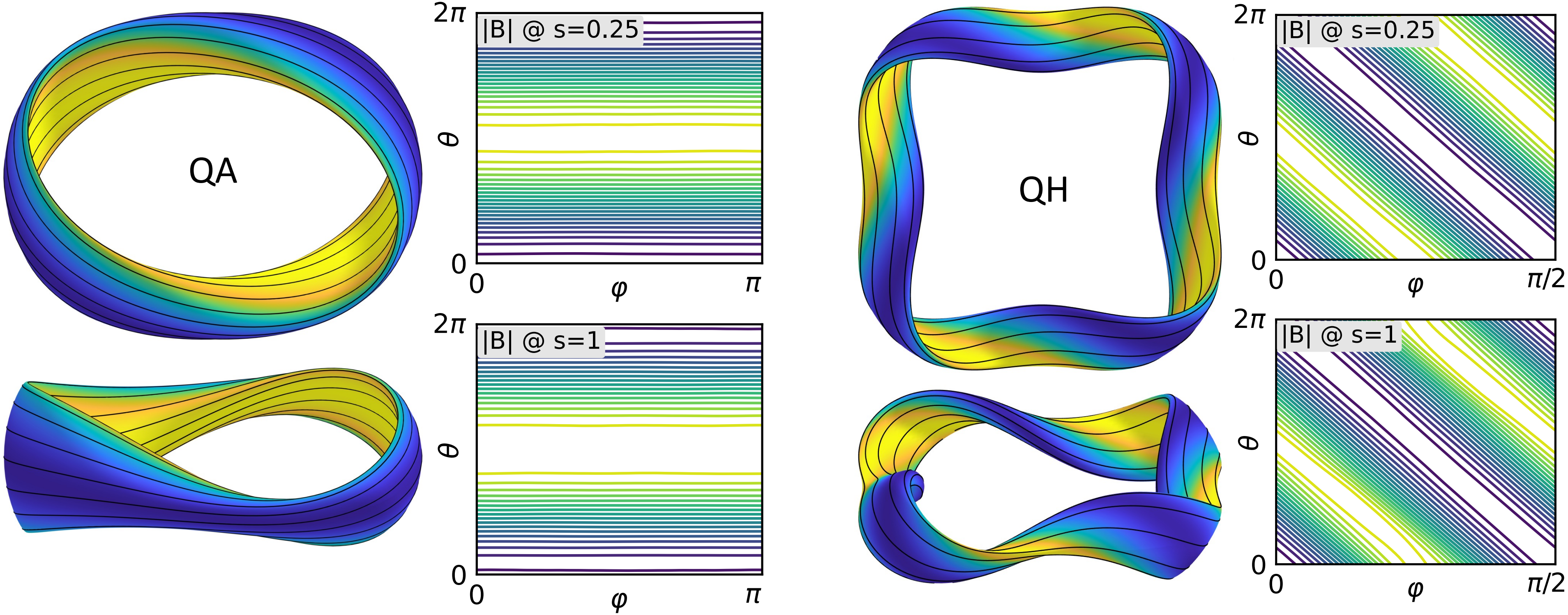


3 field periods, aspect ratio 6



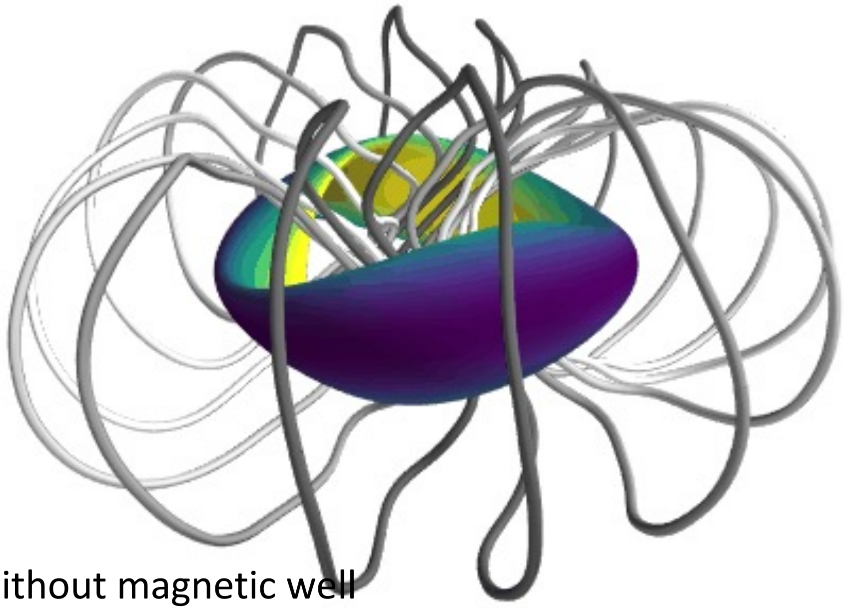
Good symmetry also exists with magnetic well

$$\frac{d^2\left(\text{flux surface volume}\right)}{d\left(\text{toroidal flux}\right)^2} < 0 \quad \text{everywhere}$$

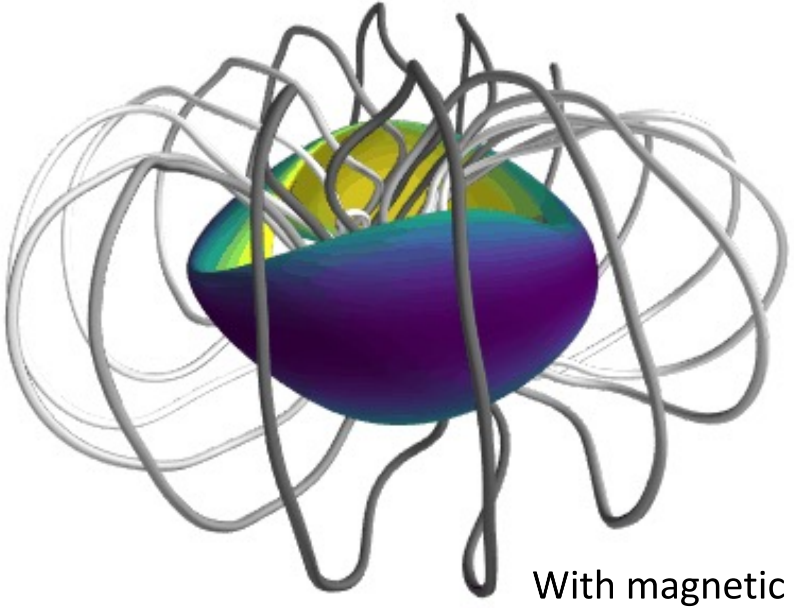


ML & Paul, PRL (2022). All input/output files and optimization scripts online at doi.org/10.5281/zenodo.5645412

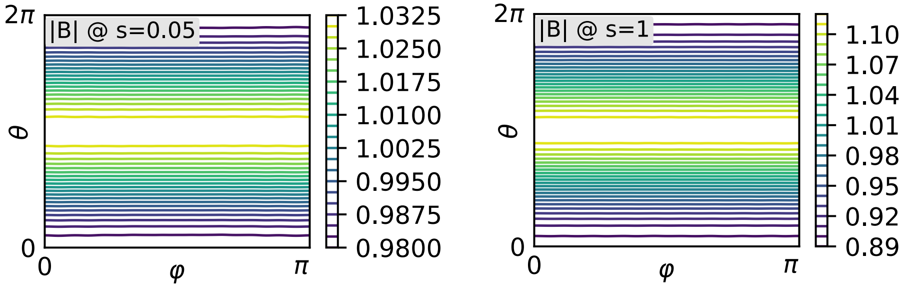
16-coil solutions have been found for the quasi-axisymmetric configurations

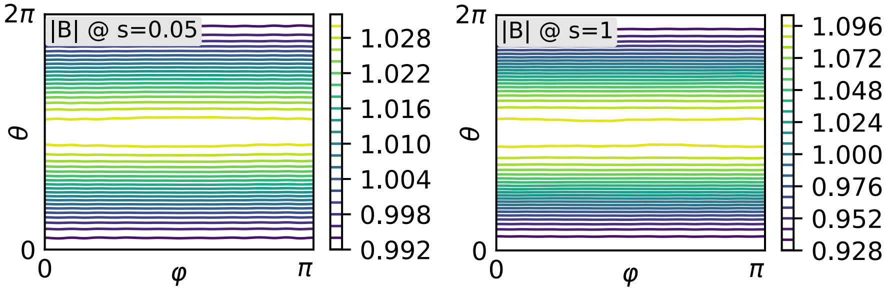


Without magnetic well



With magnetic well

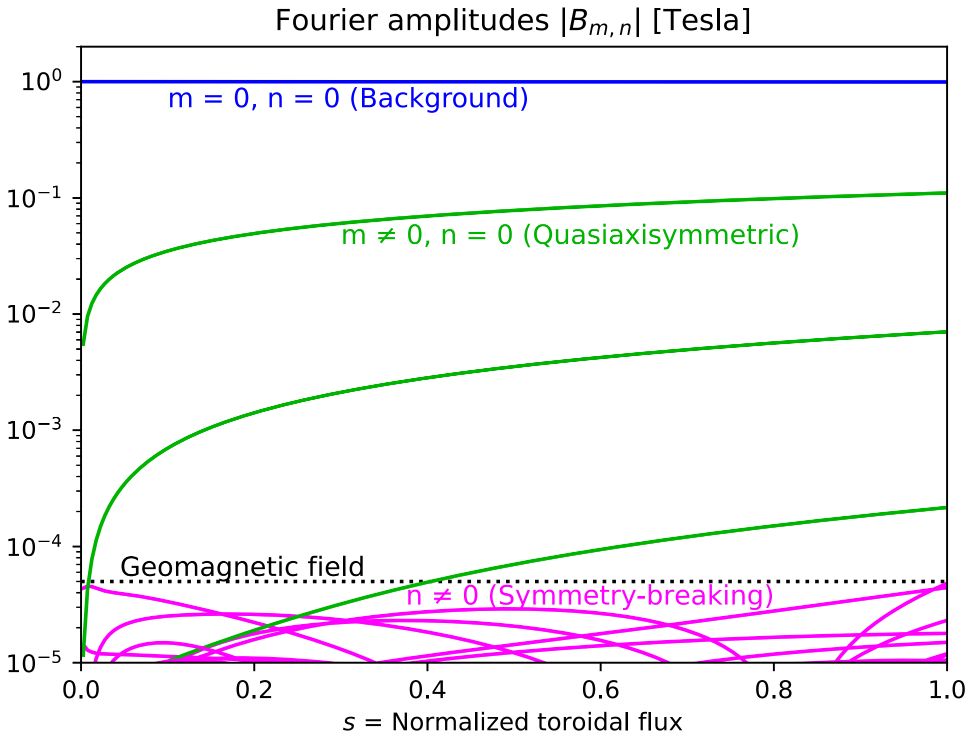


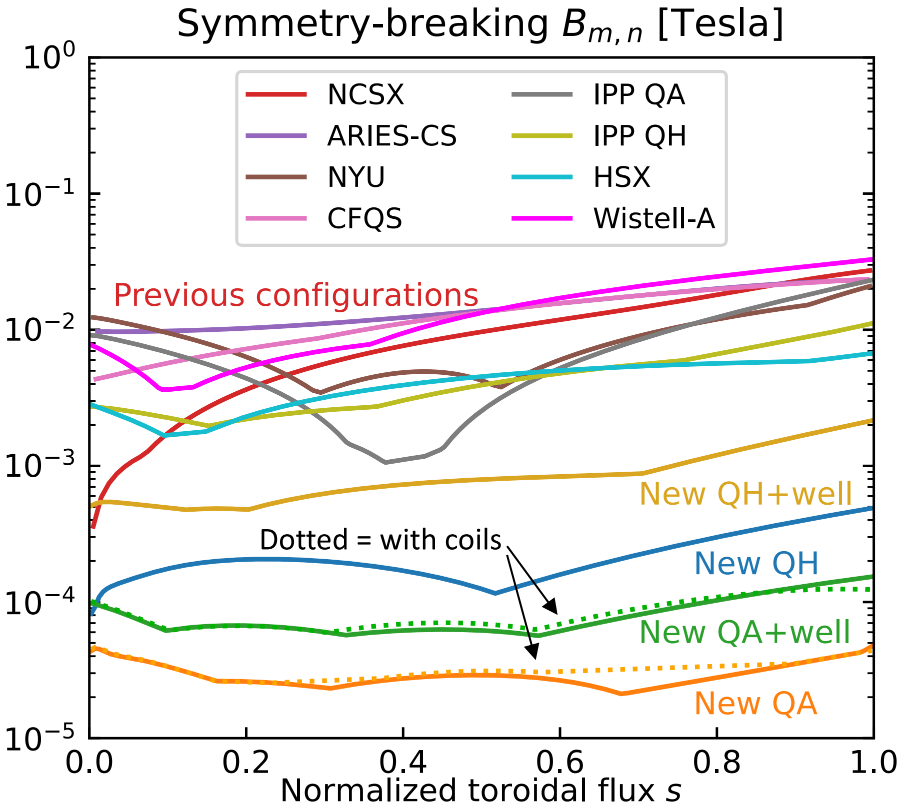


Wechsung et al, PNAS (2022). <R> / 10 between filament centers.

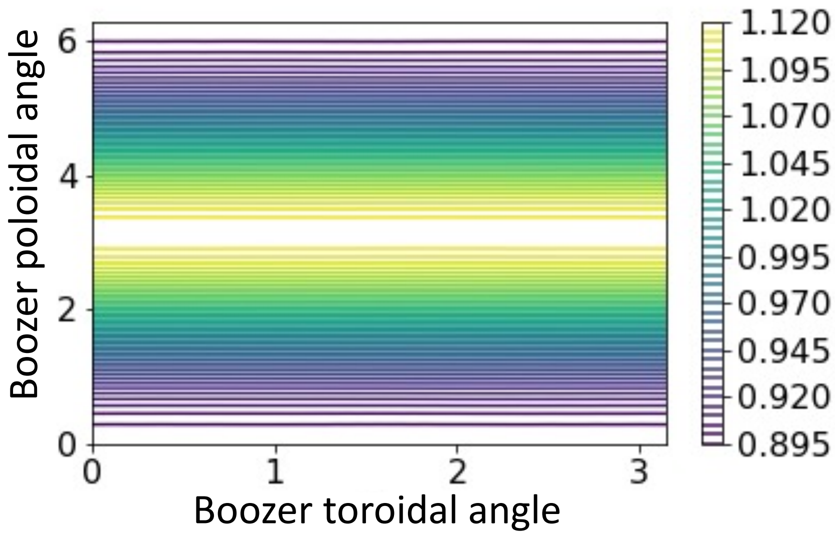
Symmetry-breaking modes can be made extremely small

New QA configuration

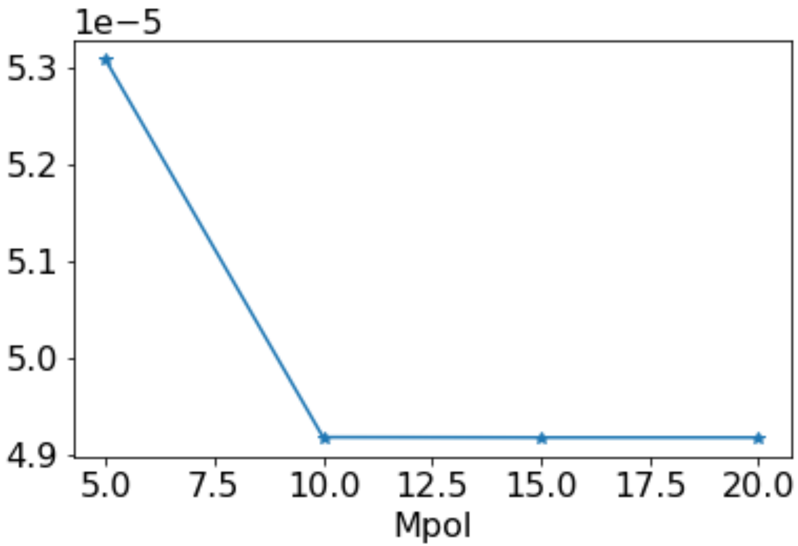




|B| in Boozer coordinates was verified by independent SPEC calculations



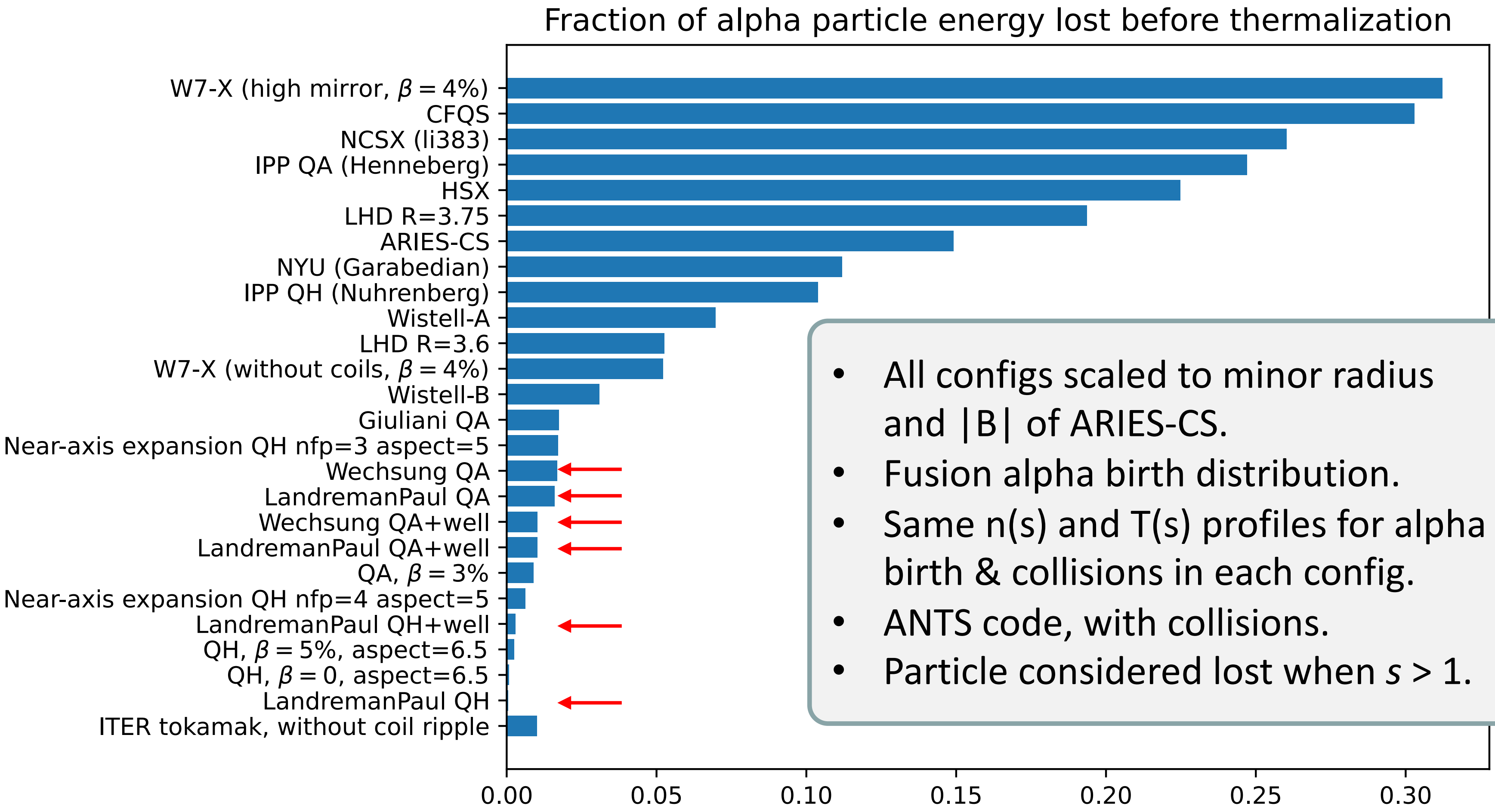
max $|B_{m,n}|$ [Tesla] with n ≠ 0



(Ntor = Mpol, Lrad = Mpol + 4)

By Elizabeth Paul

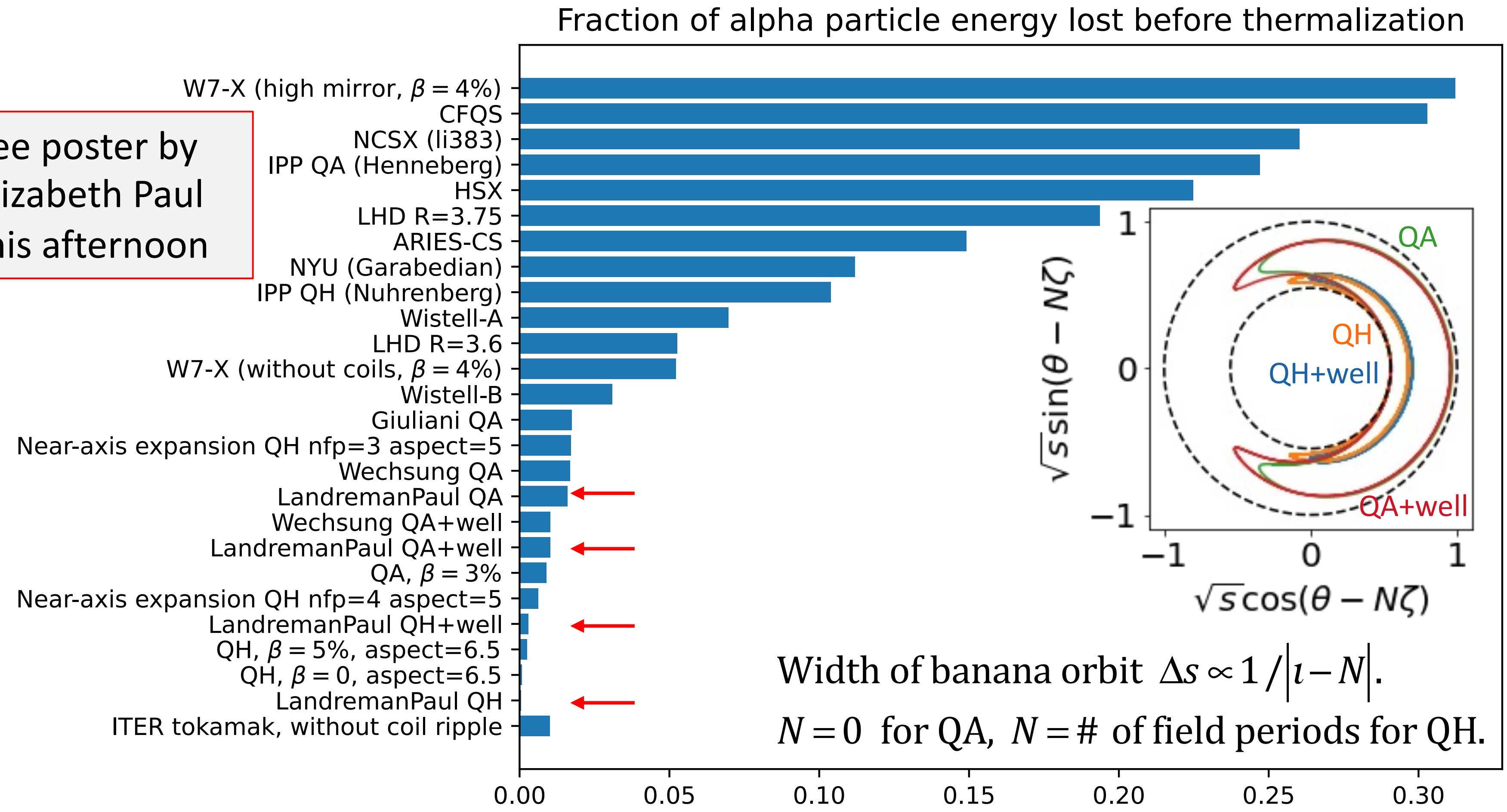
Quasisymmetry works: alpha particle confinement is significantly improved



- All configs scaled to minor radius and |B| of ARIES-CS.
- Fusion alpha birth distribution.
- Same n(s) and T(s) profiles for alpha birth & collisions in each config.
- ANTS code, with collisions.
- Particle considered lost when $s > 1$.

Alpha confinement in quasi-helical stellarators can be better than in a tokamak due to thinner bananas

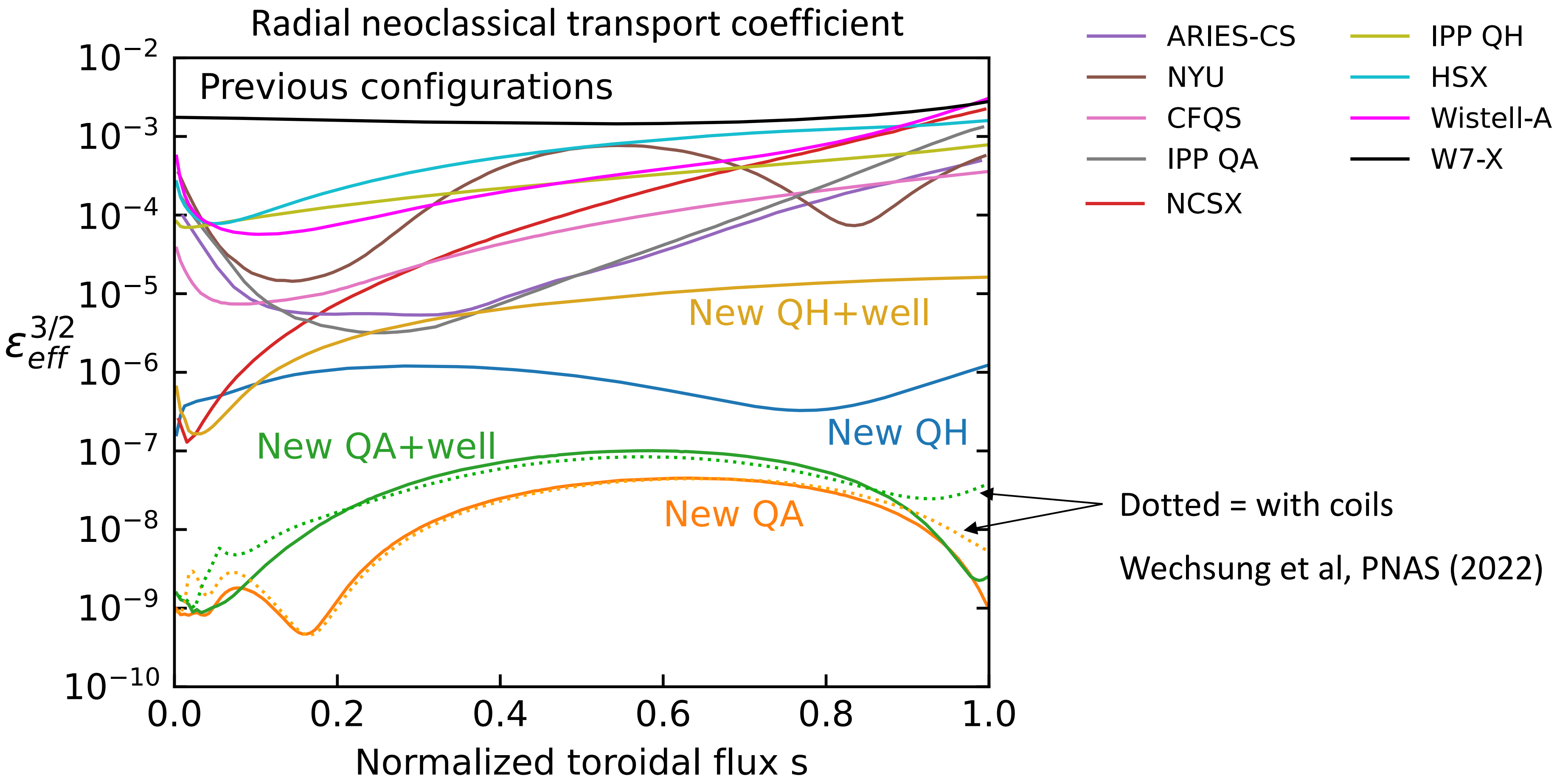
See poster by Elizabeth Paul this afternoon



Width of banana orbit $\Delta s \propto 1/|\iota - N|$.

$N = 0$ for QA, $N = \#$ of field periods for QH.

The symmetry also yields extremely low collisional transport for a thermal plasma



Wechsung et al, PNAS (2022)

Even better quasisymmetry and ε_{eff} is achieved by refinement with combined plasma-and-coil optimization

Giuliani et al, arXiv:2203.03753 (2022)

Objective:

$$f = \sum_{\text{surfaces}} (\text{QS error})^2 + (\bar{\iota} - \bar{\iota}_*)^2 + \left(\begin{array}{c} \text{coil length, mean curvature,} \\ \text{max curvature, coil-coil distance terms} \end{array} \right)$$

Parameter space: coil shapes

Derivatives: analytic + adjoint method

QA

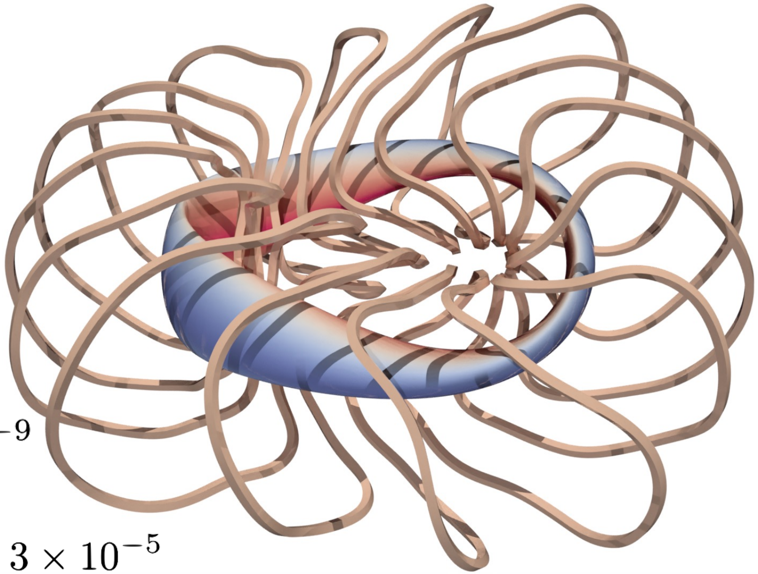
16 coils

$\iota > 0.4$

α losses < 2%

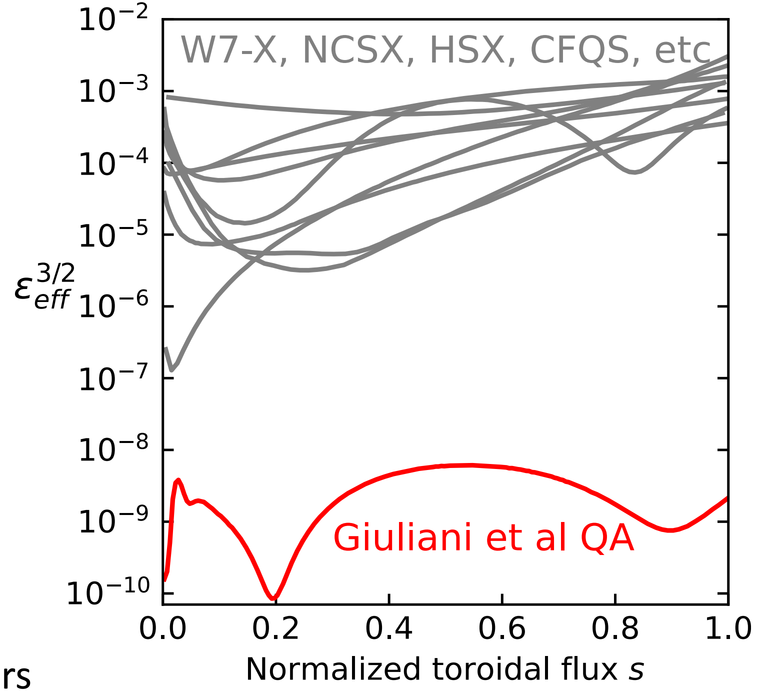
$\epsilon_{eff}^{3/2} < 5 \times 10^{-9}$

$$\left| \frac{B_{m,n}^{\text{non-QS}}}{B_0} \right| < 3 \times 10^{-5}$$



<R> / 10 between filament centers

Collisional heat flux



- Minimal optimization recipe (low β)
- Self-consistent bootstrap current at high β
- Future directions

How can bootstrap current be included self-consistently in stellarator optimization?

- Need *self-consistency* between MHD equilibrium and drift-kinetic equation.
- Previous method: fixed-point iteration, only after an optimization.

MHD equilibrium code → VMEC: given $I_0(s)$, determine $\mathbf{B}_0$.

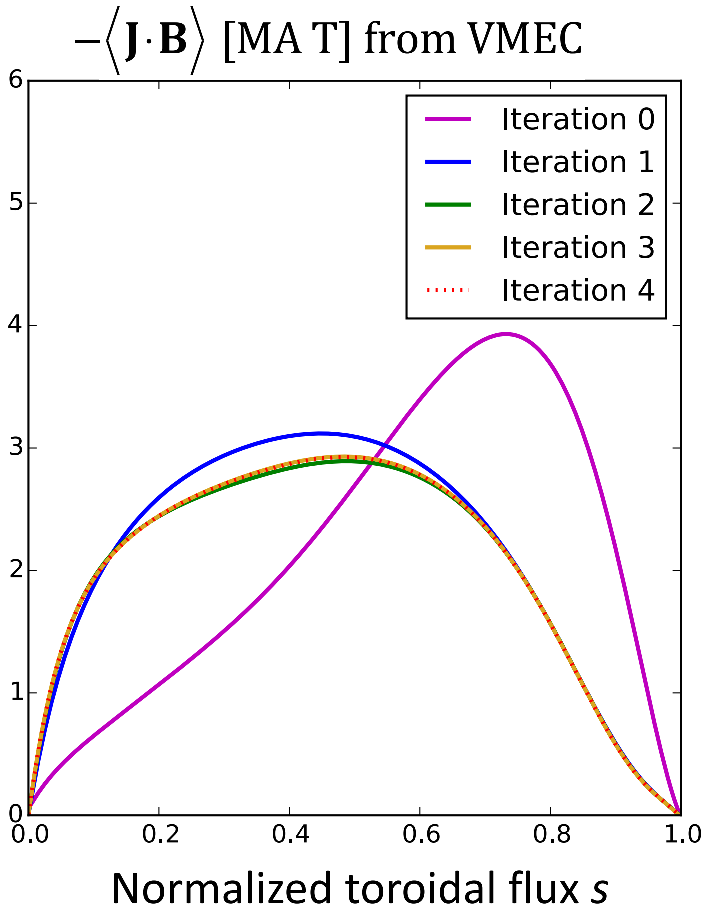
Drift-kinetic code → SFINCS: given $\mathbf{B}_0$, determine $I_1(s)$.

VMEC: given $I_1(s)$, determine $\mathbf{B}_1$.

SFINCS: given $\mathbf{B}_1$, determine $I_2(s)$.

…

- Accurate drift-kinetic bootstrap calculations in stellarators are computationally expensive. Preferably not in the optimization loop.



New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Pytte & Boozer (1981), Boozer (1983):

$$\iota \rightarrow \iota - N$$

Bootstrap current (& other quantities) in quasisymmetry are the same as in axisymmetry, up to some substitutions:

Should be accurate for the new precisely quasisymmetric configurations.

A new set of analytical formulae for the computation of the bootstrap current and the neoclassical conductivity in tokamaks

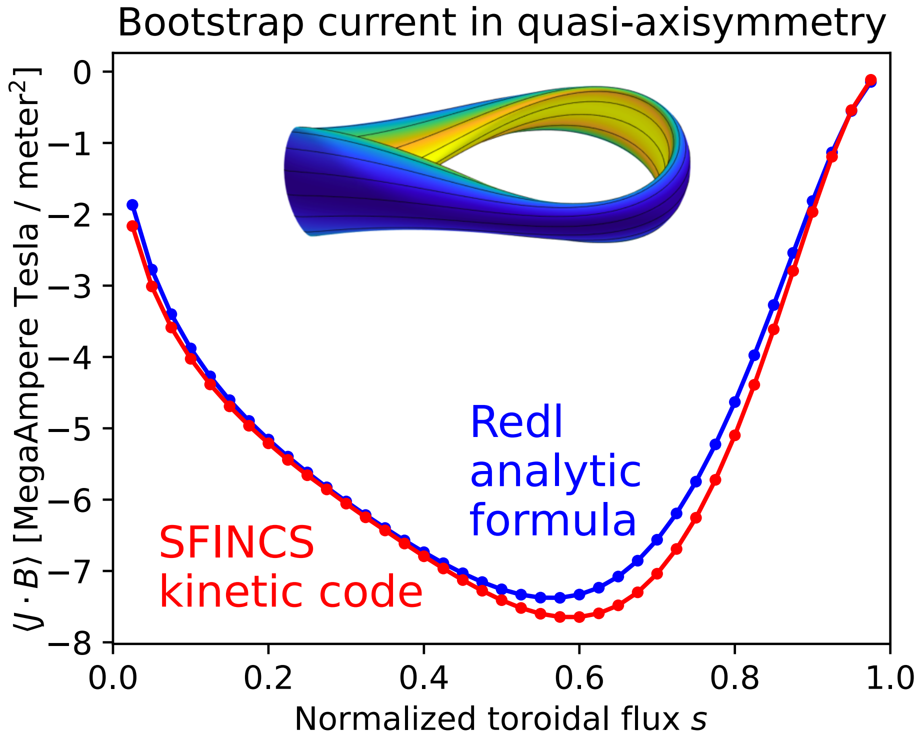
Cite as: Phys. Plasmas **28**, 022502 (2021); doi: 10.1063/5.0012664
Submitted: 6 May 2020 · Accepted: 11 December 2020 ·
Published Online: 2 February 2021

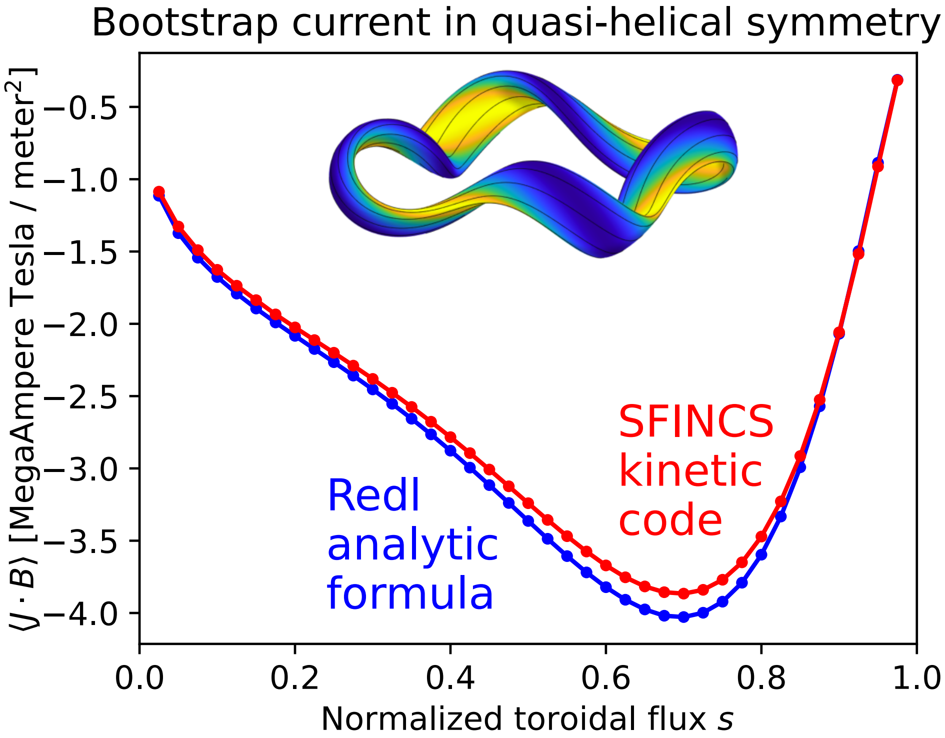
View Online　Export Citation

A. Redl,[1,2,a)] C. Angioni,[1] E. Belli,[3] O. Sauter,[4] ASDEX Upgrade Team[b)] and EUROfusion MST1 Team[c)]

Before doing new optimizations: Redl formula is accurate in previous QA & QH stellarators

$n_e = (1 - s^5)\ 4\text{x}10^{20}\ \text{m}^{-3}$, $T_e = T_i = (1 - s)\ 12\ \text{keV}$





(Not self-consistent yet)

Optimization recipe

- Objective function: $$f = f_{QS} + f_{bootstrap} + \left(A - 6.5\right)^2 + \left(a - a_{ARIES-CS}\right)^2 + \left(\langle B \rangle - \langle B \rangle_{ARIES-CS}\right)^2$$

  Boundary aspect ratio (A) — Minor radius (a)

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

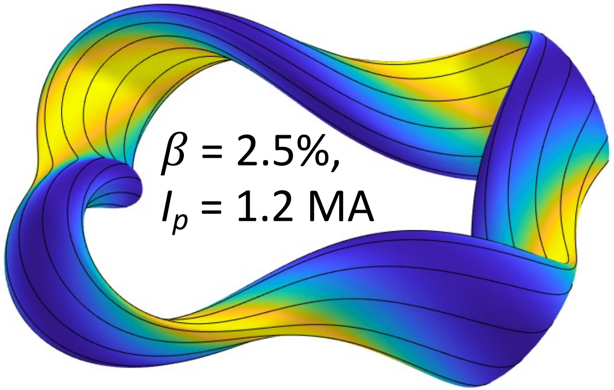
$$f_{bootstrap} = \frac{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{vmec}} - \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Redl}} \right]^2}{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{vmec}} + \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Redl}} \right]^2}$$

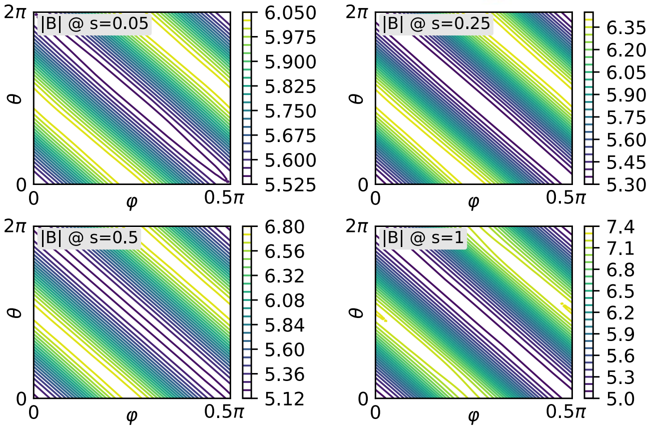
- Parameter space: {$R_{m,n}$, $Z_{m,n}$, toroidal flux, current spline values}
  or {$R_{m,n}$, $Z_{m,n}$, toroidal flux, iota spline values}
- Cold start
- Algorithm: default for least-squares in scipy (trust region reflective)
- Steps: increasing # of modes varied: m and |n/nfp| up to j in step j

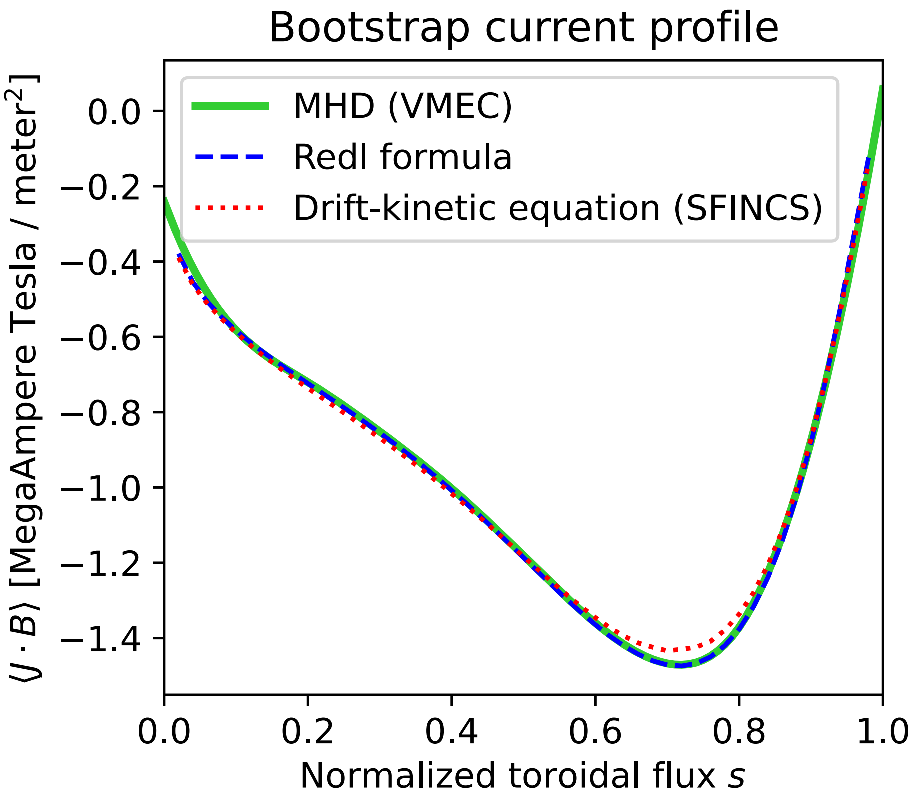
Example of optimization with self-consistent bootstrap current

n_{e0} = 2.2e20/meters^3

$T_{e0} = T_{i0}$ = 10 keV



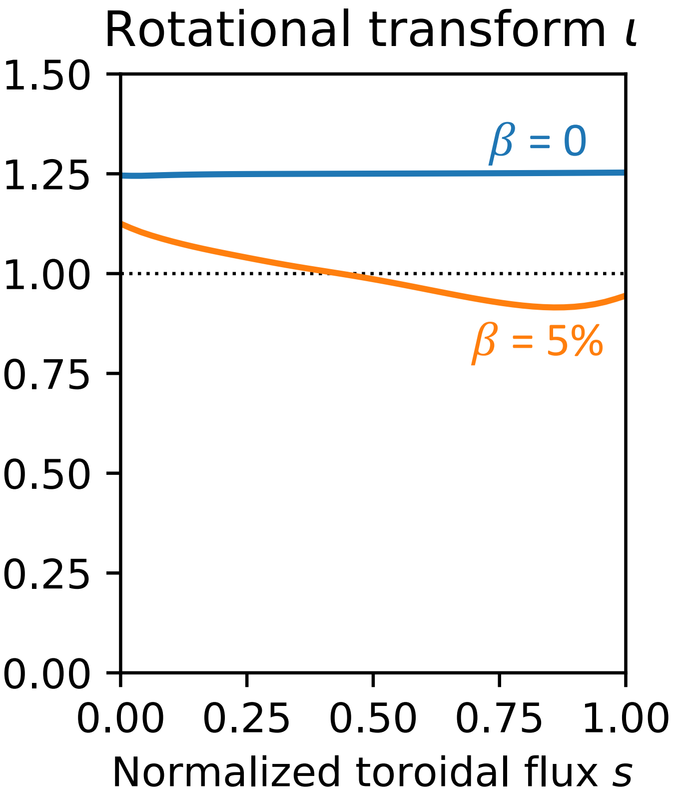




All input/output files and optimization scripts online at doi.org/10.5281/zenodo.6520103

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

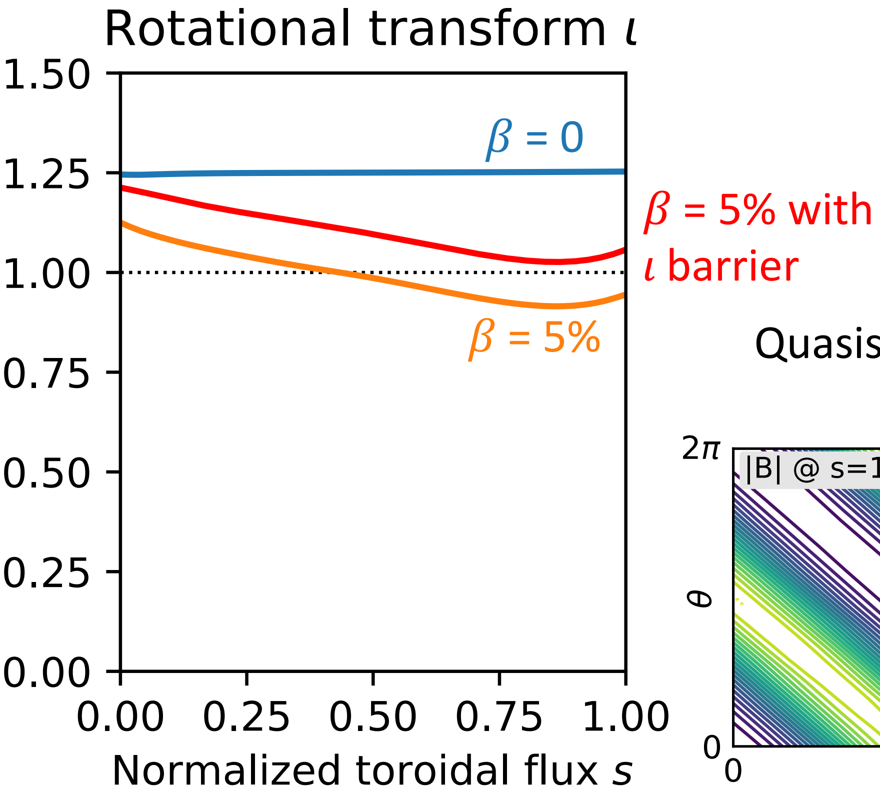
Crossing iota=1, the worst resonance, is probably unacceptable.



n_{e0} = 3e20/meters^3, $T_{e0} = T_{i0}$ = 15 keV

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

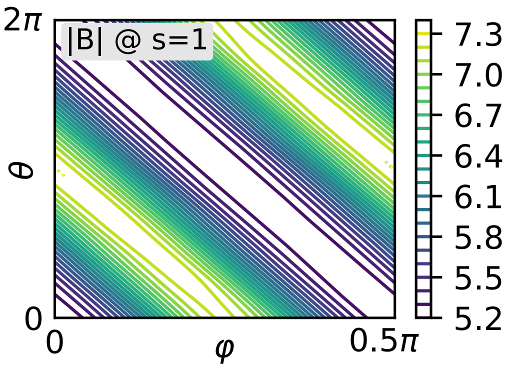
Crossing iota=1, the worst resonance, is probably unacceptable.

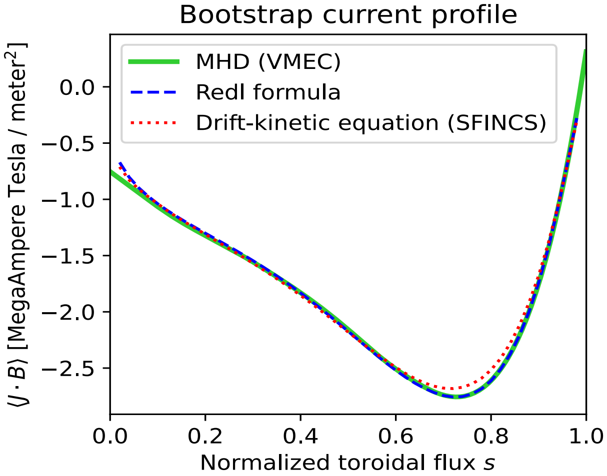


Solution: Add barrier term in objective

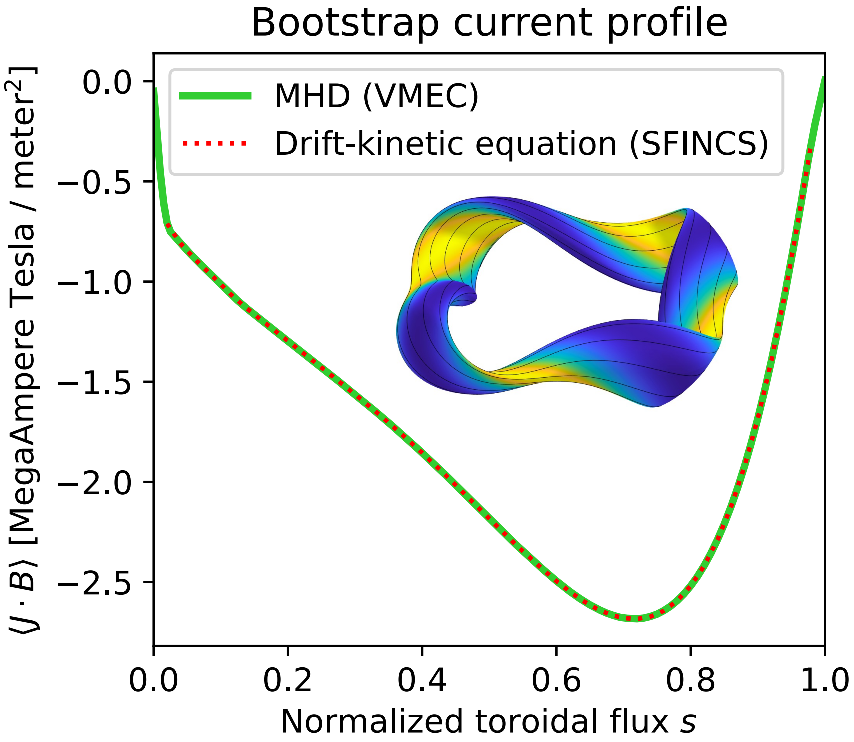
$$f \mathrel{+}= \int_0^1 ds \left[\min\left(\left|\iota(s)\right| - 1.03,\ 0\right)\right]^2$$

Quasisymmetry & bootstrap consistency remain good:





If you want *perfectly* self-consistent current, you can do a few fixed-point iterations at the end

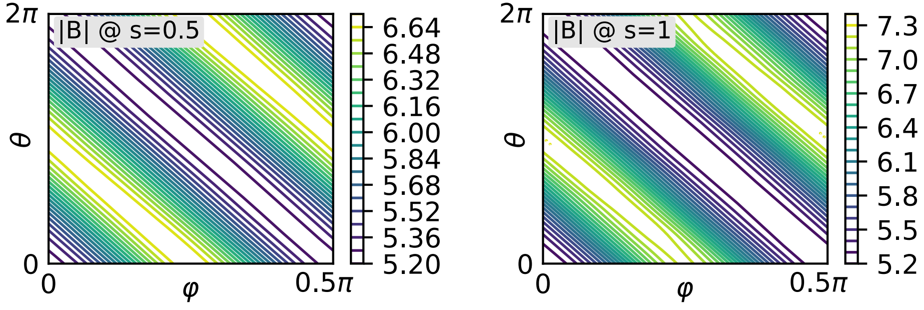


$$\langle \beta \rangle = 5\%, \qquad \varepsilon_{eff}^{3/2} < 6\times 10^{-5}$$

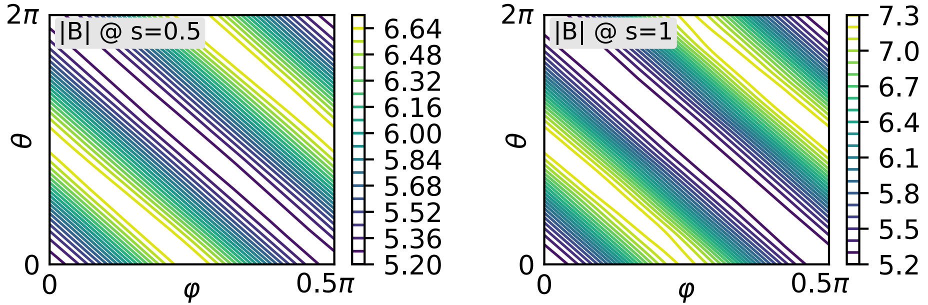
α-particle losses < 0.3%

No significant degradation in quasisymmetry:

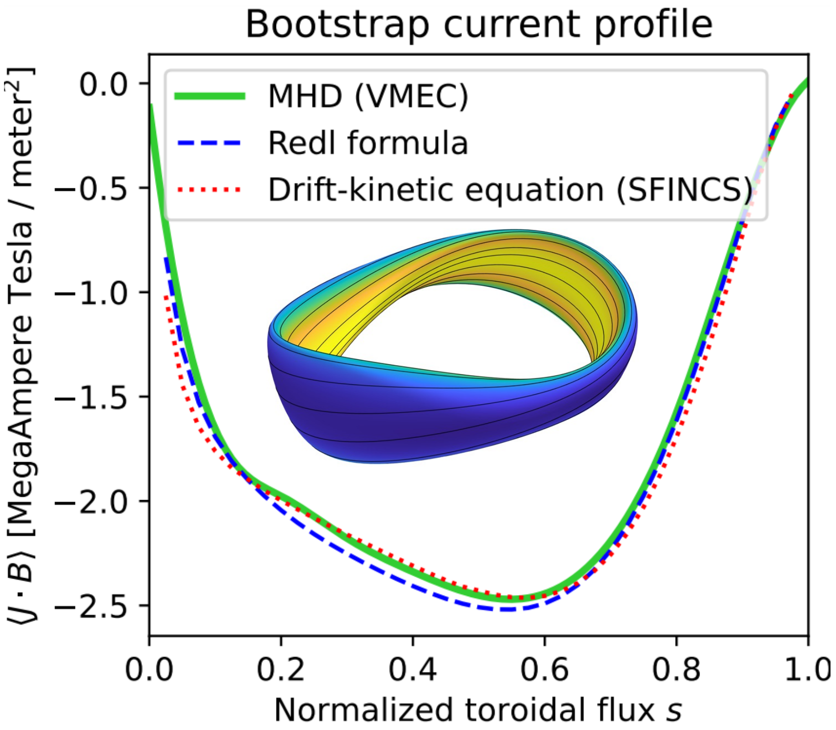
Optimization with Redl current



After SFINCS fixed-point iterations



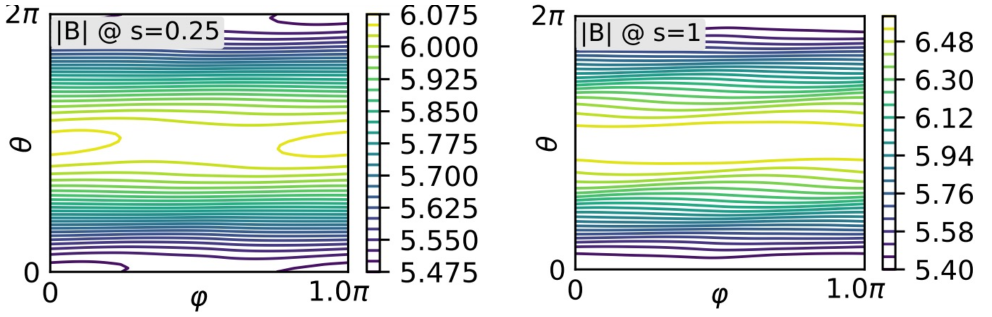
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry

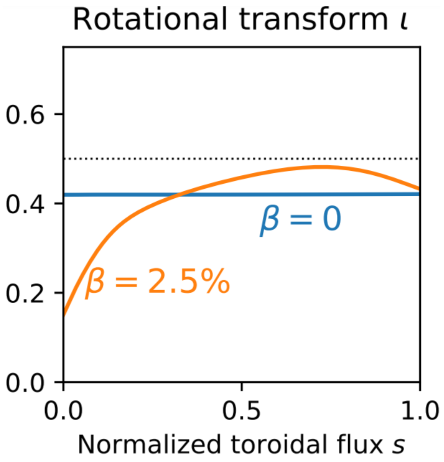


$$\langle\beta\rangle = 2.5\%, \qquad \epsilon_{eff}^{3/2} < 2\times10^{-5}$$

α-particle losses < 1.5%

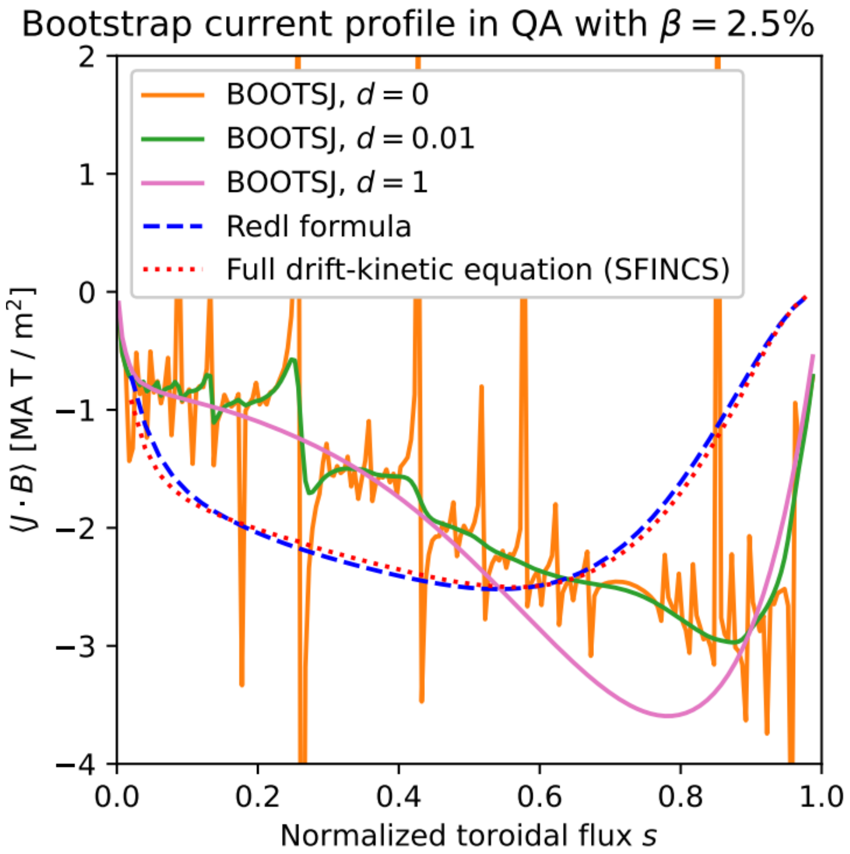
Symmetry is not as good as for vacuum, but sufficient for excellent confinement





Possible islands where ι = 1/4, 2/5, 1/3?

Redl formula is more accurate than long-mean-free-path stellarator bootstrap formula, & free of resonances



Stellarator bootstrap formulae for long-mean-free-path (low collisionality):

Shaing & Callen (1983),

Shaing et al (1989),

Helander, Parra & Newton (2017)

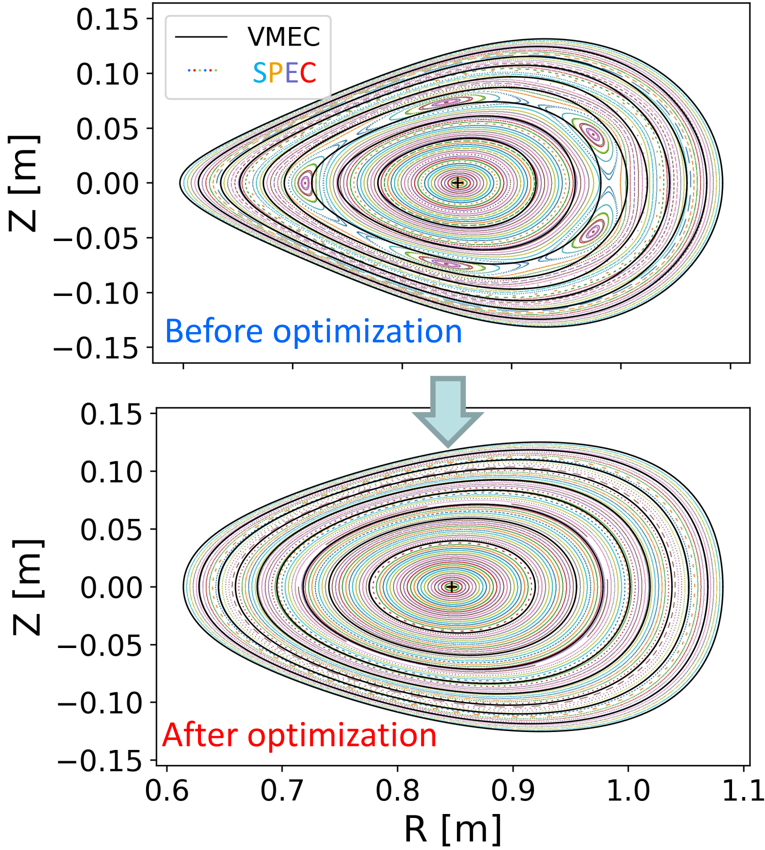
BOOTSJ ad-hoc smoothing:

$$\frac{1}{m - n/\iota} \to \frac{m - n/\iota}{(m - n/\iota)^2 + m^2 d^2}$$

- Minimal optimization recipe (low β)
- Self-consistent bootstrap current at high β
- Future directions

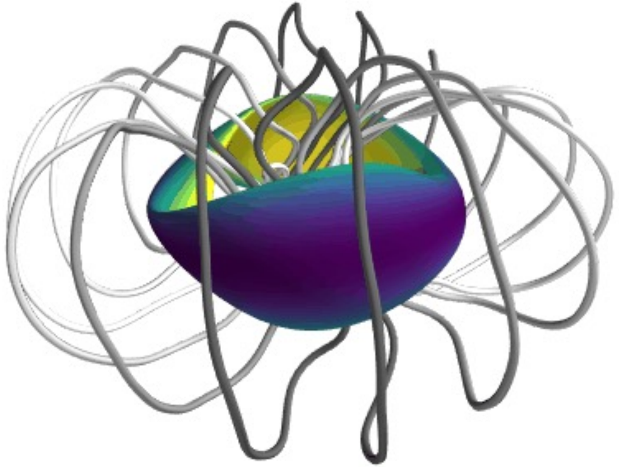
Future directions

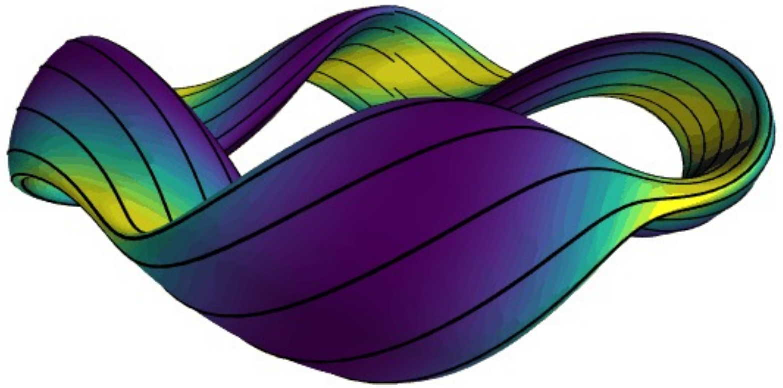
- For the high β configurations, check surface quality, & eliminate any islands.
- Coils & MHD stability for the high β configurations.
- Check robustness to uncertainty in the pressure profile.
- Similar recipes for quasi-poloidal symmetry or quasi-isodynamic?

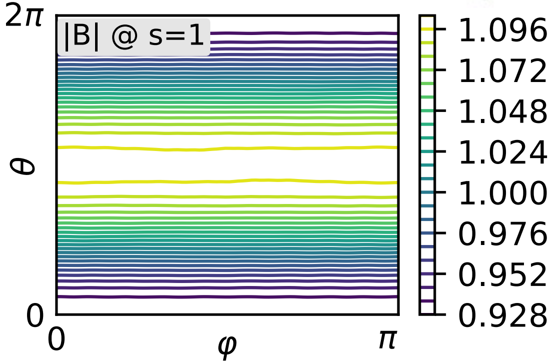


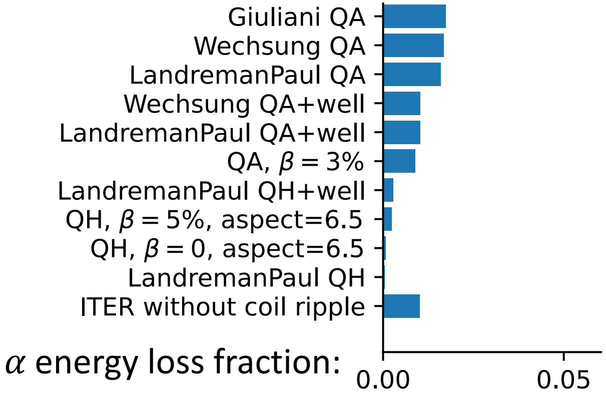
ML, Medasani & Zhu (2021), Baillod et al (2022)

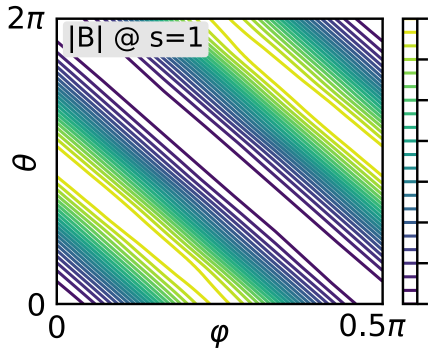
It is now possible to design stellarators with alpha confinement close to or better than a tokamak.





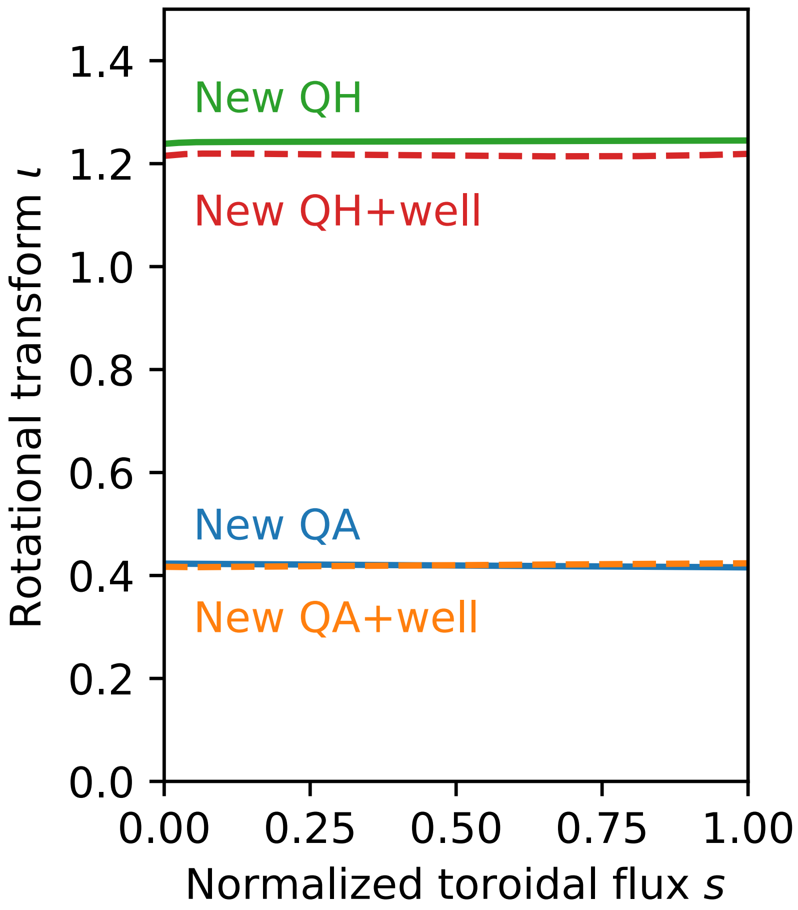


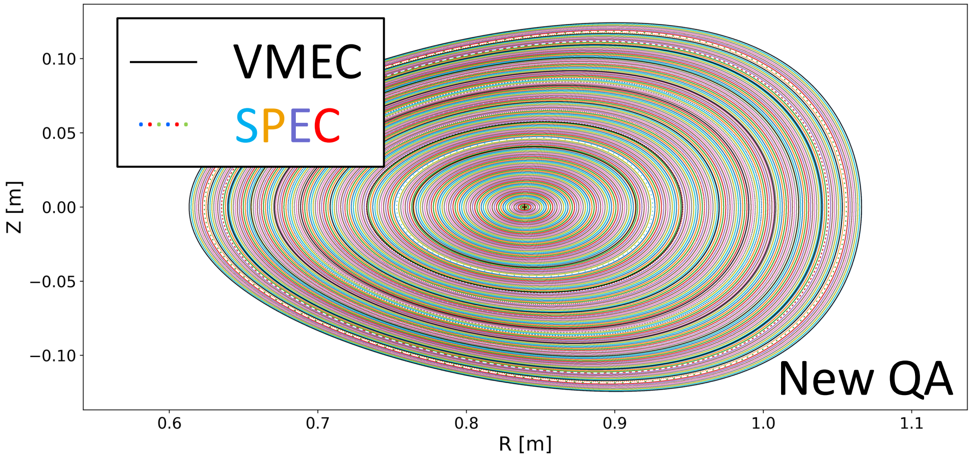


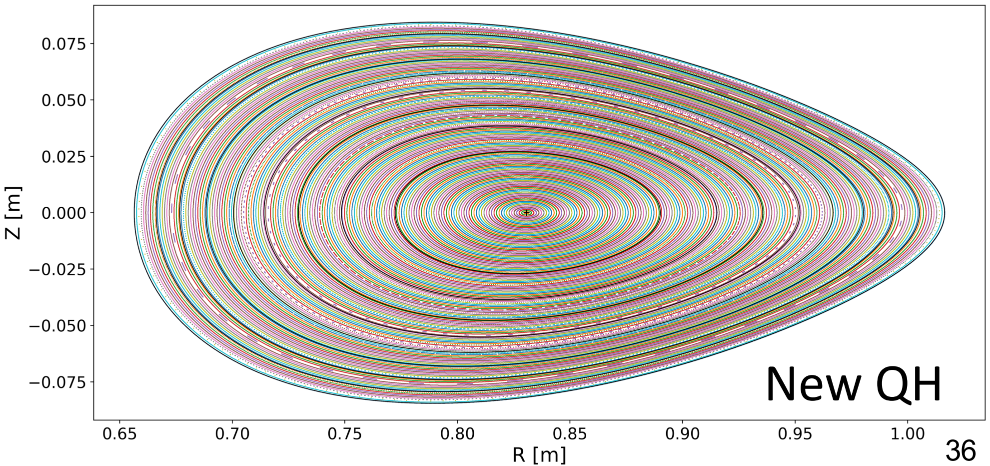


Extra slides

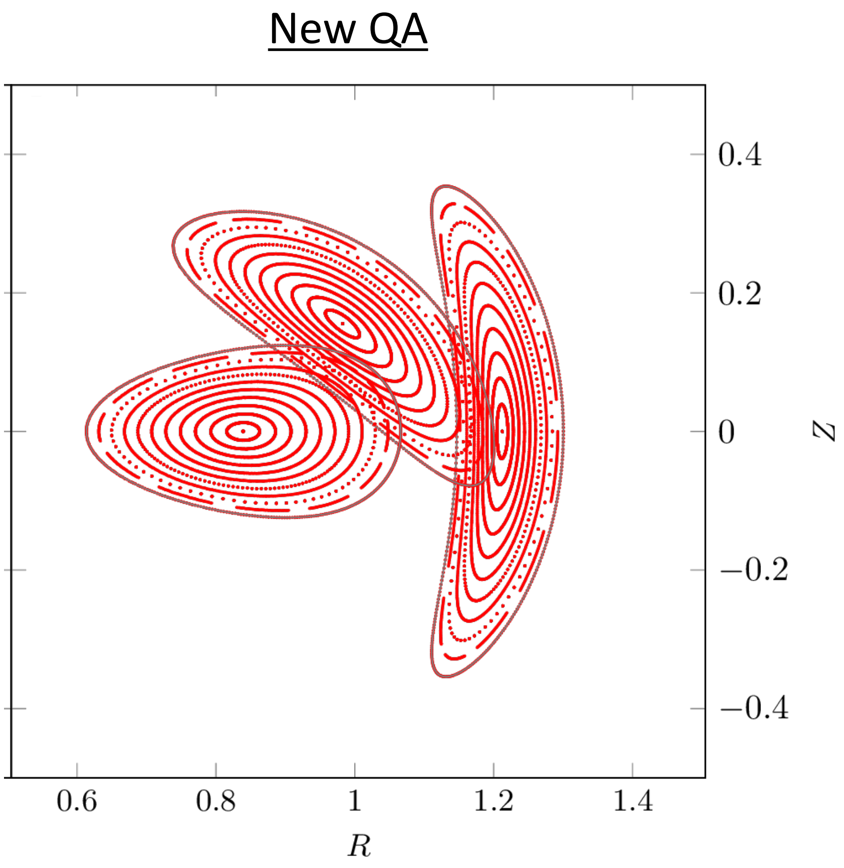
The new configurations have small magnetic shear

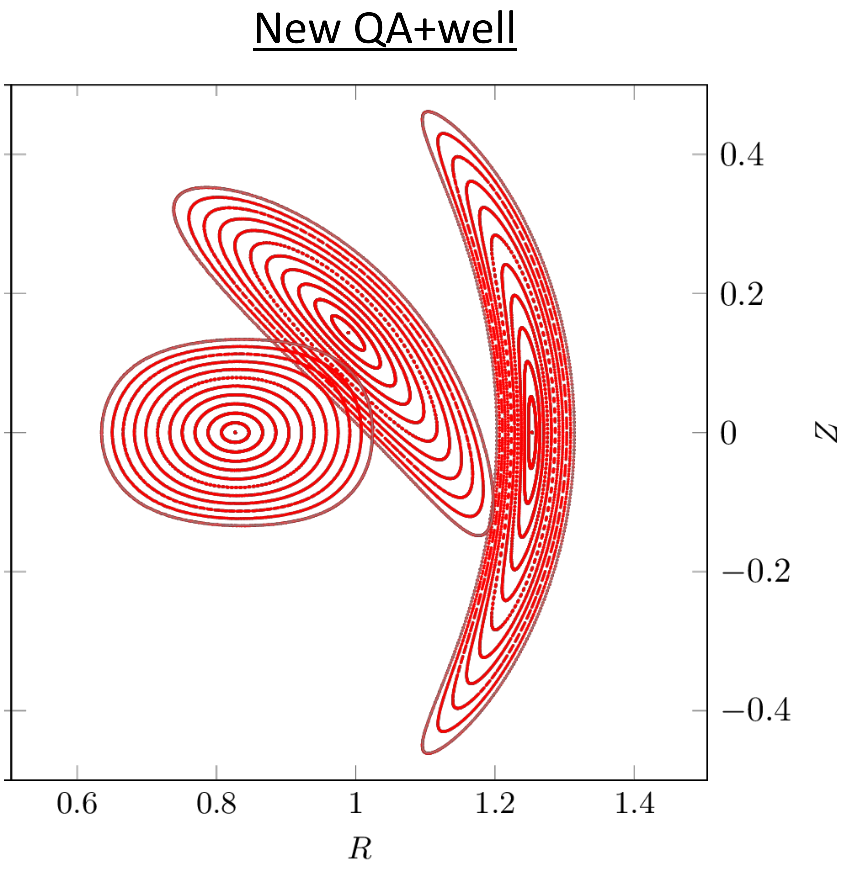


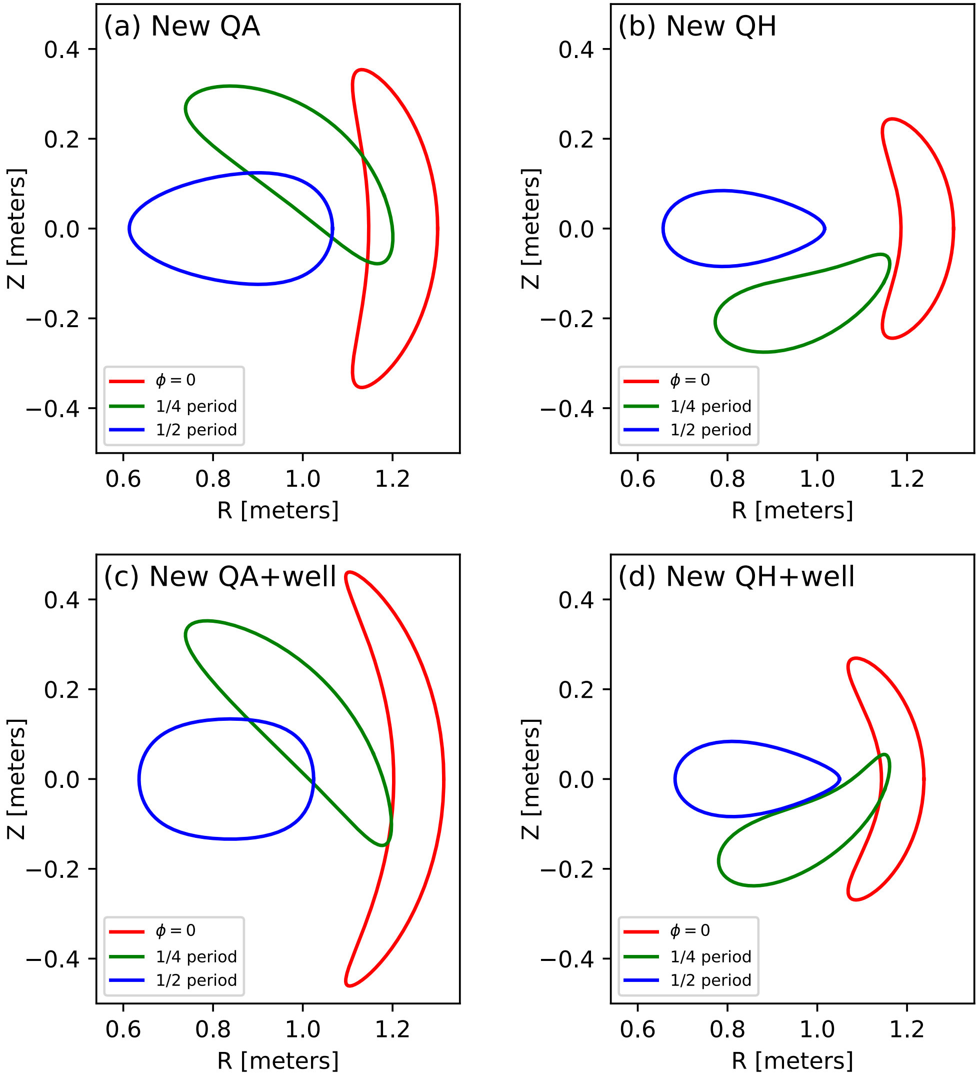




Good flux surface exist with coils

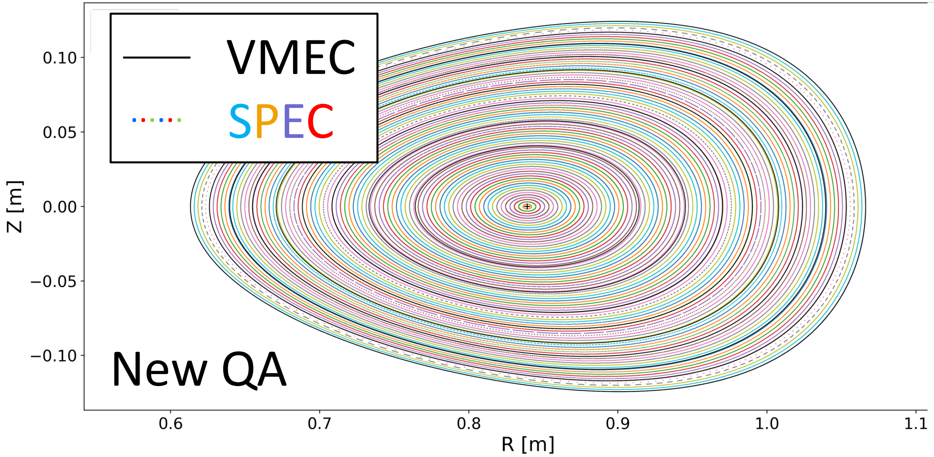


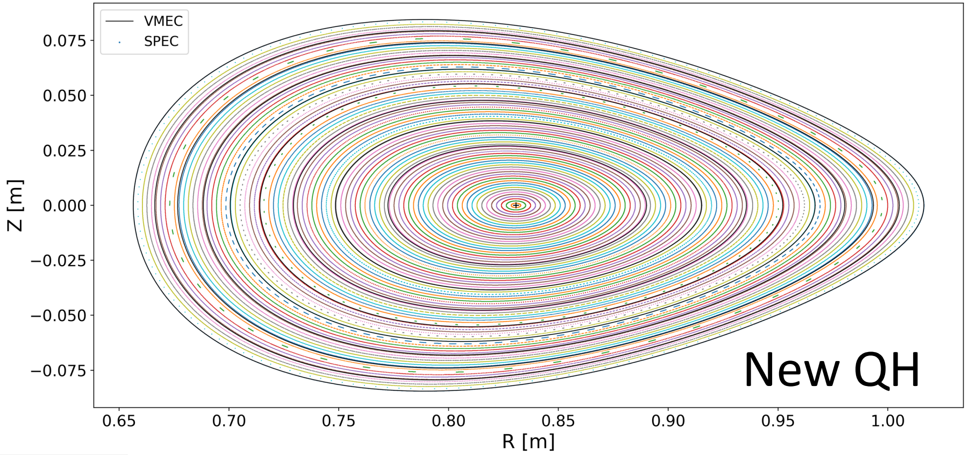


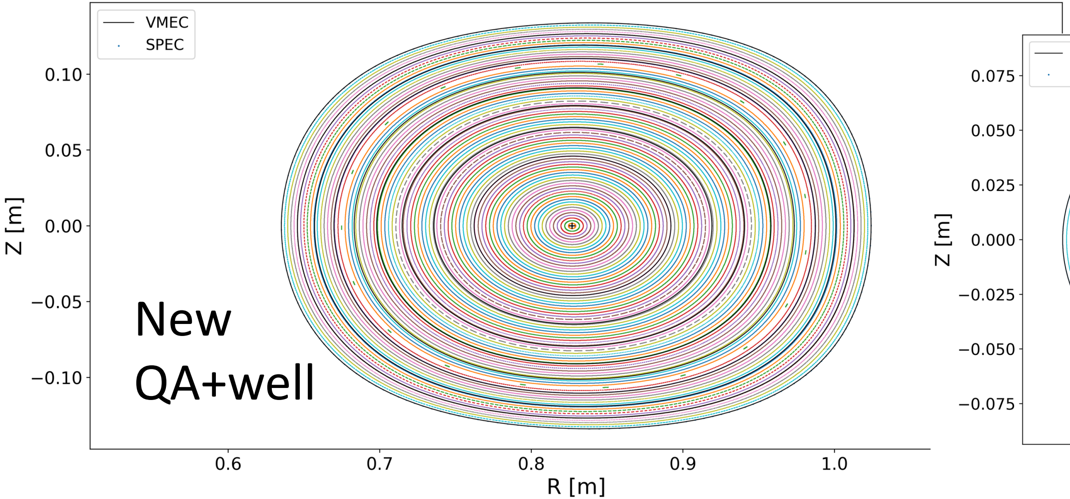


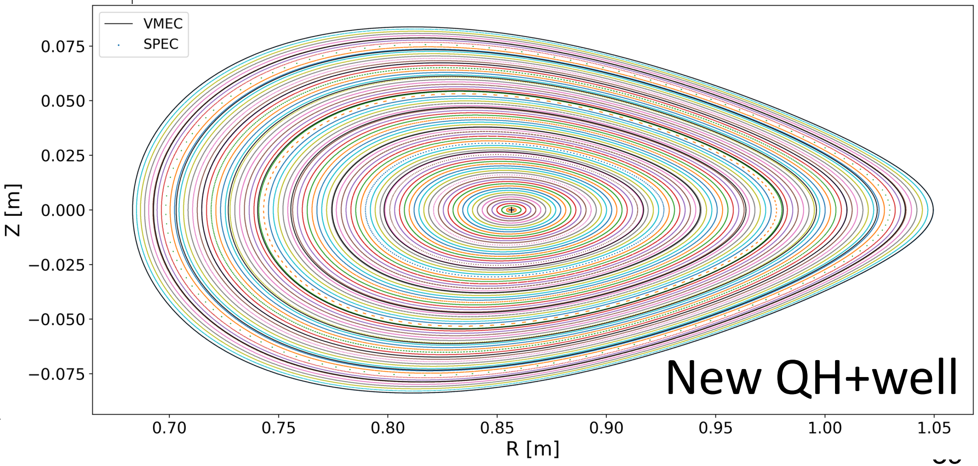
(a) New QA
(b) New QH
(c) New QA+well
(d) New QH+well
Z [meters]
R [meters]
0.4
0.2
0.0
−0.2
−0.4
0.6
0.8
1.0
1.2
$\phi = 0$
1/4 period
1/2 period

SPEC confirms the new QA/QH configurations have good surfaces

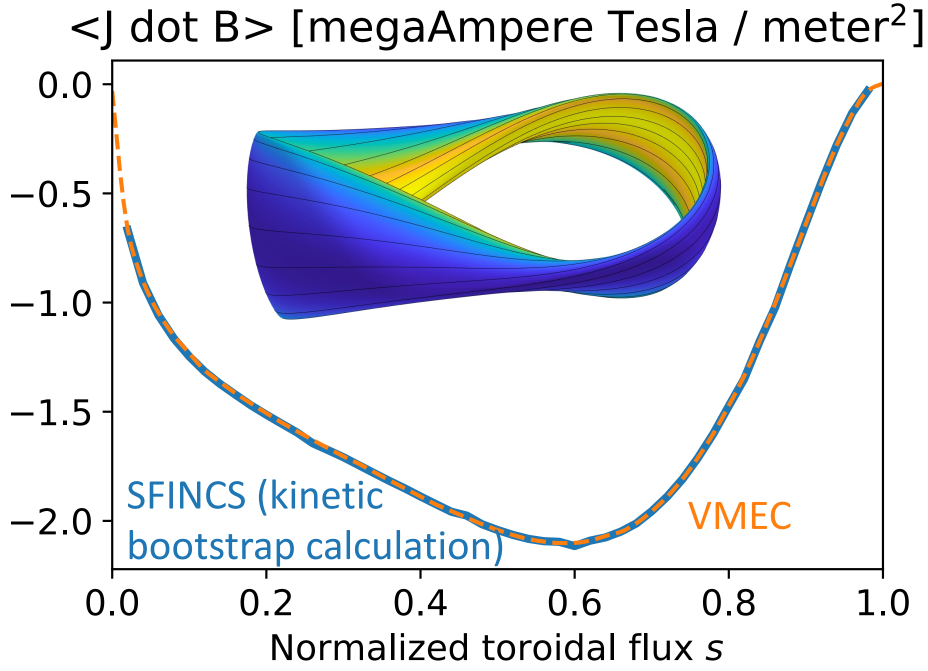








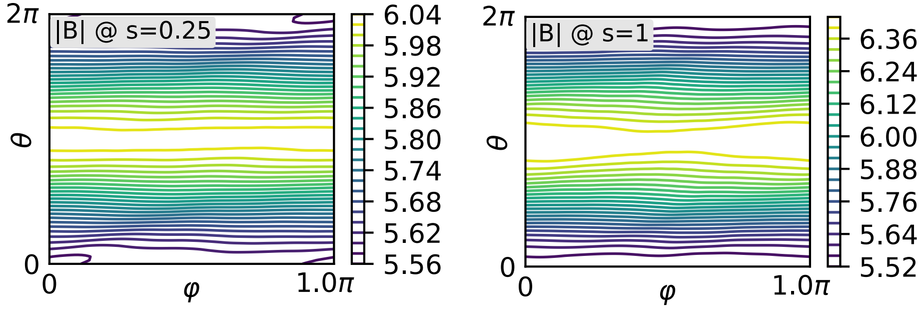
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry

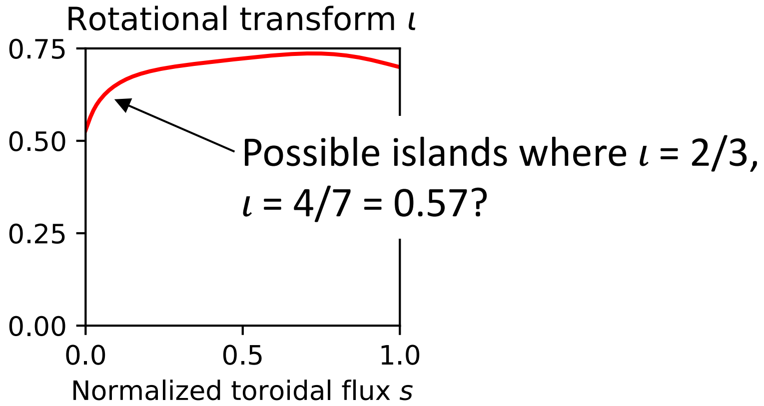


$$\langle \beta \rangle = 3\%, \quad \varepsilon_{eff}^{3/2} < 7\times10^{-6}$$

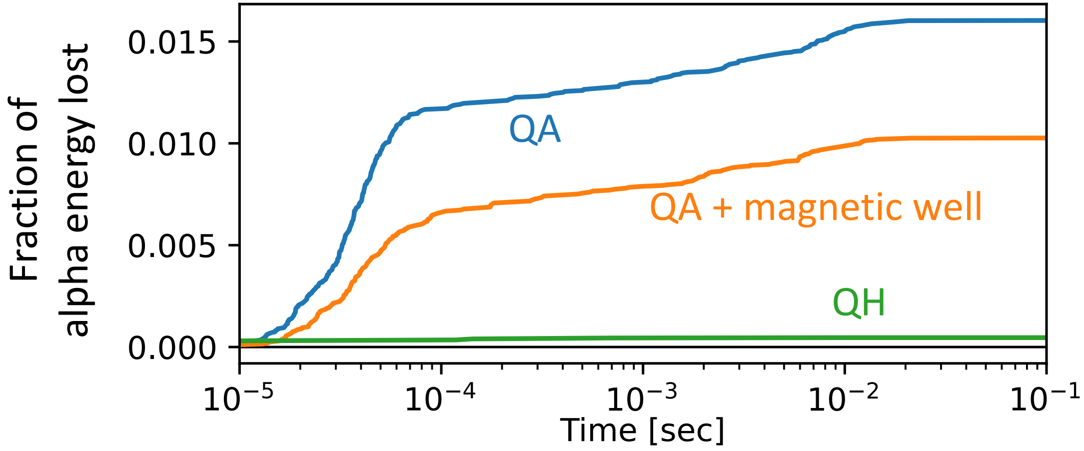
α-particle losses < 1%

Symmetry is not as good as for vacuum, but sufficient for excellent confinement

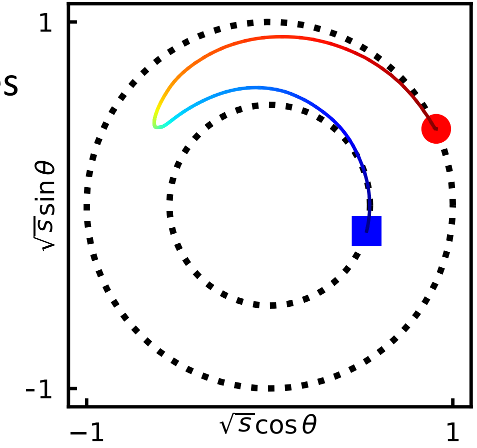




Why do the configurations with best quasisymmetry not have the best trajectory confinement?



Lost trajectories in the new QA look like this:



Width of banana orbit $\Delta s \approx \left| \frac{mvR\sqrt{2r\bar{\eta}}}{(\iota - N)\psi_{edge}Ze} \right| \propto \frac{1}{|\iota - N|}$

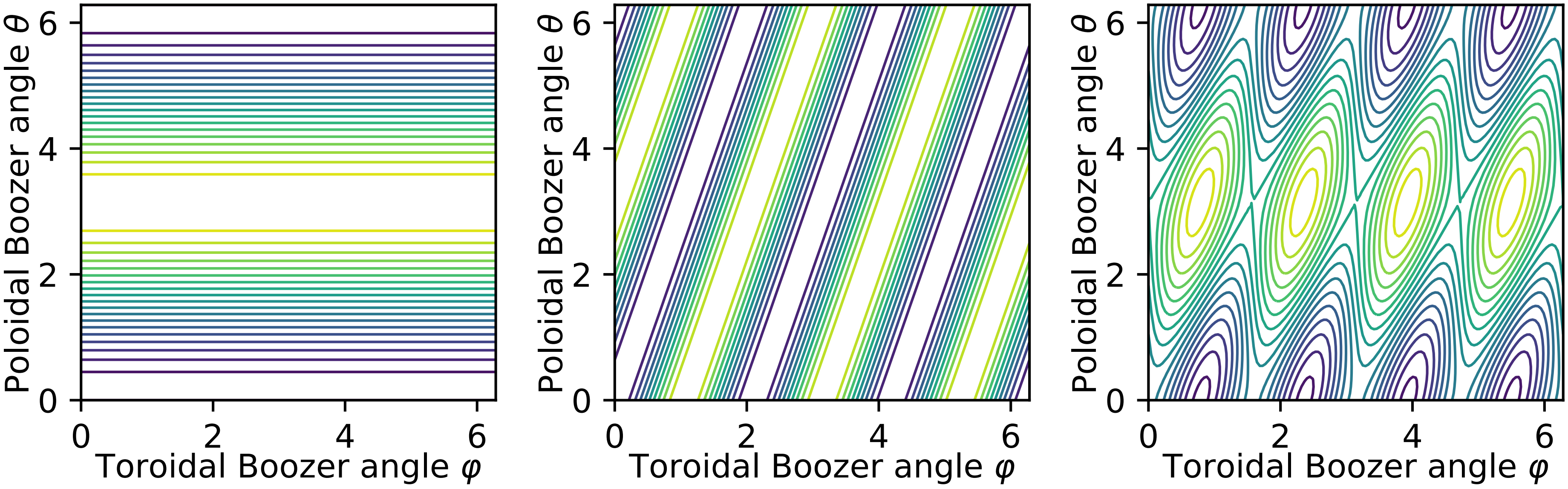
For fixed minor radius, $\frac{\Delta s_{QA}}{\Delta s_{QH}} \sim 4$

2 types of quasisymmetry

Quasi-axisymmetry (QA): $B = B(r, \theta)$

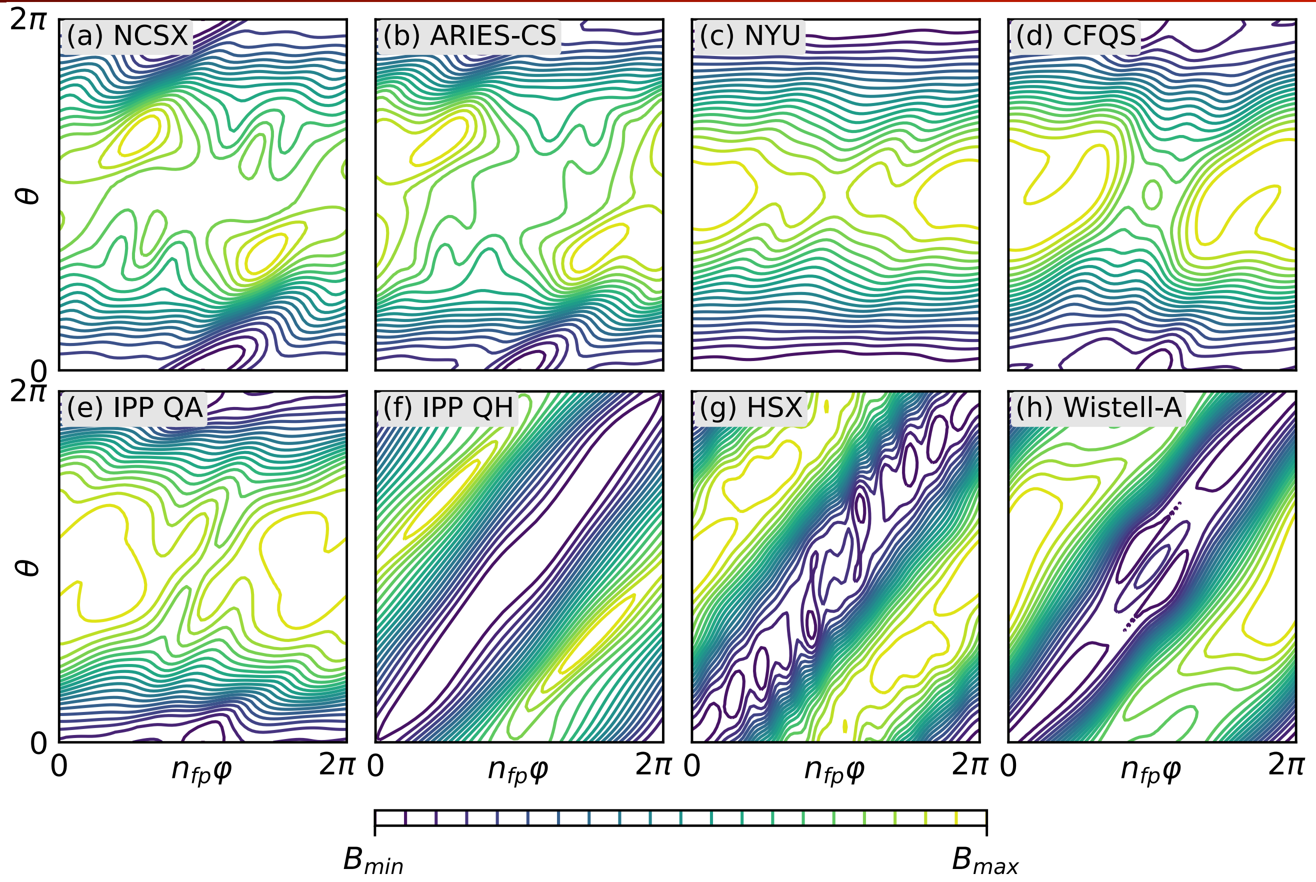
Quasi-helical symmetry (QH): $B = B(r, \theta - N\varphi)$

General stellarator (not symmetric)



Contours of $B = |\mathbf{B}|$:

Previous quasisymmetric configurations



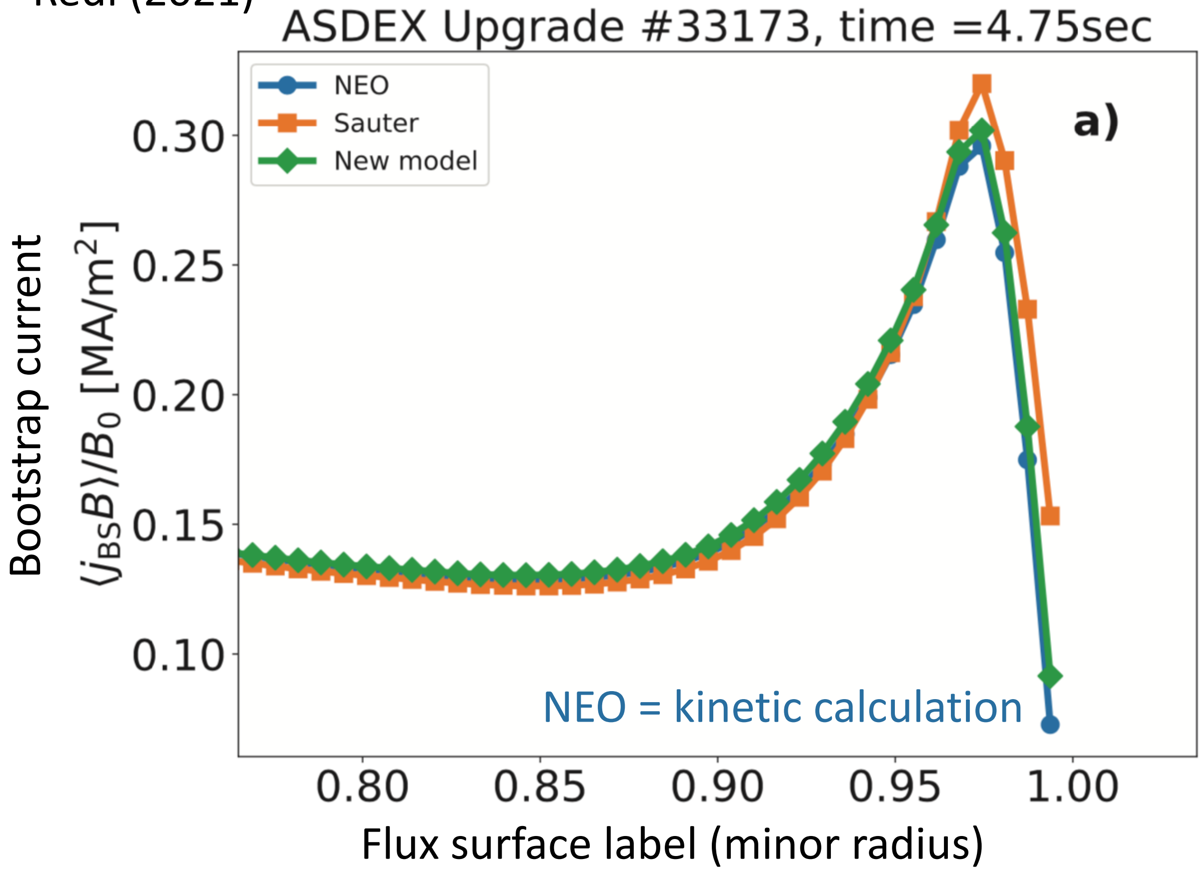
(a) Zarnstorff et al (2001)
(b) Najambadi et al (2008)
(c) Garabedian (2008)
(d) Liu et al (2018)
(e) Henneberg et al (2019)
(f) Nuhrenberg & Zille (1988)
(g) Anderson et al (1995)
(h) Bader et al (2020)

We want
$B = B(r, \theta - N\varphi)$

Is there an optimization recipe that can give consistently straight |B| contours?

New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Redl (2021)



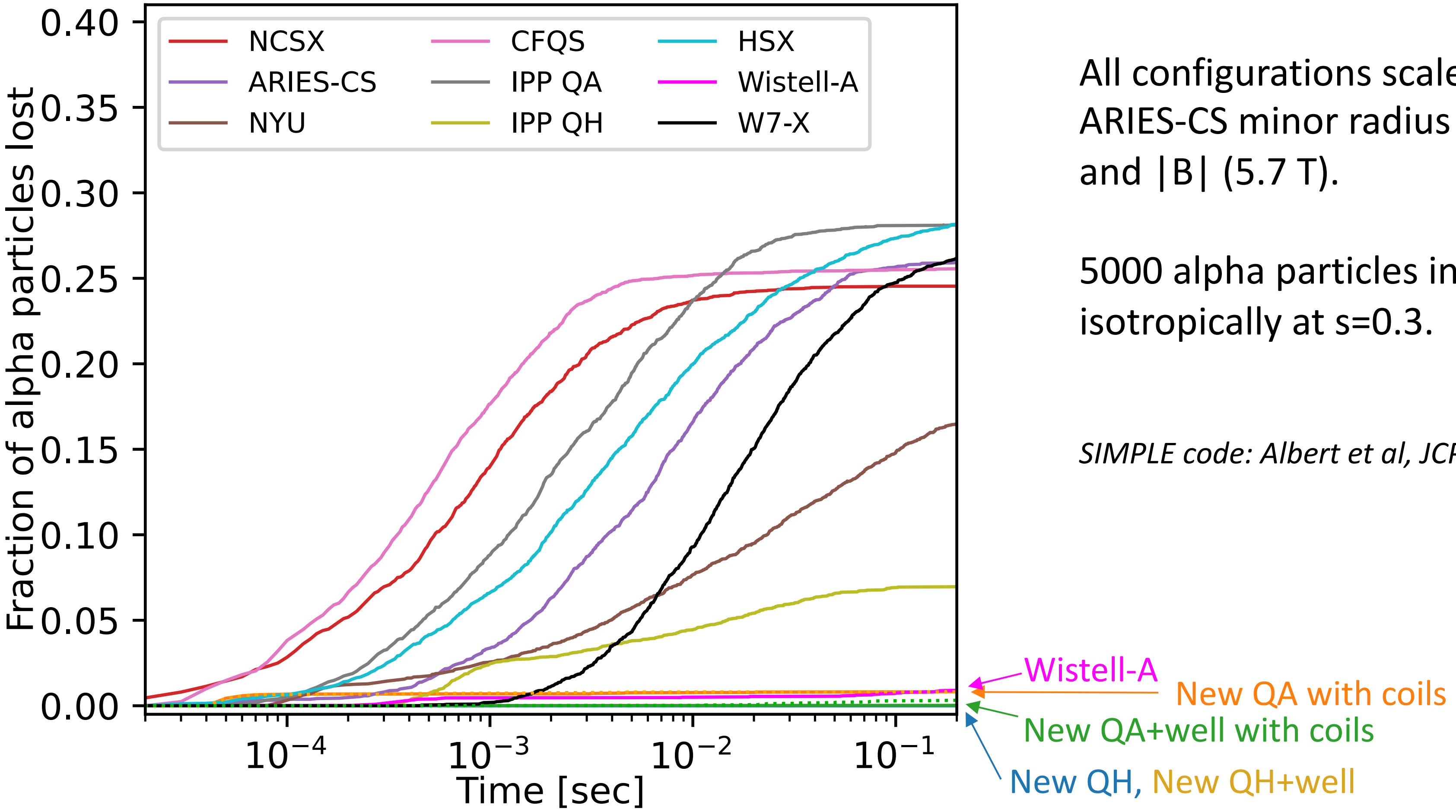
Geometry enters through

$$f_t = 1 - f_c = 1 - \frac{3}{4}\langle B^2 \rangle \int_0^{1/B_{\max}} \frac{\lambda d\lambda}{\langle \sqrt{1-\lambda B} \rangle}.$$

$$\nu_{e*} = 6.921 \cdot 10^{-18} \frac{qRn_e Z \ln \Lambda_e}{T_e^2 \epsilon^{3/2}},$$

$$\nu_{i*} = 4.90 \cdot 10^{-18} \frac{qRn_i Z^4 \ln \Lambda_{ii}}{T_i^2 \epsilon^{3/2}},$$

The symmetry yields extremely good confinement of collisionless trajectories

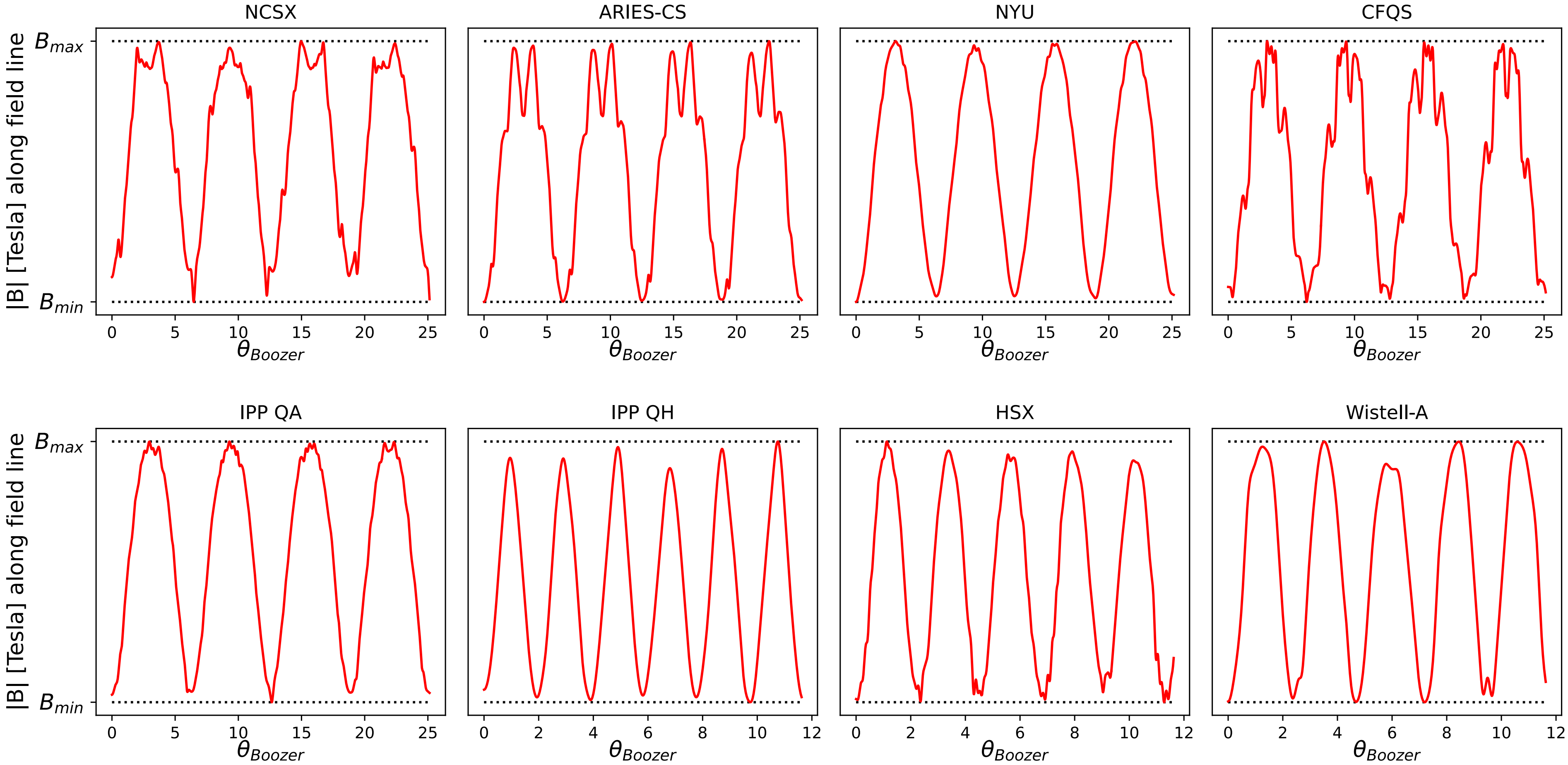


All configurations scaled to ARIES-CS minor radius (1.7 m) and |B| (5.7 T).

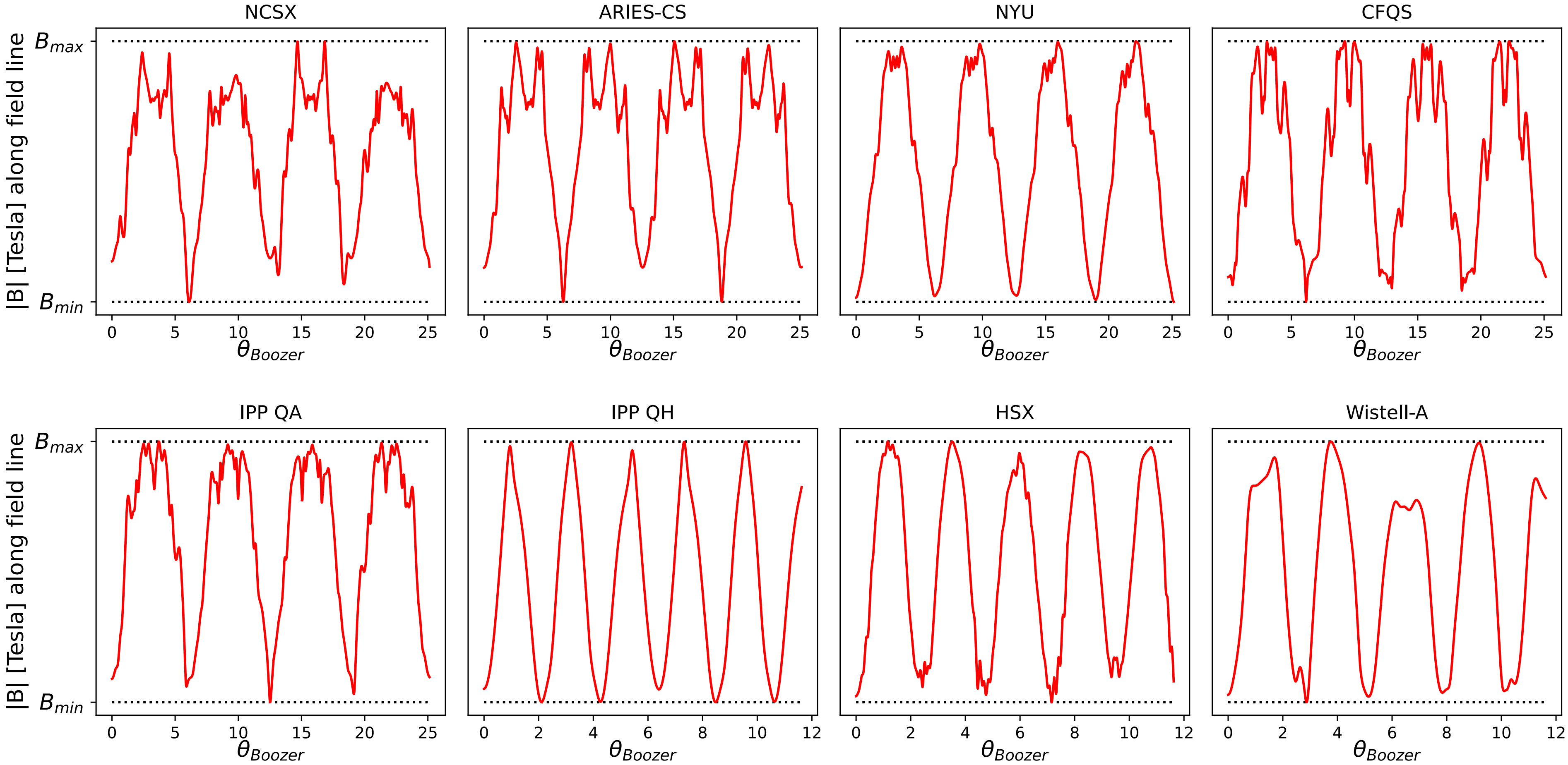
5000 alpha particles initialized isotropically at s=0.3.

SIMPLE code: Albert et al, JCP (2020).

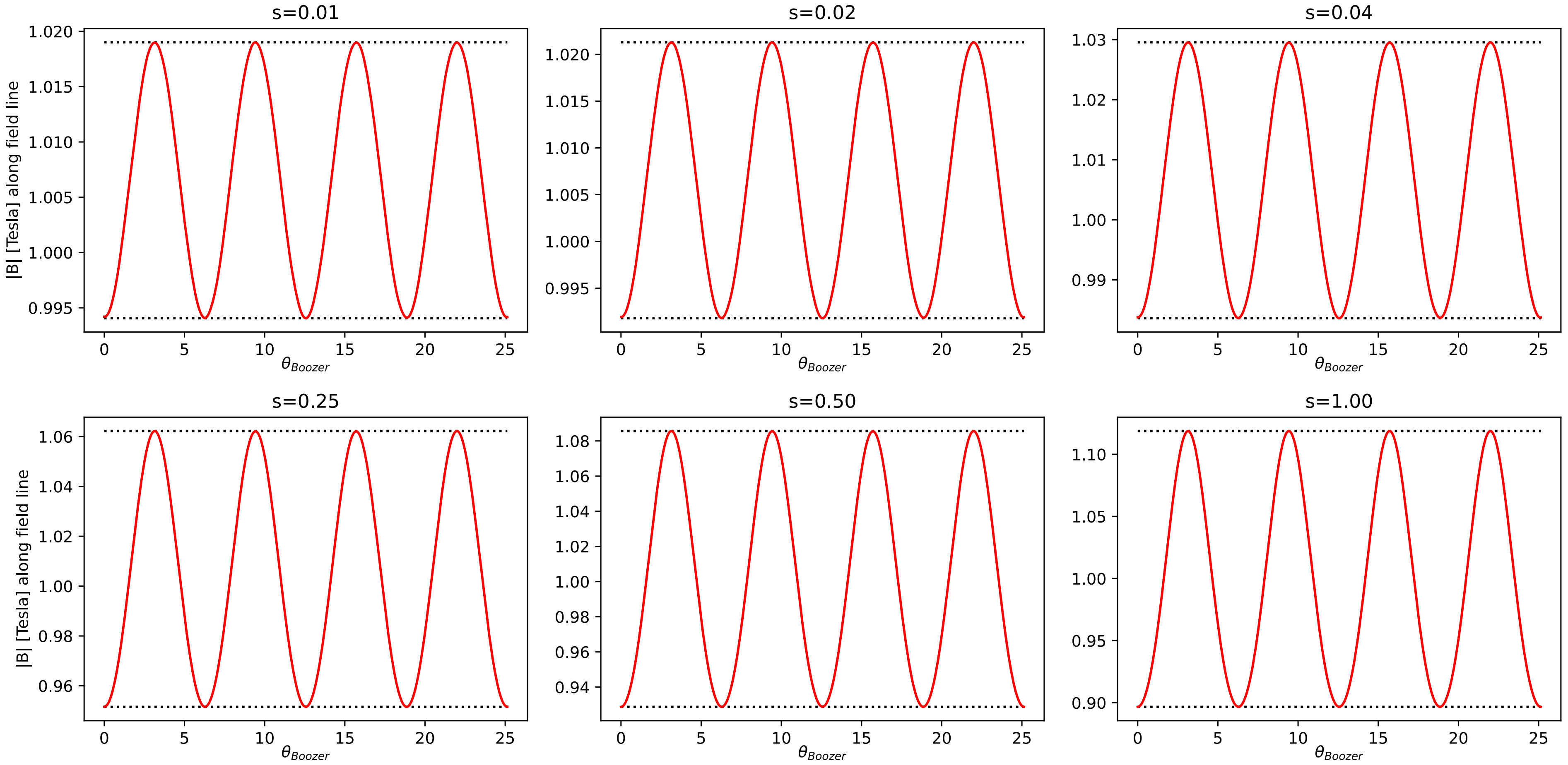
Previous quasisymmetric configurations (s=0.5)



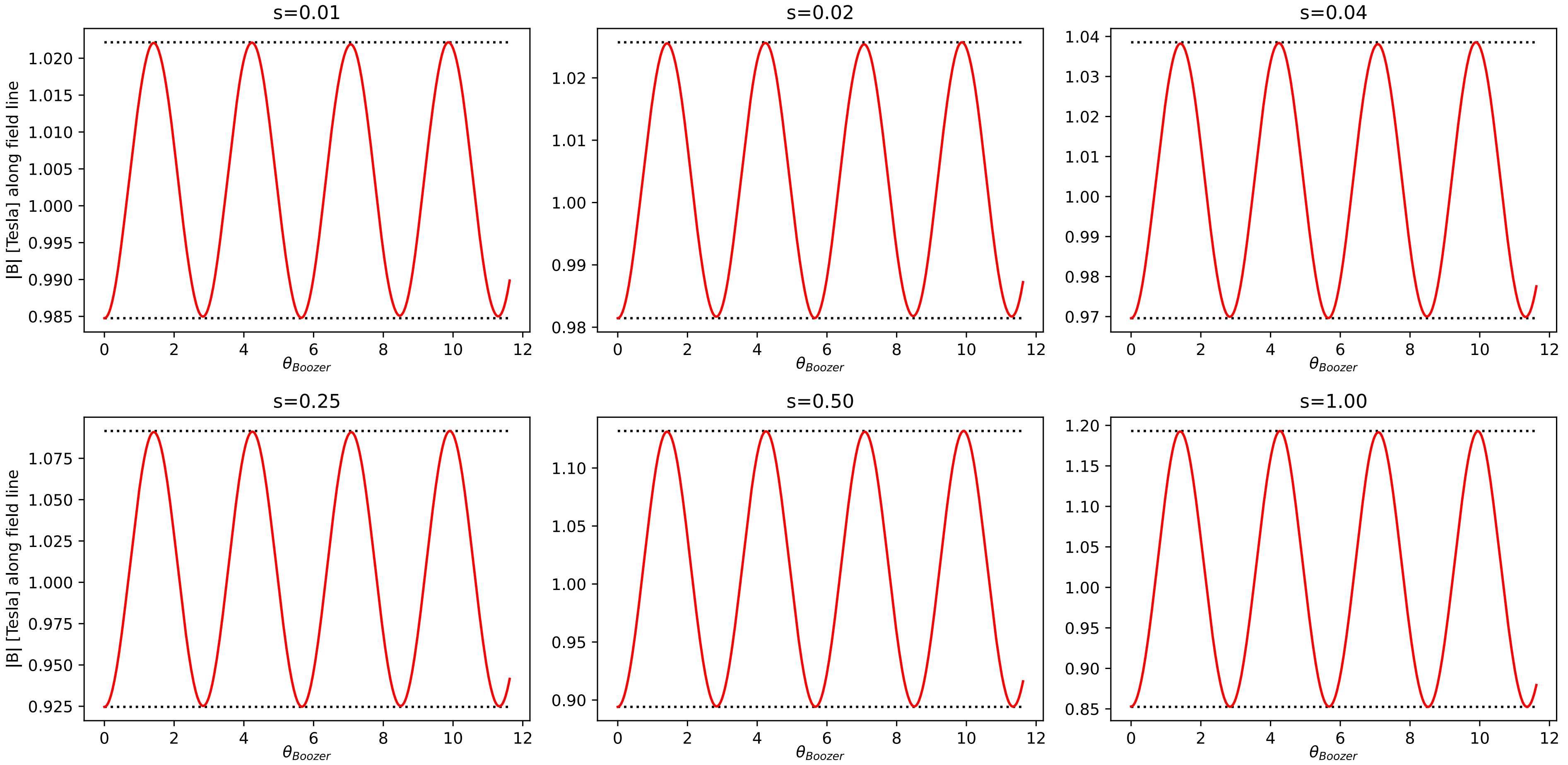
Previous quasisymmetric configurations (s=1)



|B| along a field line for new QA



|B| along a field line for new QH



|B| along a field line for new QA with magnetic well