Achieving energetic particle confinement in stellarators with precise quasisymmetry

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Landreman & Paul, PRL (2022), Wechsung et al, PNAS (2022)
Advantages of stellarators: steady-state, no disruptions, no Greenwald density limit, no power recirculated for current drive.

But, alpha particle losses & collisional transport would be too large unless you carefully choose the geometry.
Remarkable progress in stellarator confinement in the last year. All configurations scaled to same minor radius and $|B|$. See also Bader et al, Nuclear Fusion (2021).
Advantages of stellarators: steady-state, no disruptions, no Greenwald density limit, no power recirculated for current drive.

But, alpha particle losses & collisional transport would be too large unless you carefully choose the geometry.

A solution: **quasisymmetry**

\[ B = B(s, \theta - N \varphi) \]

\[ \int (v_d \cdot \nabla s) dt = 0 \]
Since 2021

Goal: $B = B(s, \theta - N \varphi)$

- ML & Paul, Phys Rev Lett (2022)
- Wechsung et al, PNAS (2022)
- Giuliani et al, 1-stage, arXiv (2022)
- Nies & Paul Adjoint method

Near-axis expansion
5% $\beta$, Self-consistent plasma current

Quasi-axisymmetry (QA) $N = 0$

Quasi-helical symmetry (QH) $N \neq 0$
• Optimizing stellarator geometry for precise quasisymmetry
• Self-consistent bootstrap current
• Conclusions
• Optimizing stellarator geometry for precise quasisymmetry

• Self-consistent bootstrap current

• Conclusions
Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

\[
\begin{align*}
\mathfrak{f}_{QH} &= (A - A^*)^2 + \mathfrak{f}_{QS} \\
\mathfrak{f}_{QA} &= (A - A^*)^2 + \left( \iota^* - \int_0^1 \iota ds \right)^2 + \mathfrak{f}_{QS}
\end{align*}
\]

Boundary aspect ratio

Goal: \( B = B(s, \theta - N \varphi). \)

For quasi-axisymmetry, \( N = 0. \)

For quasi-helical symmetry, \( N \) is the “number of field periods”, e.g. \( N = 4 \) here.
Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:
  \[
  f_{QS} = \int d^3x \left( \frac{1}{B^3} \left[ (N - \ell) \mathbf{B} \times \nabla \mathbf{B} \cdot \nabla \psi - (G + NI) \mathbf{B} \cdot \nabla \mathbf{B} \right] \right)^2
  \]

  \[
  f_{QH} = (A - A^*)^2 + f_{QS}
  \]

  Boundary aspect ratio

  \[
  f_{QA} = (A - A^*)^2 + \left( \ell^* - \int_0^1 \ell ds \right)^2 + f_{QS}
  \]

- Parameter space: \( R_{m,n} \) & \( Z_{m,n} \) defining a toroidal boundary
  \[
  R(\theta, \phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \quad Z(\theta, \phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)
  \]

- Codes used: SIMSOPT with VMEC
- Cold start: circular cross-section torus
- Vacuum fields at first, allowing precise checks
- Algorithm: default for least-squares in scipy (trust region reflective)
- 6 steps: increasing # of modes varied & VMEC resolution
- Run many optimizations, pick the best
Straight $|B|$ contours are possible for quasi-axisymmetry.

Aspect = 6

Straight $|B|$ contours are possible for quasi-helical symmetry, as shown in the images. The aspect ratio of the flux surfaces is indicated as 8. The results are from ML & Paul, PRL (2022).
Good symmetry also exists with magnetic well

\[ \frac{d^2 (\text{flux surface volume})}{d(\text{toroidal flux})^2} < 0 \text{ everywhere} \]

QA

\[ B @ s = 0.25 \]

\[ B @ s = 1 \]

QH

16-coil solutions have been found for the quasi-axisymmetric configurations


With magnetic well

Without magnetic well

<Haven’t looked at the QHs yet

<\textbf{R} / 10 between filament centers.>
Symmetry-breaking modes can be made extremely small

New QA configuration

Fourier amplitudes $|B_{m,n}|$ [Tesla]

- $m = 0, n = 0$ (Background)
- $m \neq 0, n = 0$ (Quasi-axisymmetric)

Geomagnetic field

$$s = \text{Normalized toroidal flux}$$
Quasisymmetry works: alpha particle confinement is significantly improved

- All configs scaled to minor radius and $|B|$ of ARIES-CS
- Fusion alpha birth distribution
- ANTS code, with collisions
- Particle considered lost when $s > 1$

Fraction of alpha particle energy lost before thermalization
Alpha confinement in quasi-helical stellarators can be better than in a tokamak due to thinner bananas.

\[ \Delta s \propto \frac{1}{|t - N|}. \]

\( N = 0 \) for QA, \( N = \# \) of field periods for QH.

See poster by Elizabeth Paul (session 3)
• Optimizing stellarator geometry for precise quasisymmetry

• Self-consistent bootstrap current

• Conclusions
How can bootstrap current be included self-consistently in stellarator optimization?

- Need *self-consistency* between MHD equilibrium and drift-kinetic equation.
- Previous method: fixed-point iteration, only after an optimization.
- Accurate drift-kinetic bootstrap calculations in stellarators are computationally expensive. Preferably not in the optimization loop.
New idea: exploit quasisymmetry & use analytic expressions for tokamaks


Bootstrap current (& other quantities) in quasisymmetry are the same as in axisymmetry, up to some substitutions:

\[ t \rightarrow t - N \]

Should be accurate for the new precisely quasisymmetric configurations.

Bootstrap current (& other quantities) in quasisymmetry are the same as in axisymmetry, up to some substitutions: $\iota \rightarrow \iota - N$

Should be accurate for the new precisely quasisymmetric configurations.
Before doing new optimizations: Redl formula is accurate in previous QA & QH stellarators

\[ n_e = (1 - s^5) \times 10^{20} \text{ m}^{-3}, \quad T_e = T_i = (1 - s) \times 12 \text{ keV} \]

QA (Landreman & Paul (2021))

QH (Landreman & Paul (2021))

(Not self-consistent yet)
Optimization recipe

- **Objective function:**
  \[ f = f_{QS} + f_{bootstrap} + \left( A - 6.5 \right)^2 + \left( a - a_{ARIES-CS} \right)^2 + \left( \left\langle B \right\rangle - \left\langle B \right\rangle_{ARIES-CS} \right)^2 \]
  \[ f_{QS} = \int d^3x \left( \frac{1}{B^3} \left[ \left( N - I \right) \mathbf{B} \times \nabla \mathbf{B} \cdot \nabla \psi - \left( G + NI \right) \mathbf{B} \cdot \nabla \mathbf{B} \right] \right)^2 \]
  \[ f_{bootstrap} = \frac{\int_0^1 ds \left[ \left\langle \mathbf{j} \cdot \mathbf{B} \right\rangle_{vme} - \left\langle \mathbf{j} \cdot \mathbf{B} \right\rangle_{Redl} \right]^2}{\int_0^1 ds \left[ \left\langle \mathbf{j} \cdot \mathbf{B} \right\rangle_{vme} + \left\langle \mathbf{j} \cdot \mathbf{B} \right\rangle_{Redl} \right]^2} \]

- **Parameter space:** \( \{ R_{m,n}, Z_{m,n}, \) toroidal flux, current spline values\}
  or \( \{ R_{m,n}, Z_{m,n}, \) toroidal flux, iota spline values\}

- **Cold start**
- **Algorithm:** default for least-squares in scipy (trust region reflective)
- **Steps:** increasing # of modes varied: \( m \) and \( |n/nfp| \) up to \( j \) in step \( j \)
Example of optimization with self-consistent bootstrap current

\[ n_{e0} = 2.5 \times 10^{20}/\text{meters}^3 \]

\[ T_{e0} = T_{i0} = 10 \text{ keV} \]

\[ \beta = 2.8\%, \quad I_p = 1.3 \text{ MA} \]
If you want perfectly self-consistent current, you can do a few fixed-point iterations at the end.

\[ \langle J \cdot B \rangle \text{ [megaAmpere Tesla / meter}^2\text{]} \]

No significant degradation in quasisymmetry:

Optimization with Redl current

\[ \beta = 5\% \]

\[ \alpha \text{ energy losses } < 0.3\% \]
• Optimizing stellarator geometry for precise quasisymmetry

• Self-consistent bootstrap current

• Conclusions
It is now possible to design stellarators with alpha confinement comparable to or better than a tokamak.

More to do: coils, surface quality, & MHD stability at high $\beta$, robustness to pressure profile, ...
Extra slides
The bootstrap current arises in tokamaks & stellarators when the density & temperature become significant

- Ions and electrons have different trajectories. Different mean flows = electric current.
- Current depends on geometry, density, & temperature.
- For $\beta > 0$, we don’t know $B$ until we include this effect.
- Need self-consistency between MHD equilibrium and kinetic equation.
- How can a self-consistent bootstrap current calculation be integrated with stellarator optimization?
The symmetry also yields extremely low collisional transport for a thermal plasma.
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry.

Symmetry is not as good as for vacuum, but sufficient for excellent confinement.

Possible islands where $\iota = 2/3, \iota = 4/7 = 0.57$.

$\langle \beta \rangle = 3\%$, \( \varepsilon_{\text{eff}}^{3/2} < 7 \times 10^{-6} \)

\( \alpha \)-particle losses < 1%
The absolute value of $|B_{m,n}|$ in Tesla was verified by independent SPEC calculations.

$max |B_{m,n}|$ [Tesla] with $n \neq 0$

By Elizabeth Paul

(Ntor = Mpol, Lrad = Mpol + 4)
To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included. Crossing iota=1, the worst resonance, is probably unacceptable.

\[ n_{e0} = 3 \times 10^{20} \text{meters}^3, \quad T_{e0} = T_{i0} = 15 \text{ keV} \]
To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included.

Crossing iota=1, the worst resonance, is probably unacceptable.

Solution: Add barrier term in objective

\[ f' = \int_0^1 ds \left[ \min \left( |\iota(s)| - 1.03, 0 \right) \right]^2 \]

Rotational transform \( \iota \)

\( \beta = 0 \)

\( \beta = 5\% \) with \( \iota \) barrier

Quasisymmetry & bootstrap consistency remain good:
Why do the configurations with best quasisymmetry not have the best trajectory confinement?

Lost trajectories in the new QA look like this:

Width of banana orbit \( \Delta s \approx \frac{mvR\sqrt{2r\eta}}{(t-N)\psi_{edge} Ze} \) \( \propto \frac{1}{t-N} \)

For fixed minor radius, \( \frac{\Delta s_{QA}}{\Delta s_{QH}} \sim 4 \)
2 types of quasisymmetry

Quasi-axisymmetry (QA): $B = B(r, \theta)$

Quasi-helical symmetry (QH): $B = B(r, \theta - N\varphi)$

General stellarator (not symmetric)

Contours of $B = |B|$: $B_{\text{min}}$ to $B_{\text{max}}$
Is there an optimization recipe that can give consistently straight $|B|$ contours?

We want

$$B = B(r, \theta - N \varphi)$$
The new configurations have small magnetic shear.

New QH

New QH+well

New QA

New QA+well
New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Redl (2021)

ASDEX Upgrade #33173, time = 4.75sec

Geometry enters through

\[ f_t = 1 - f_c = 1 - \frac{3}{4} \langle B^2 \rangle \int_0^{1/B_{\text{max}}} \frac{\lambda d\lambda}{\langle \sqrt{1 - \lambda B} \rangle} \]

\[ \nu_{e*} = 6.921 \cdot 10^{-18} \frac{q R n_e Z \ln \Lambda_e}{T_e \epsilon^{3/2}} \]

\[ \nu_{i*} = 4.90 \cdot 10^{-18} \frac{q R n_i Z^4 \ln \Lambda_{ii}}{T_i \epsilon^{3/2}} \]

NEO = kinetic calculation
The symmetry yields extremely good confinement of collisionless trajectories.

All configurations scaled to ARIES-CS minor radius (1.7 m) and |B| (5.7 T).

5000 alpha particles initialized isotropically at s=0.3.

*SIMPLE code: Albert et al, JCP (2020).*
Previous quasisymmetric configurations \((s=0.5)\)
Previous quasisymmetric configurations (s=1)
$|B|$ along a field line for new QA

$s=0.01$

$s=0.02$

$s=0.04$

$s=0.25$

$s=0.50$

$s=1.00$
$|B|$ along a field line for new QH
$|B|$ along a field line for new QA with magnetic well.
SPEC confirms the new QA/QH configurations have good surfaces
Good flux surface exist with coils

New QA

New QA+well
Future directions

- Eliminate islands at high $\beta$ (plasma pressure & current)
- Coils & MHD stability for the high $\beta$ configurations
- Quasi-isodynamic ($|B|$ contours close poloidally)
- Understand trade-offs using multi-objective optimization
- Robustness to uncertainty in the pressure profile
- Interact with systems codes & reactor studies
- Combined coil + confinement (1-stage) optimization
- Optimization of turbulence & divertor

Before optimization

After optimization

ML, Medasani & Zhu (2021)