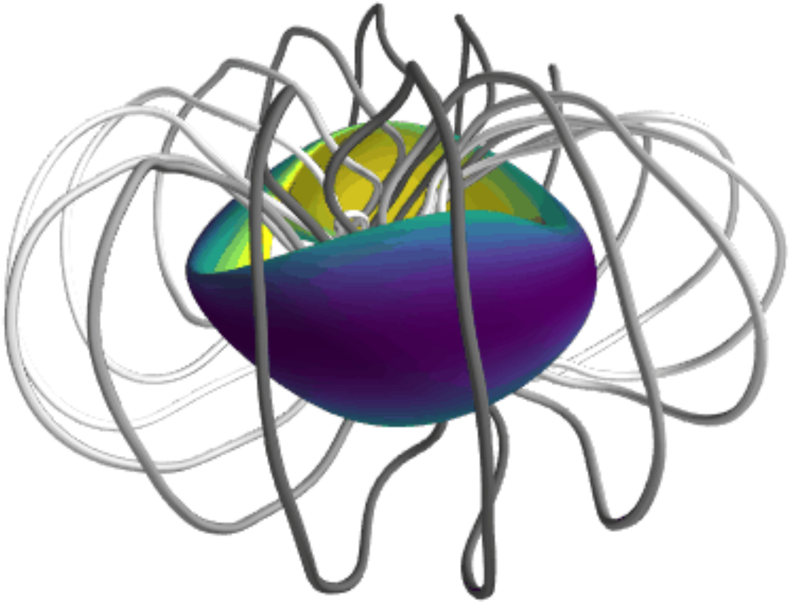
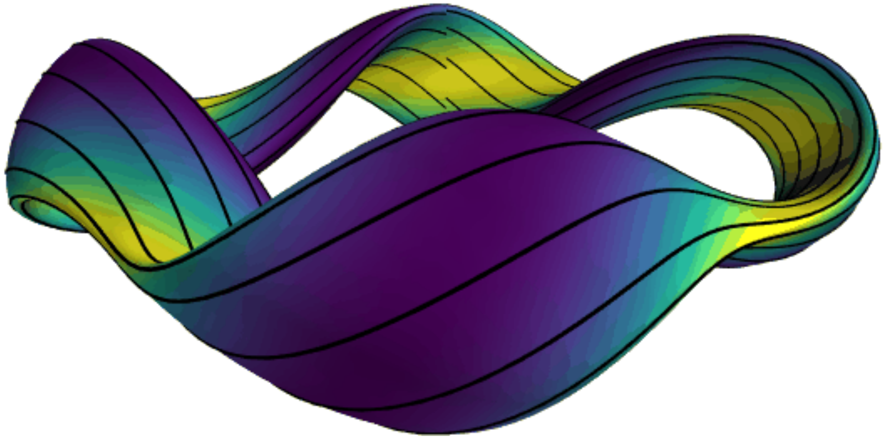
Achieving energetic particle confinement in stellarators with precise quasisymmetry





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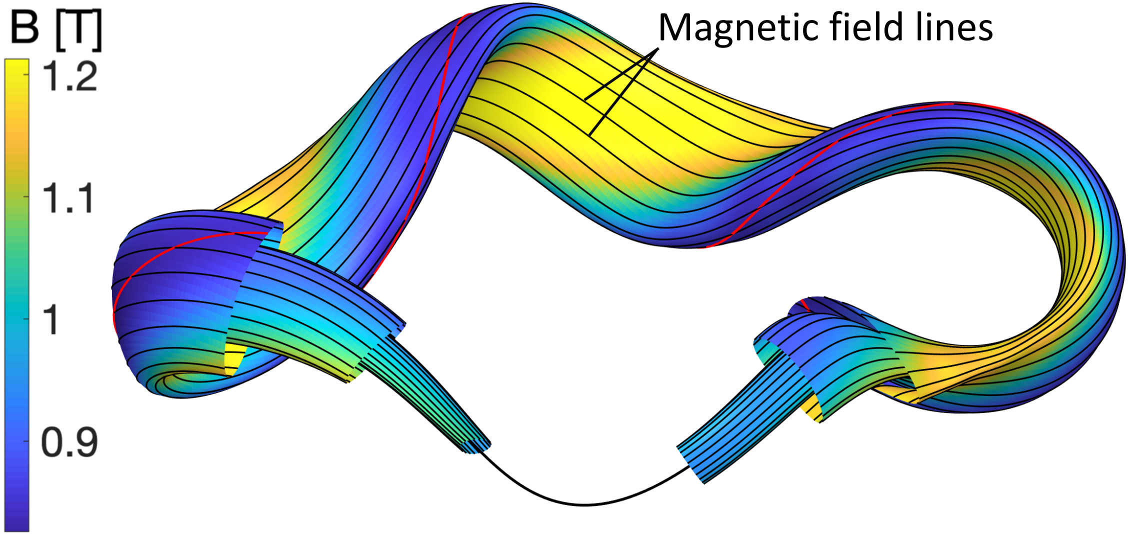
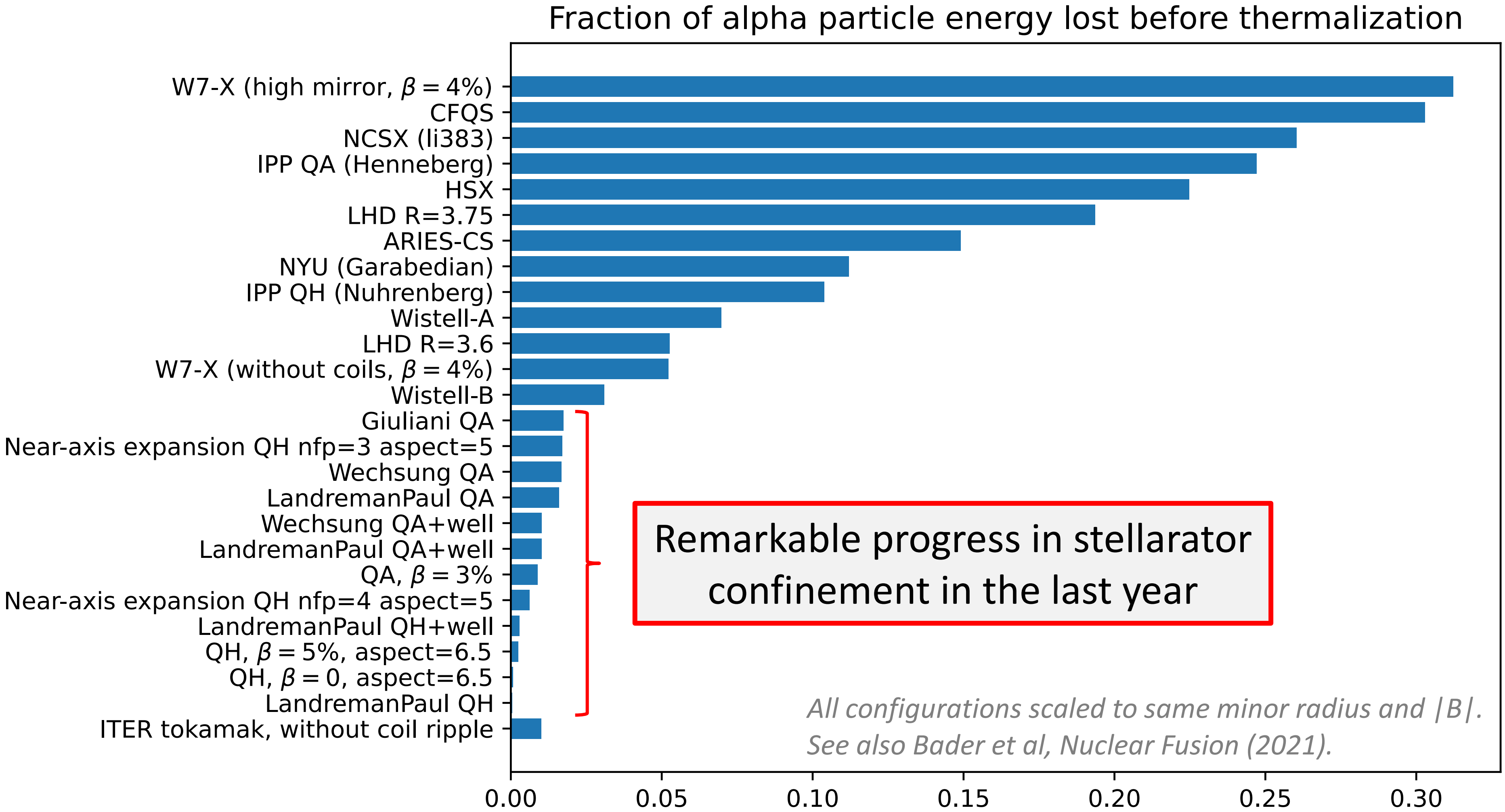
Landreman & Paul, PRL (2022), Wechsung et al, PNAS (2022)

Advantages of stellarators: steady-state, no disruptions, no Greenwald density limit, no power recirculated for current drive.

But, alpha particle losses & collisional transport would be too large unless you carefully choose the geometry.

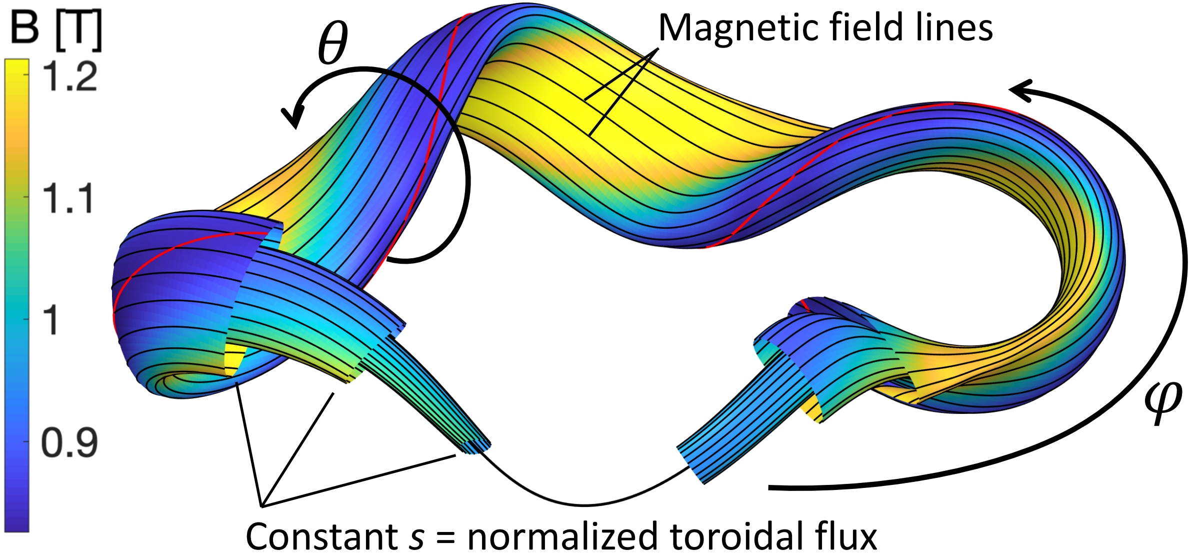



Fraction of alpha particle energy lost before thermalization
W7-X (high mirror, $\beta = 4\%$)
CFQS
NCSX (li383)
IPP QA (Henneberg)
HSX
LHD R=3.75
ARIES-CS
NYU (Garabedian)
IPP QH (Nuhrenberg)
Wistell-A
LHD R=3.6
W7-X (without coils, $\beta = 4\%$)
Wistell-B
Giuliani QA
Near-axis expansion QH nfp=3 aspect=5
Wechsung QA
LandremanPaul QA
Wechsung QA+well
LandremanPaul QA+well
QA, $\beta = 3\%$
Near-axis expansion QH nfp=4 aspect=5
LandremanPaul QH+well
QH, $\beta = 5\%$, aspect=6.5
QH, $\beta = 0$, aspect=6.5
LandremanPaul QH
ITER tokamak, without coil ripple
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Remarkable progress in stellarator confinement in the last year
All configurations scaled to same minor radius and |B|.
See also Bader et al, Nuclear Fusion (2021).

Advantages of stellarators: steady-state, no disruptions, no Greenwald density limit, no power recirculated for current drive.

But, alpha particle losses & collisional transport would be too large unless you carefully choose the geometry.

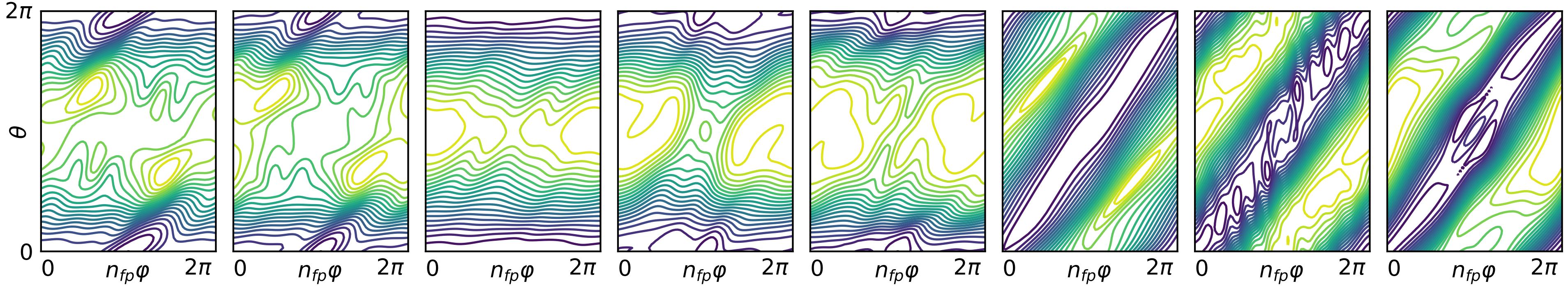
A solution: **quasisymmetry**



$$B = B(s, \theta - N\varphi)$$

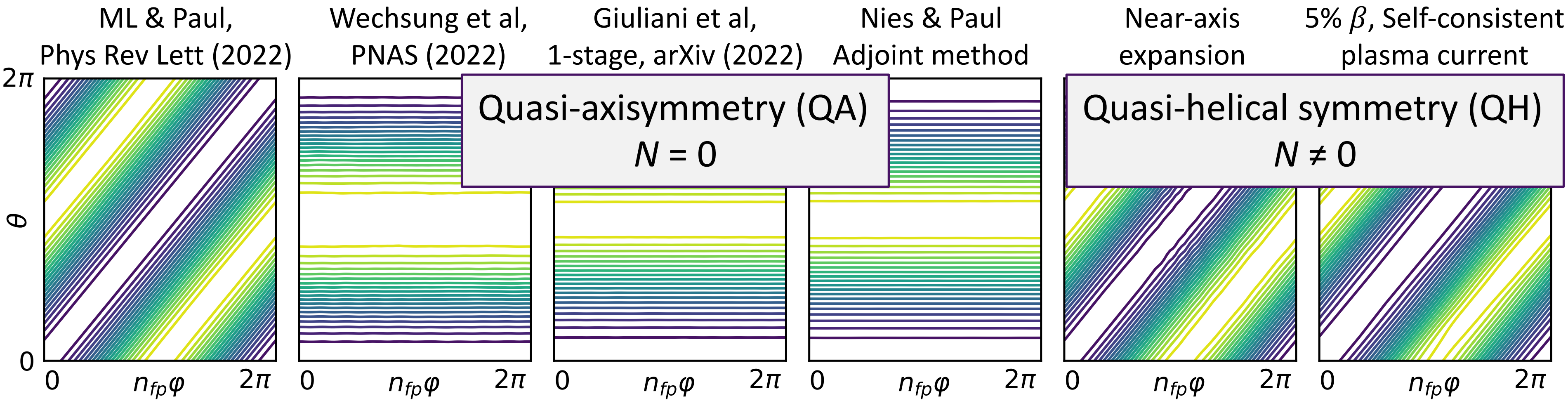
Boozer angles

$$\Rightarrow \quad \oint \left(\mathbf{v}_d \cdot \nabla s \right) dt = 0$$



Goal: $B = B(s, \theta - N\,\varphi)$

Since 2021



- Optimizing stellarator geometry for precise quasisymmetry
- Self-consistent bootstrap current
- Conclusions

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Optimization problem

- 2 stage approach, as for W7-X: First optimize shape of boundary surface, then coils.
- Objective functions:

$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

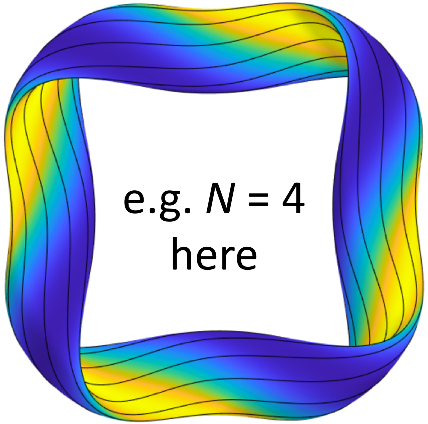
$$f_{QH} = \left(A - A_*\right)^2 + f_{QS} \qquad\qquad f_{QA} = \left(A - A_*\right)^2 + \left(\iota_* - \int_0^1 \iota\, ds\right)^2 + f_{QS}$$

Boundary aspect ratio

Goal: $B = B(s, \theta - N\varphi)$.

For quasi-axisymmetry,
$N = 0$.

For quasi-helical symmetry,
N is the "number of field periods",



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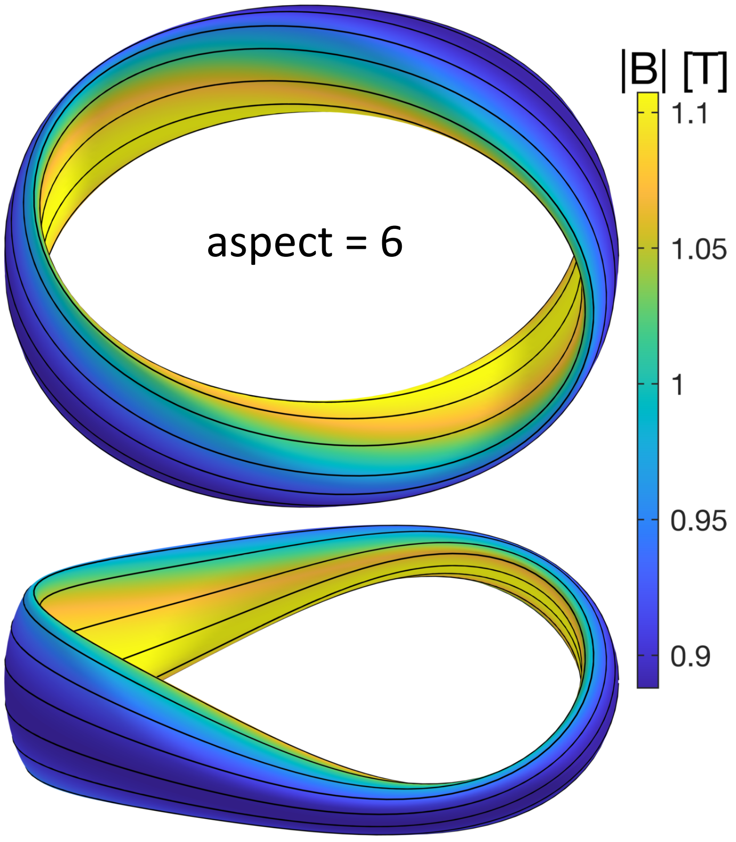
Boundary aspect ratio

- Parameter space: $R_{m,n}$ & $Z_{m,n}$ defining a toroidal boundary

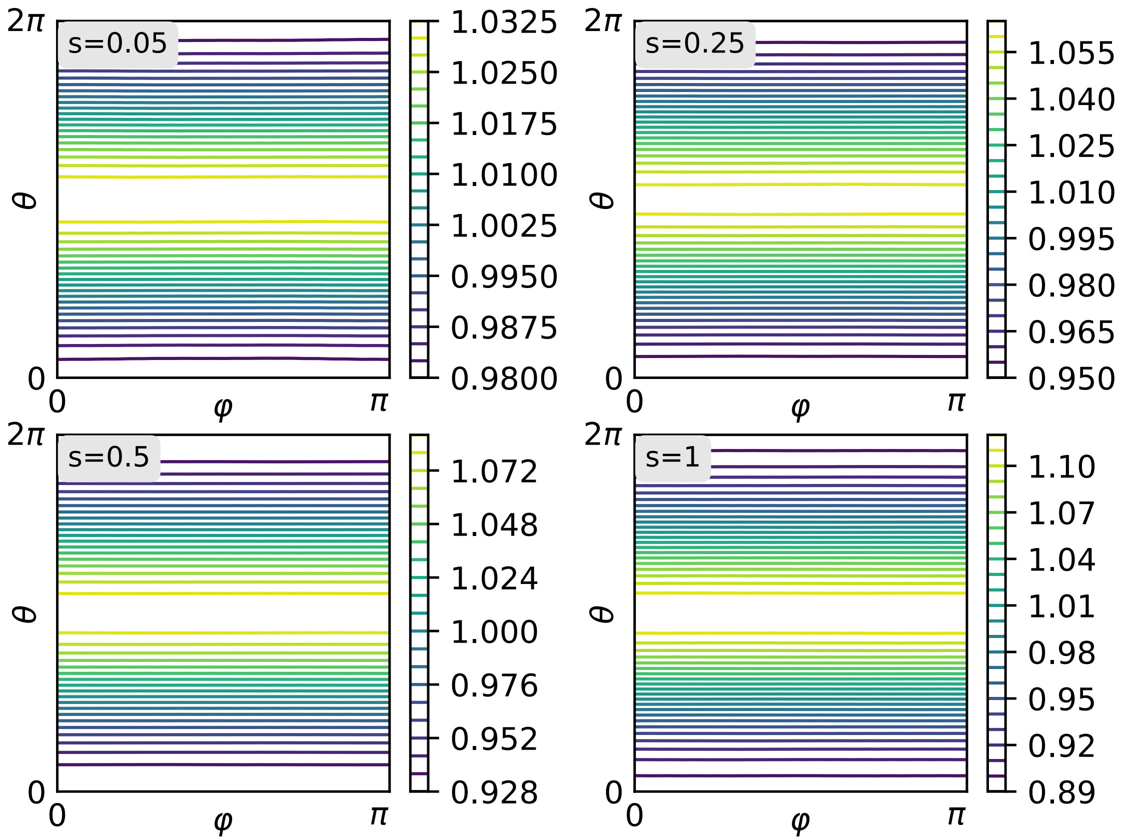
$$R(\theta,\phi) = \sum_{m,n} R_{m,n} \cos(m\theta - n\phi), \qquad Z(\theta,\phi) = \sum_{m,n} Z_{m,n} \sin(m\theta - n\phi)$$

- Codes used: SIMSOPT with VMEC
- Cold start: circular cross-section torus
- Vacuum fields at first, allowing precise checks
- Algorithm: default for least-squares in scipy (trust region reflective)
- 6 steps: increasing # of modes varied & VMEC resolution
- Run many optimizations, pick the best

Straight |B| contours are possible for quasi-axisymmetry

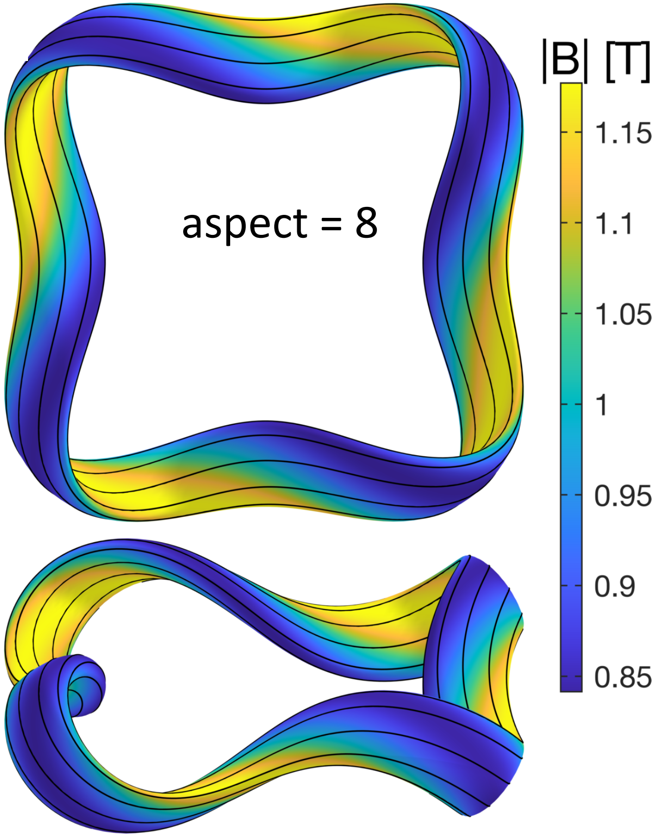


|B| on flux surfaces of the quasi-axisymmetric field



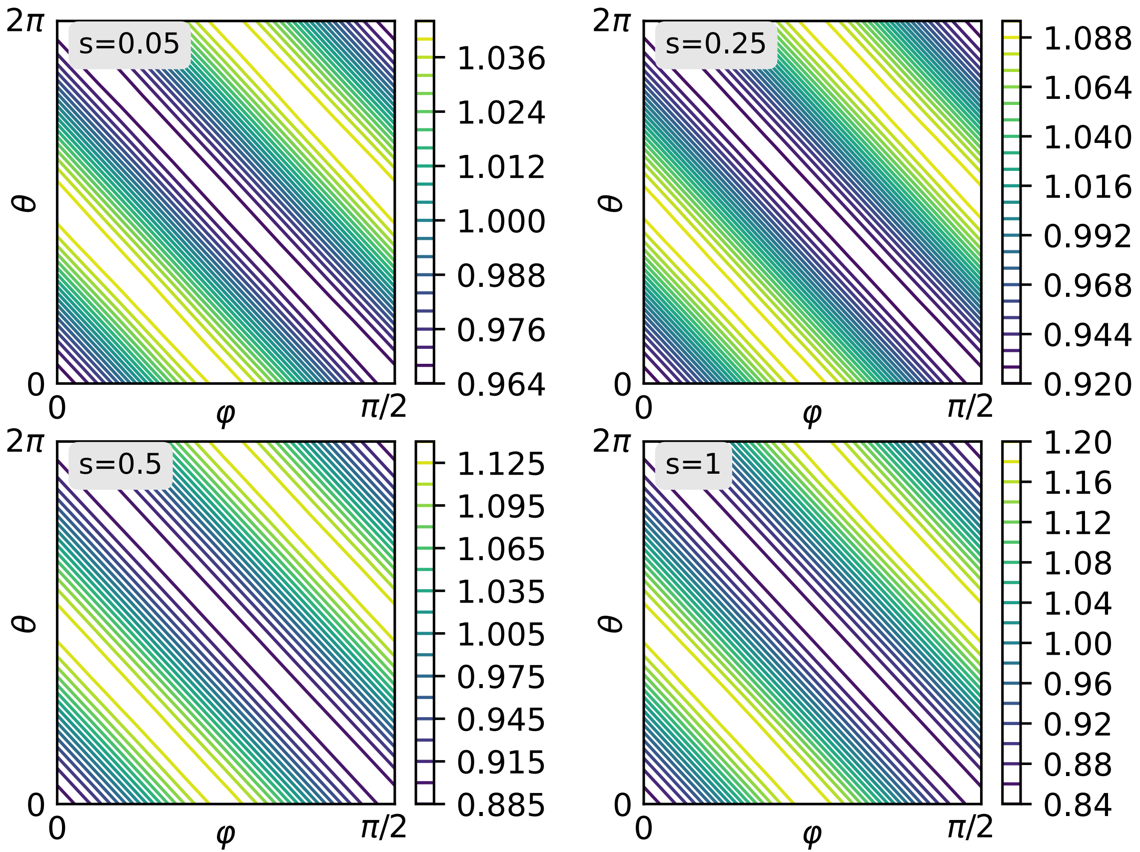
ML & Paul, PRL (2022).

Straight |B| contours are possible for quasi-helical symmetry



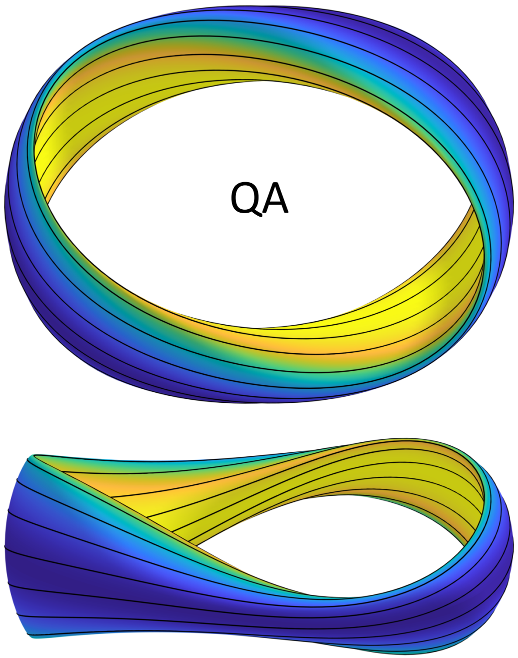
ML & Paul, PRL (2022).

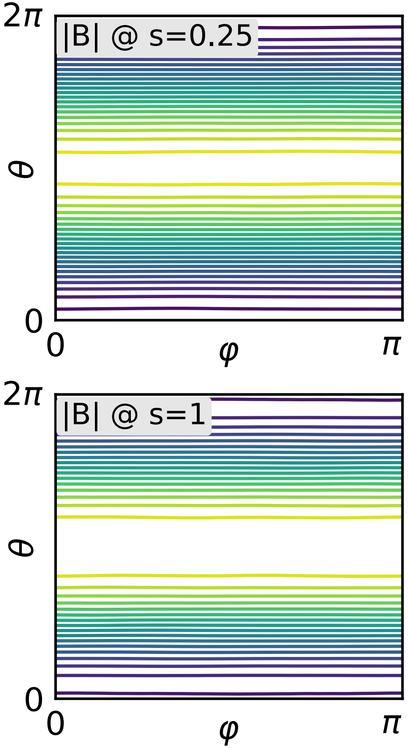
|B| on flux surfaces of the quasi-helically symmetric field

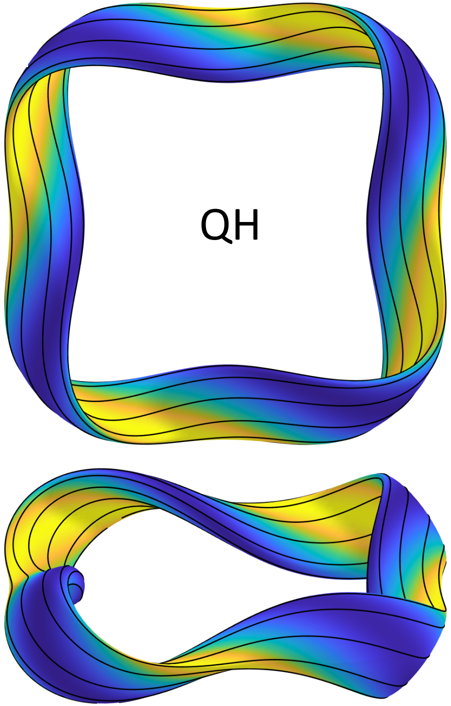


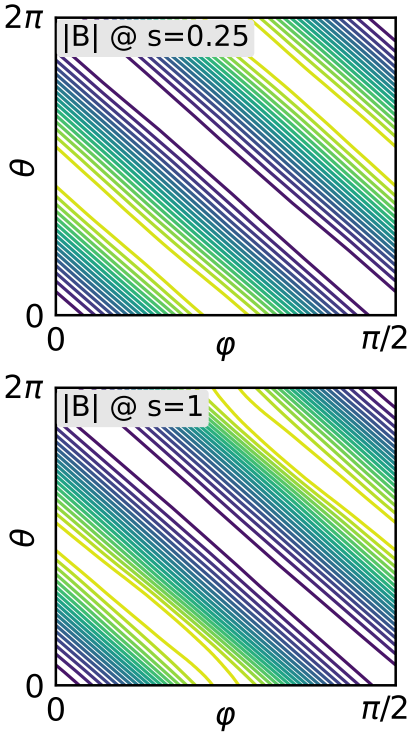
Good symmetry also exists with magnetic well

$$\frac{d^2\left(\text{flux surface volume}\right)}{d\left(\text{toroidal flux}\right)^2} < 0 \ \ \text{everywhere}$$



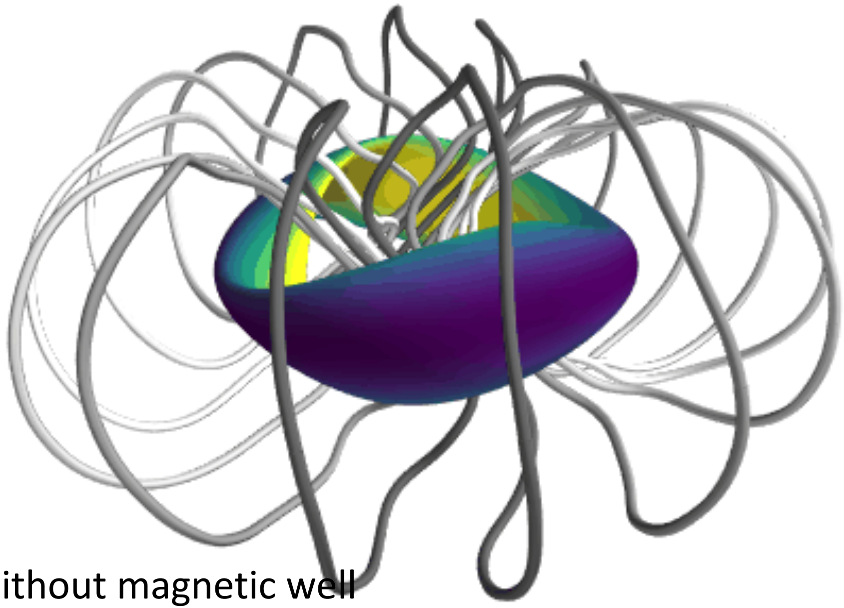




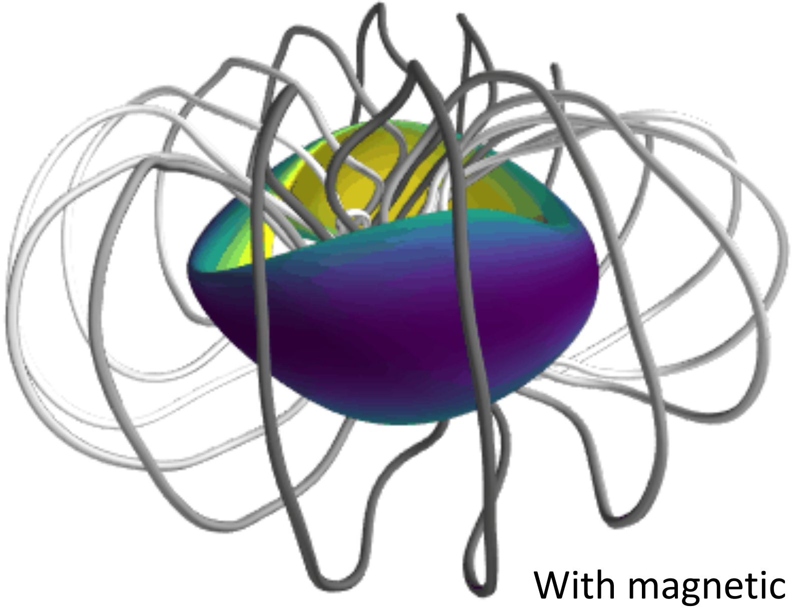


ML & Paul, PRL (2022).

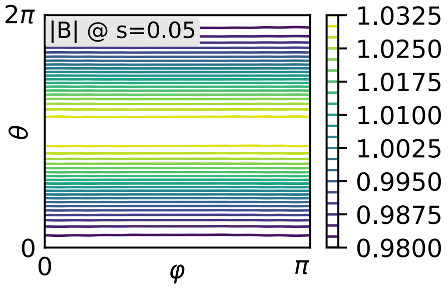
16-coil solutions have been found for the quasi-axisymmetric configurations

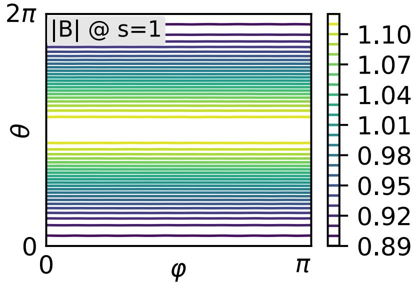


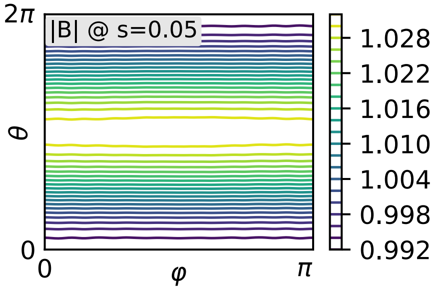
Without magnetic well

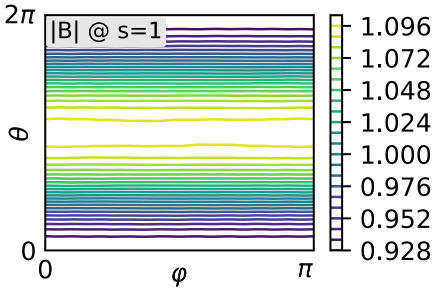


With magnetic well





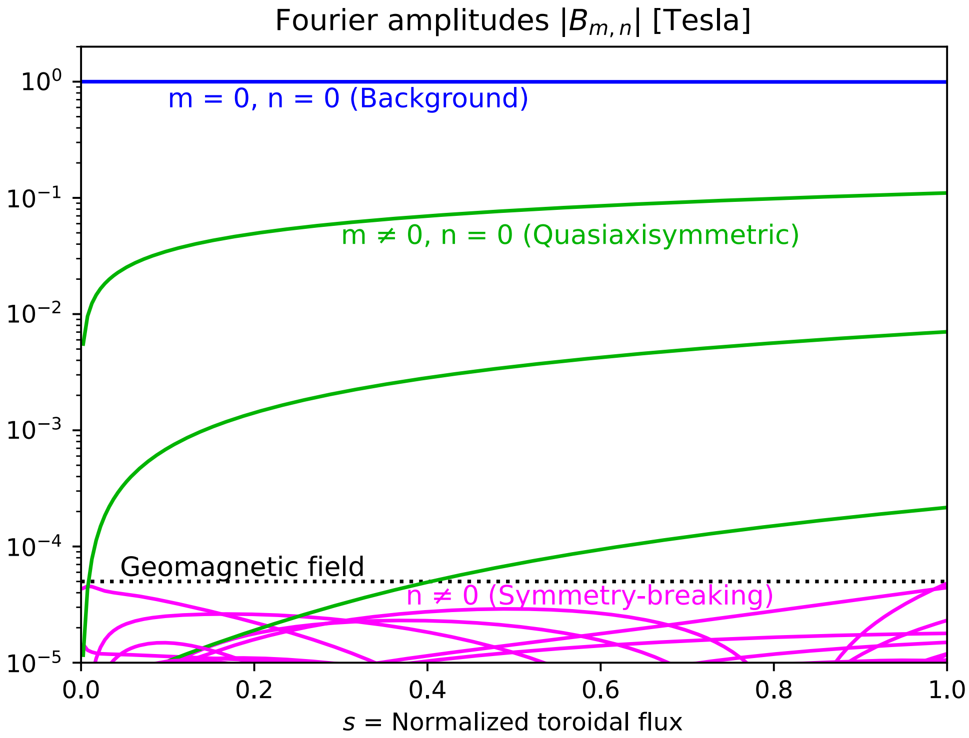


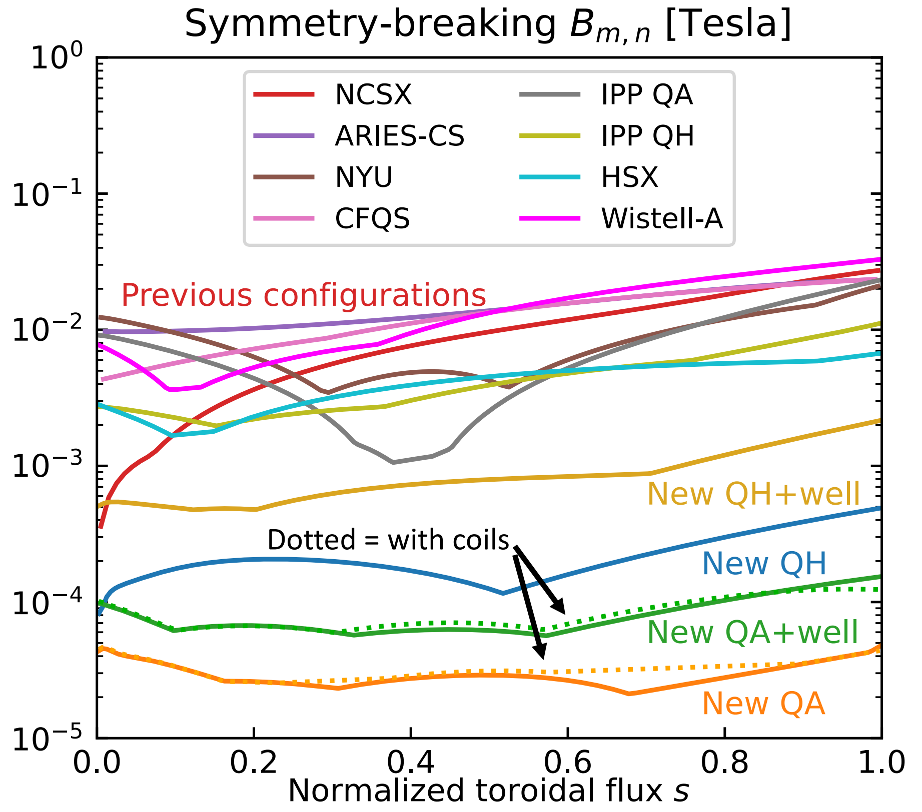


Wechsung et al, PNAS (2022). <R> / 10 between filament centers. Haven't looked at the QHs yet

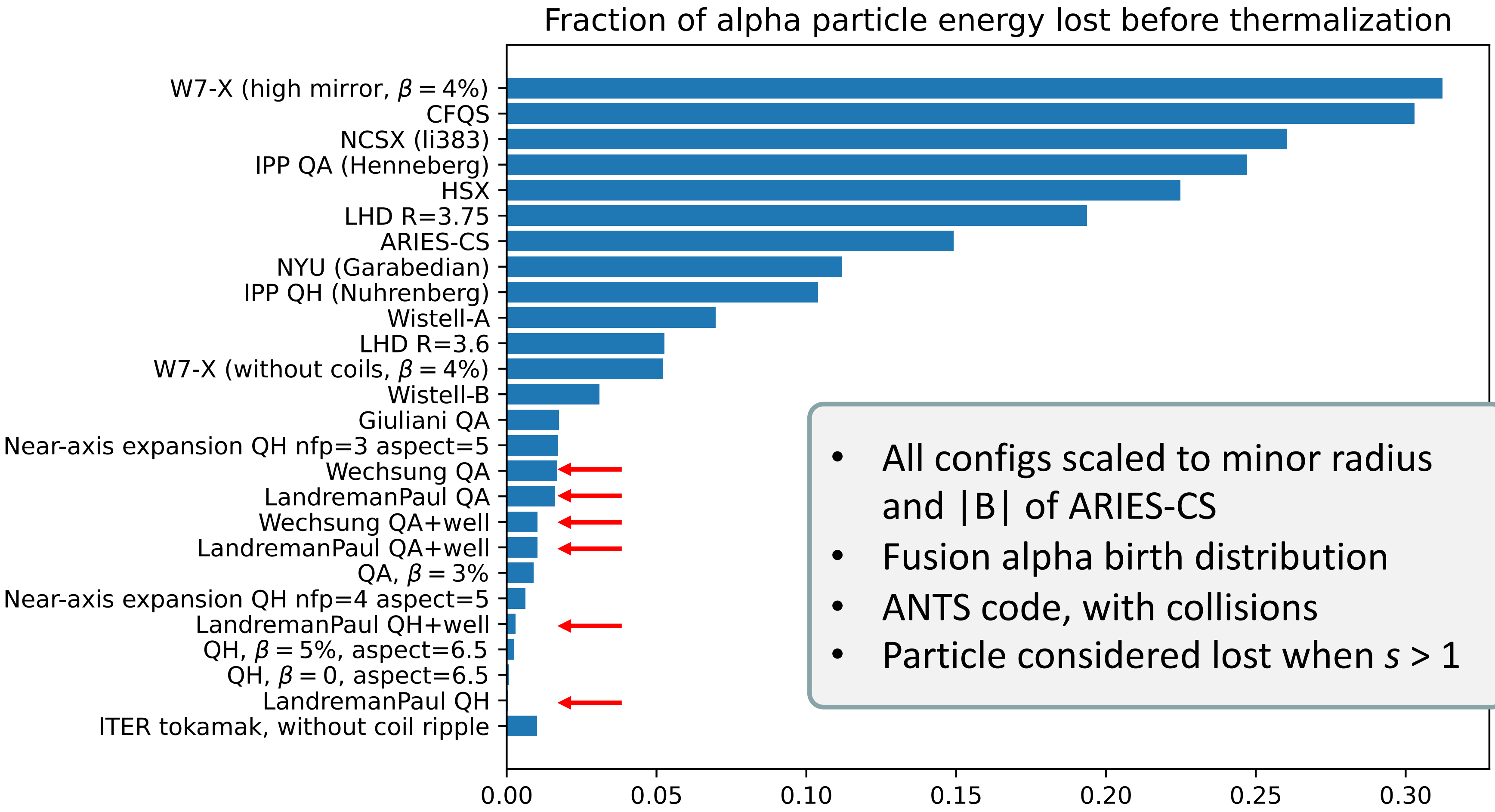
Symmetry-breaking modes can be made extremely small

New QA configuration





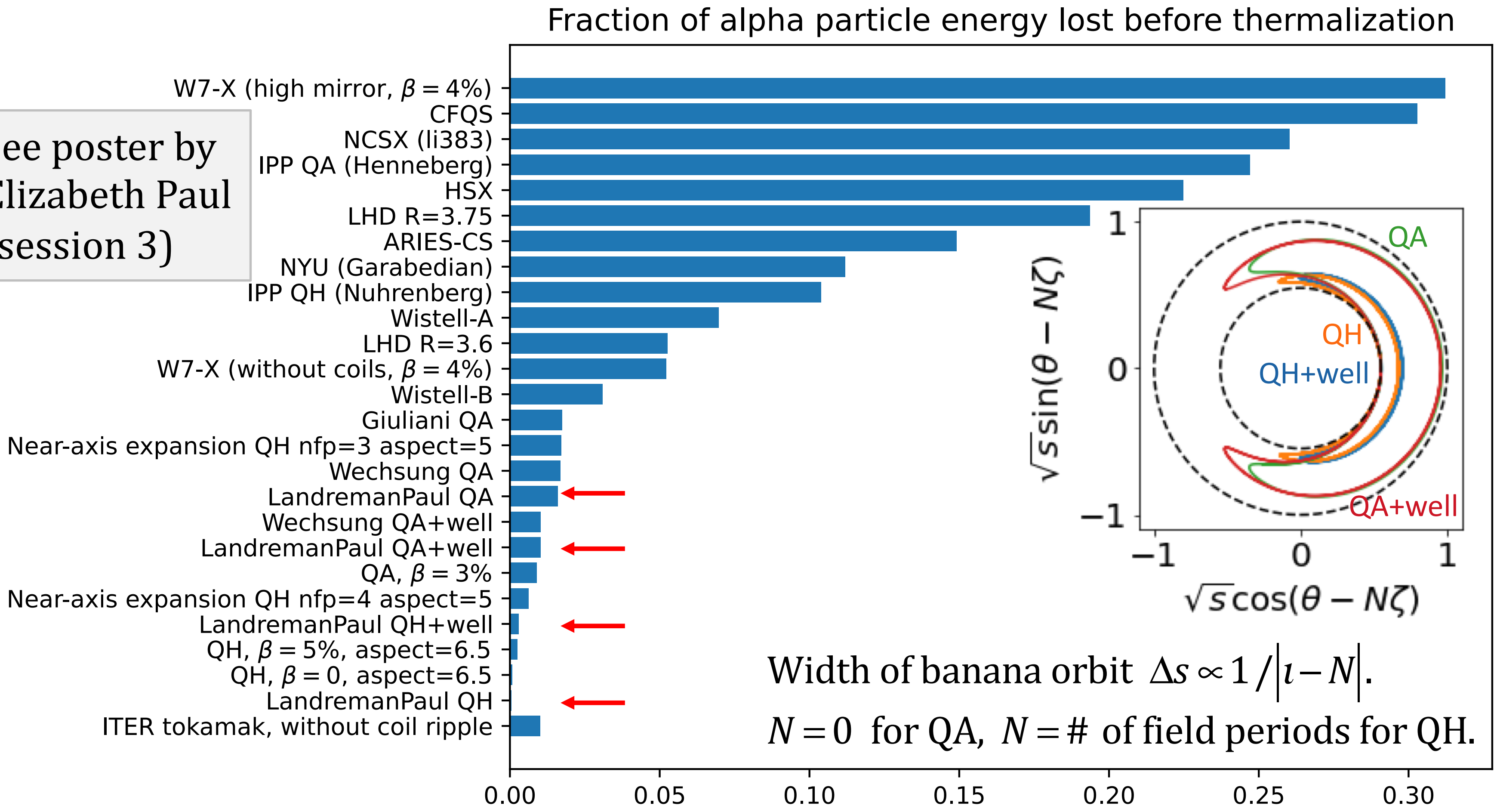
Quasisymmetry works: alpha particle confinement is significantly improved



- All configs scaled to minor radius and |B| of ARIES-CS
- Fusion alpha birth distribution
- ANTS code, with collisions
- Particle considered lost when $s > 1$

Alpha confinement in quasi-helical stellarators can be better than in a tokamak due to thinner bananas

See poster by Elizabeth Paul (session 3)



Width of banana orbit $\Delta s \propto 1/|\iota - N|$.

$N = 0$ for QA, $N = \#$ of field periods for QH.

- Optimizing stellarator geometry for precise quasisymmetry
- Self-consistent bootstrap current
- Conclusions

How can bootstrap current be included self-consistently in stellarator optimization?

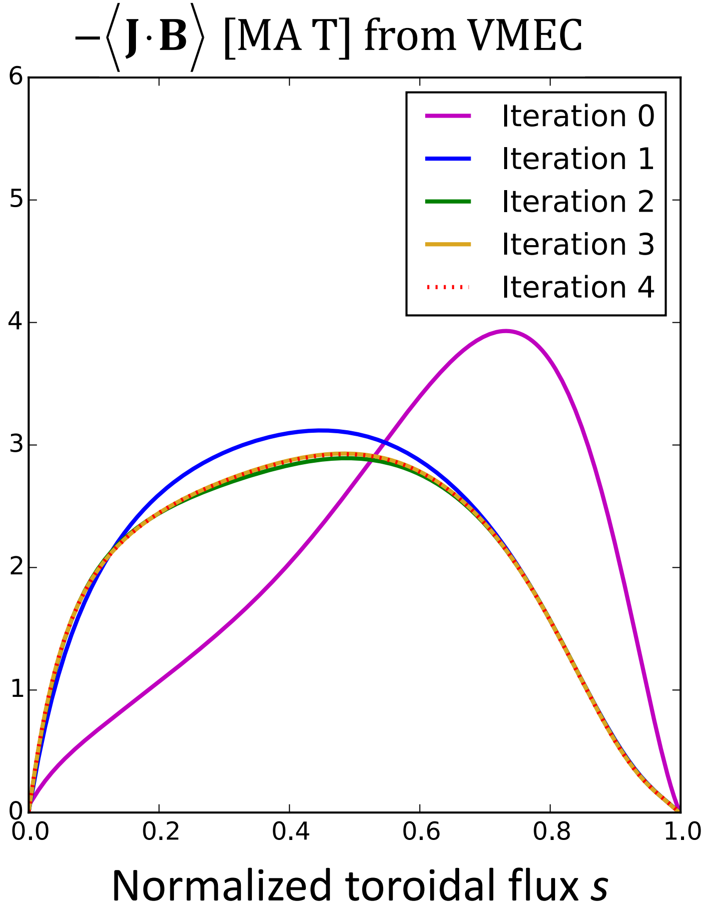
- Need *self-consistency* between MHD equilibrium and drift-kinetic equation.
- Previous method: fixed-point iteration, only after an optimization.

MHD equilibrium code

Drift-kinetic code

VMEC: given $I_0(s)$, determine $\mathbf{B}_0$.
SFINCS: given $\mathbf{B}_0$, determine $I_1(s)$.
VMEC: given $I_1(s)$, determine $\mathbf{B}_1$.
SFINCS: given $\mathbf{B}_1$, determine $I_2(s)$.
…

- Accurate drift-kinetic bootstrap calculations in stellarators are computationally expensive. Preferably not in the optimization loop.



New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Pytte & Boozer (1981), Boozer (1983):

$$\iota \rightarrow \iota - N$$

Bootstrap current (& other quantities) in quasisymmetry are the same as in axisymmetry, up to some substitutions:

Should be accurate for the new precisely quasisymmetric configurations.

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A new set of analytical formulae for the computation of the bootstrap current and the neoclassical conductivity in tokamaks

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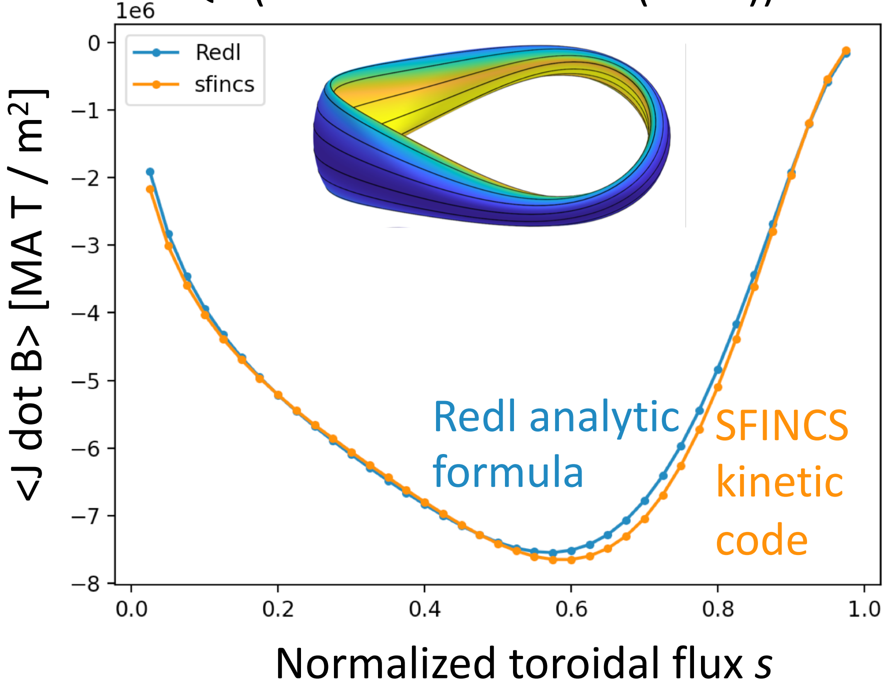
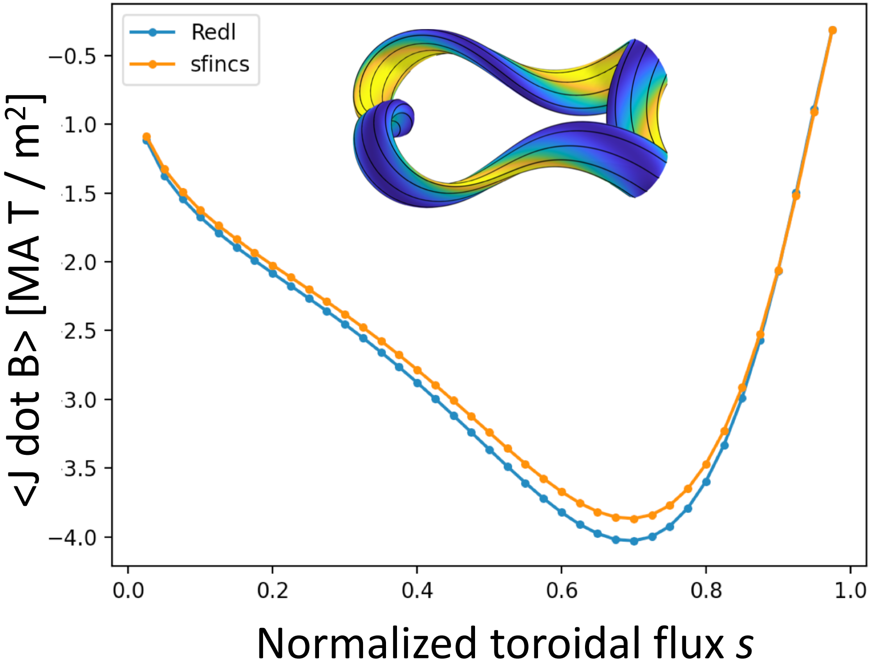
Before doing new optimizations: Redl formula is accurate in previous QA & QH stellarators

$n_e = (1 - s^5)\ 4\text{x}10^{20}\ \text{m}^{-3}, \quad T_e = T_i = (1 - s)\ 12\ \text{keV}$

QA (Landreman & Paul (2021))



QH (Landreman & Paul (2021))



(Not self-consistent yet)

Optimization recipe

- Objective function: $$f = f_{QS} + f_{bootstrap} + \left(A - 6.5\right)^2 + \left(a - a_{ARIES-CS}\right)^2 + \left(\langle B \rangle - \langle B \rangle_{ARIES-CS}\right)^2$$

Boundary aspect ratio

Minor radius

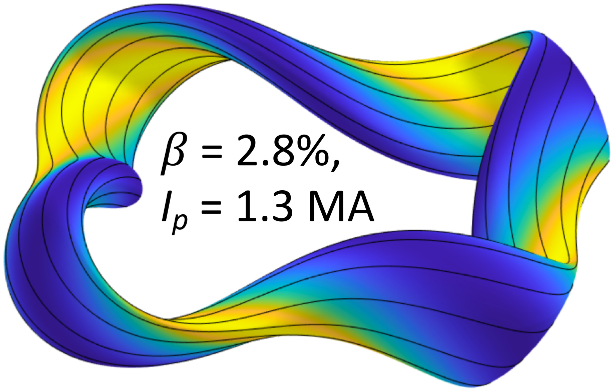
$$f_{QS} = \int d^3x \left(\frac{1}{B^3} \left[\left(N - \iota\right) \mathbf{B} \times \nabla B \cdot \nabla \psi - \left(G + NI\right) \mathbf{B} \cdot \nabla B \right] \right)^2$$

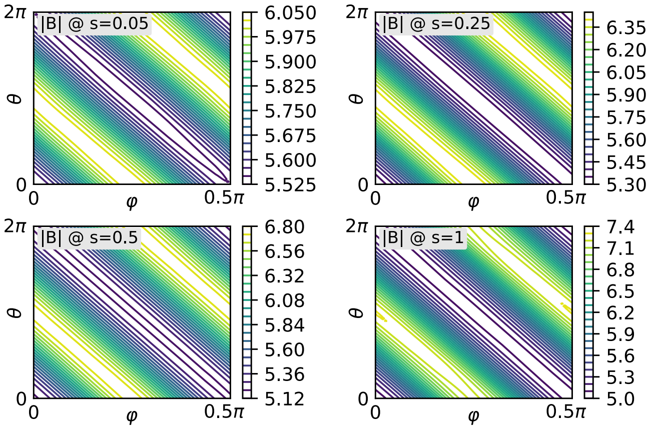
$$f_{bootstrap} = \frac{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{vmec}} - \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Redl}} \right]^2}{\int_0^1 ds \left[\langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{vmec}} + \langle \mathbf{j} \cdot \mathbf{B} \rangle_{\text{Redl}} \right]^2}$$

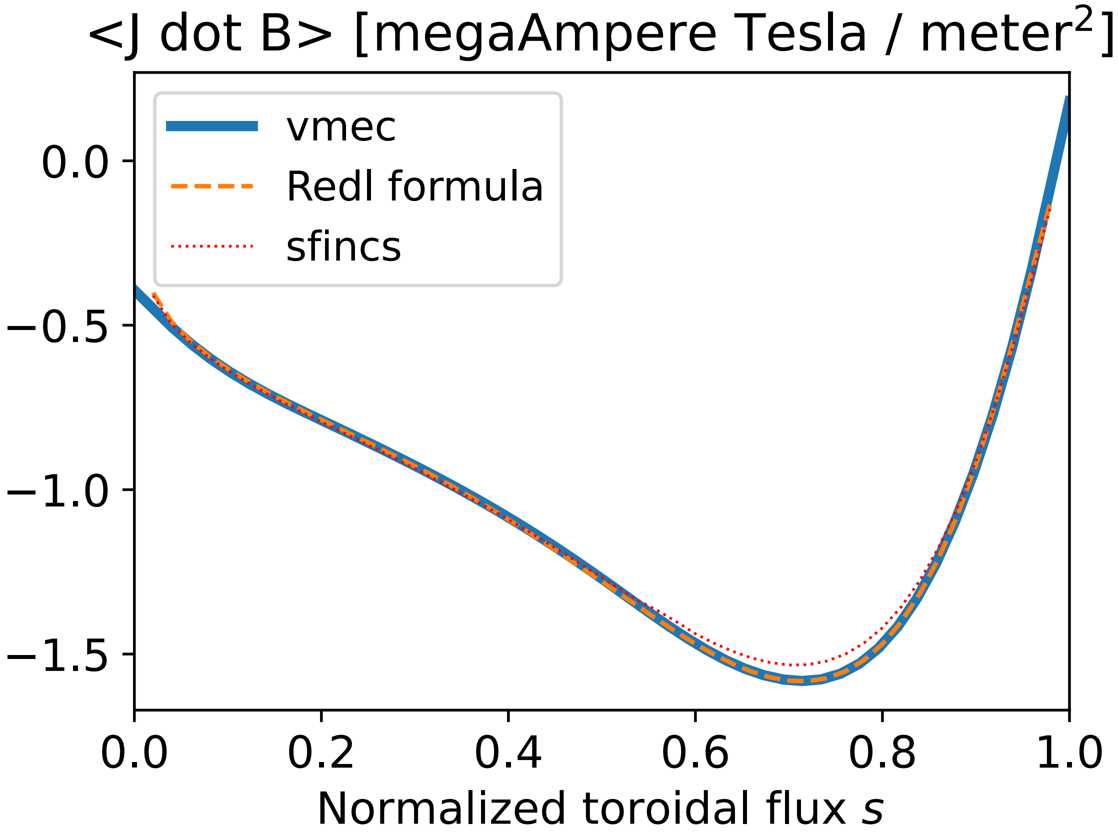
- Parameter space: {$R_{m,n}$, $Z_{m,n}$, toroidal flux, current spline values}
  or {$R_{m,n}$, $Z_{m,n}$, toroidal flux, iota spline values}
- Cold start
- Algorithm: default for least-squares in scipy (trust region reflective)
- Steps: increasing # of modes varied: m and |n/nfp| up to j in step j

Example of optimization with self-consistent bootstrap current

n_{e0} = 2.5e20/meters^3
$T_{e0} = T_{i0}$ = 10 keV

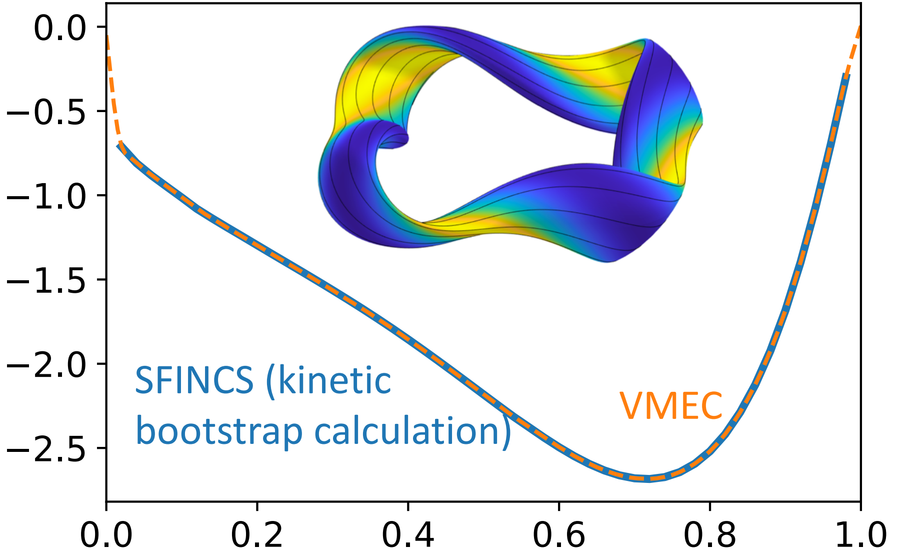






If you want *perfectly* self-consistent current, you can do a few fixed-point iterations at the end

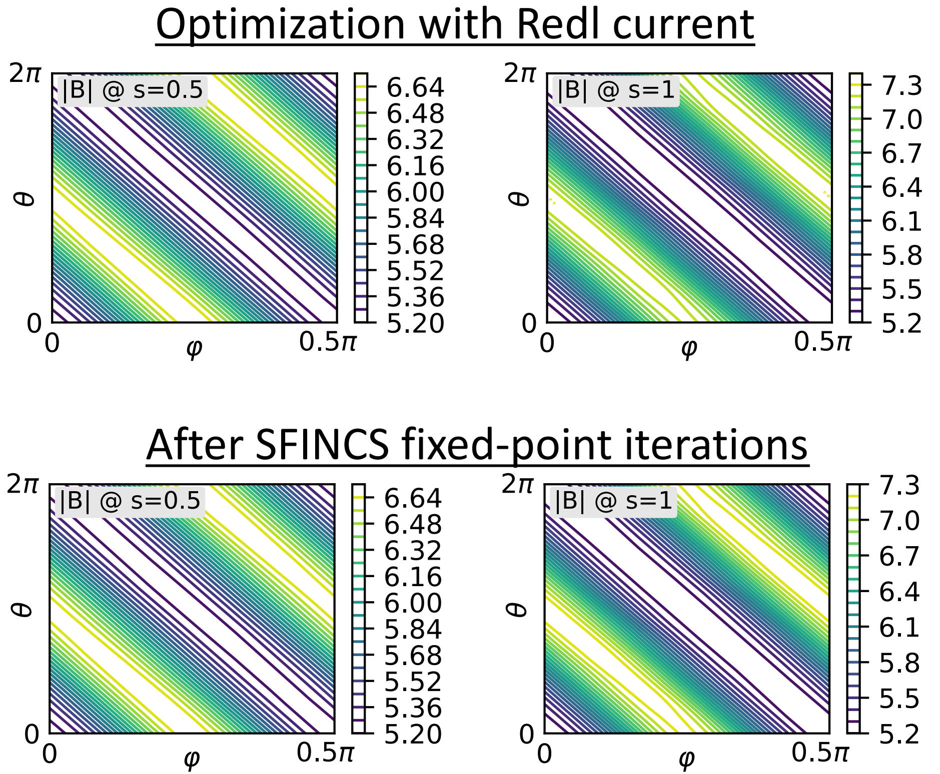
<J dot B> [megaAmpere Tesla / meter^2]



$$\langle \beta \rangle = 5\%$$

α energy losses < 0.3%

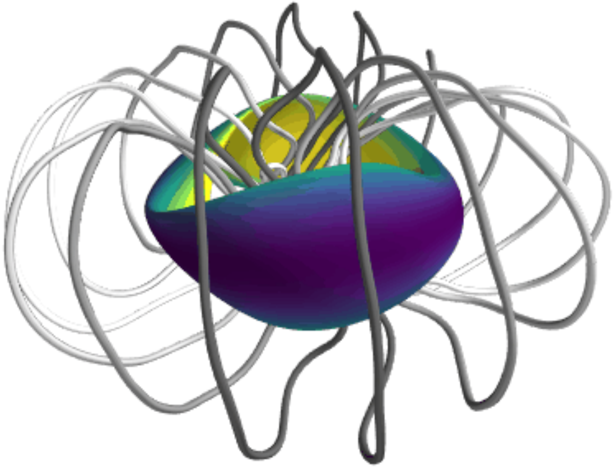
No significant degradation in quasisymmetry:

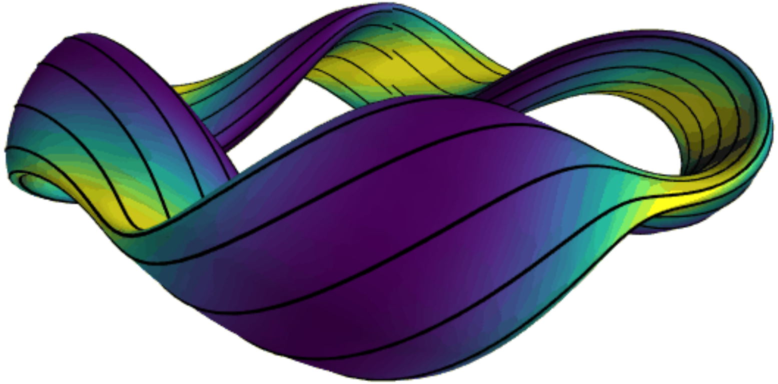


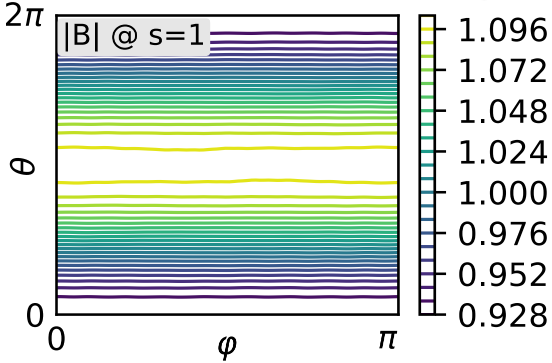
- Optimizing stellarator geometry for precise quasisymmetry
- Self-consistent bootstrap current
- Conclusions

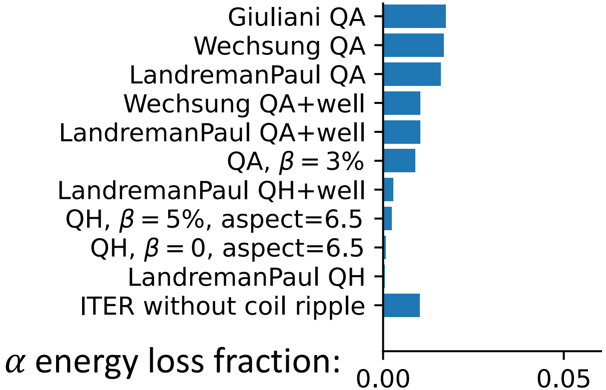
It is now possible to design stellarators with alpha confinement comparable to or better than a tokamak.

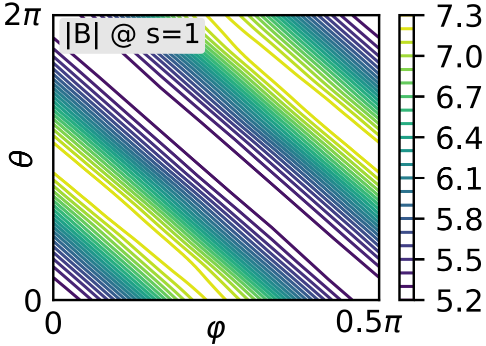
More to do: coils, surface quality, & MHD stability at high β, robustness to pressure profile, ...







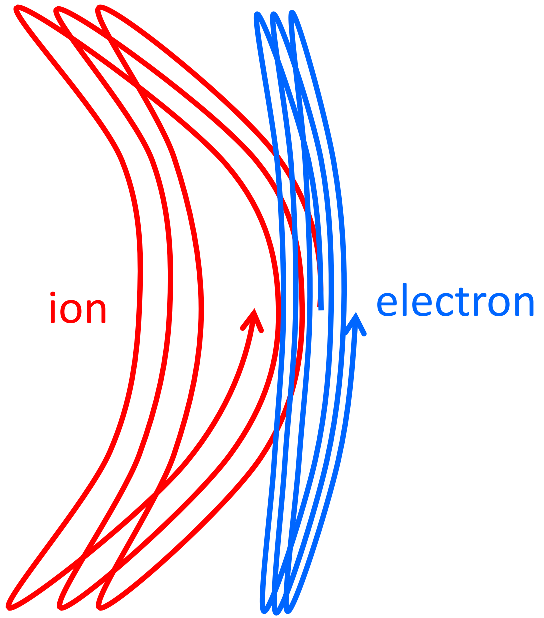




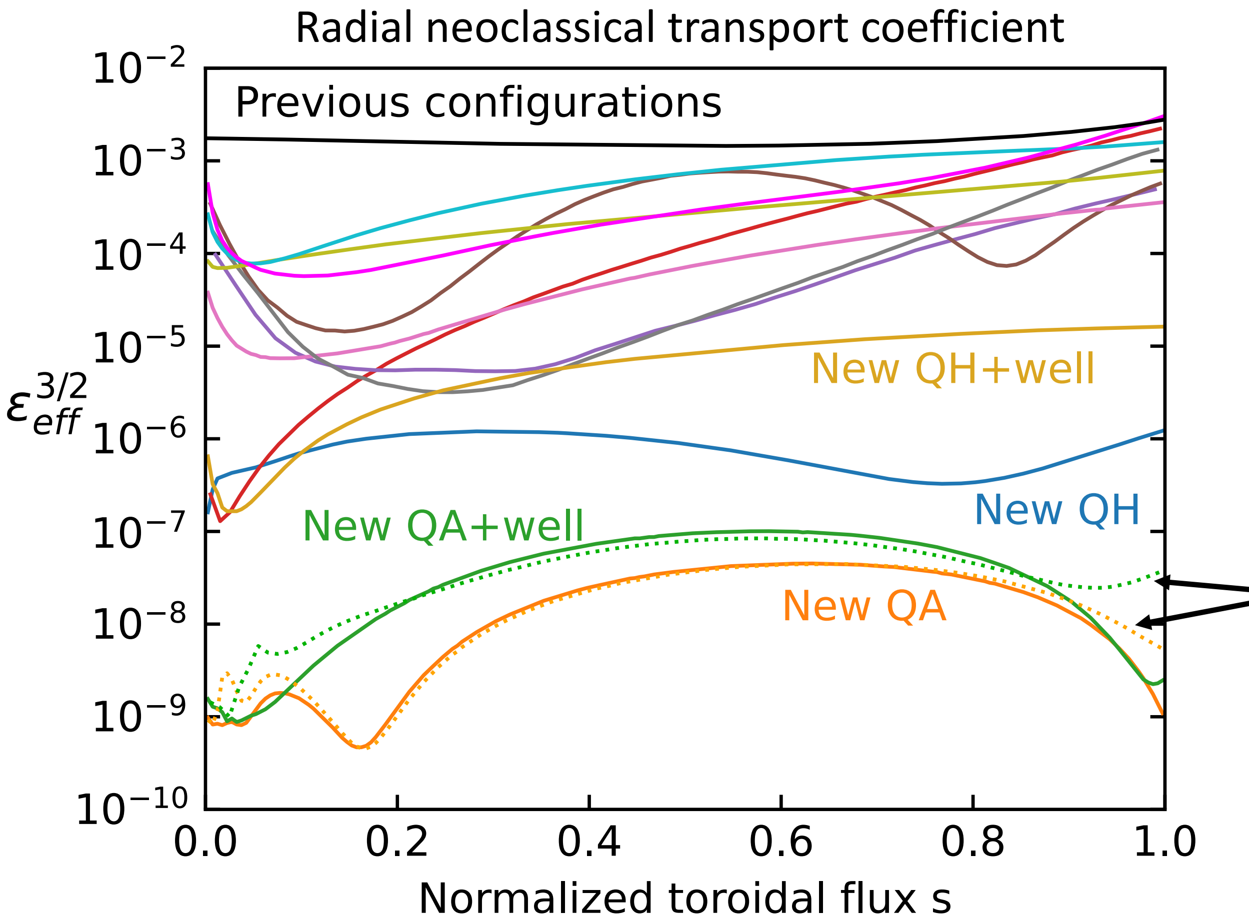
Extra slides

The bootstrap current arises in tokamaks & stellarators when the density & temperature become significant

- Ions and electrons have different trajectories. Different mean flows = electric current.
- Current depends on geometry, density, & temperature.
- For $\beta > 0$, we don't know **B** until we include this effect.
- Computing the bootstrap current requires kinetic theory: coupled 4D advection-dominated integro-differential equations.
- Need *self-consistency* between MHD equilibrium and kinetic equation.
- How can a self-consistent bootstrap current calculation be integrated with stellarator optimization?



The symmetry also yields extremely low collisional transport for a thermal plasma

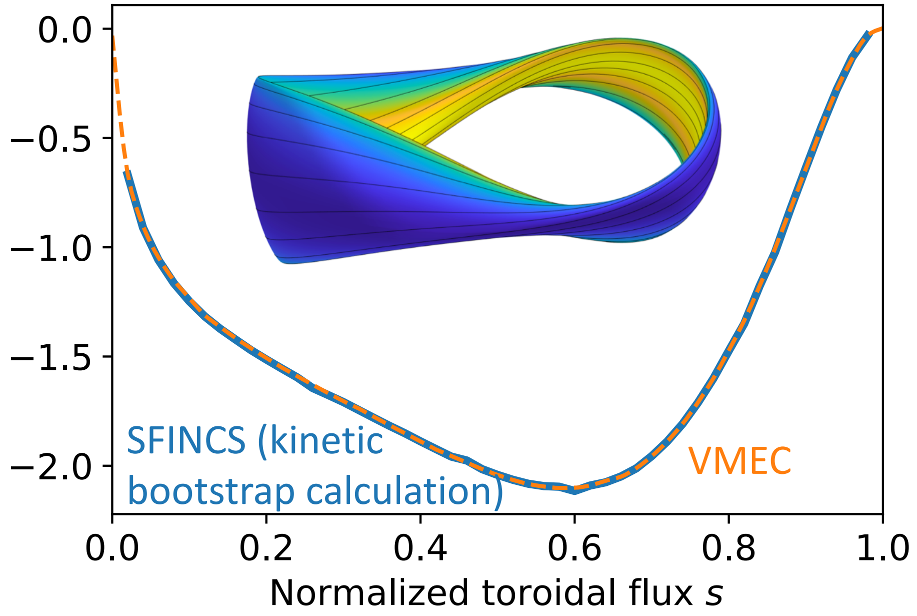


Dotted = with coils

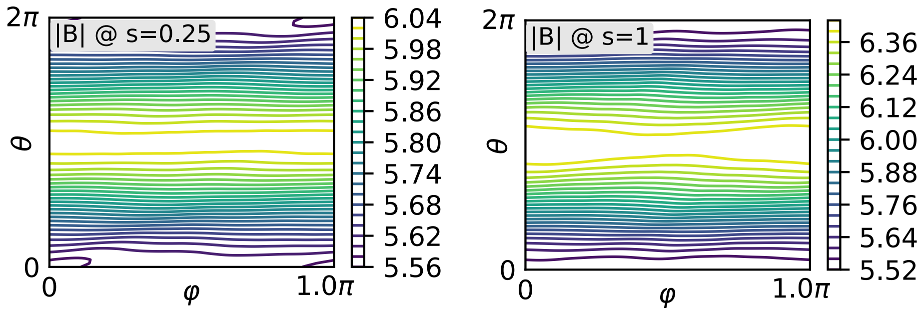
Wechsung et al, PNAS (2022)

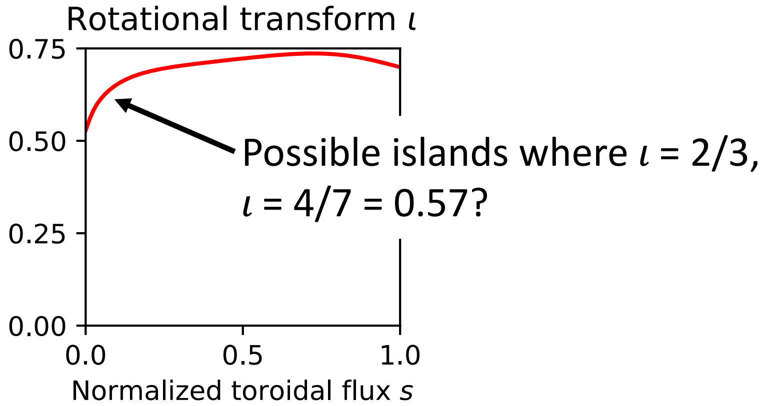
The optimization with self-consistent bootstrap current also works for quasi-axisymmetry

<J dot B> [megaAmpere Tesla / meter^2]



Symmetry is not as good as for vacuum, but sufficient for excellent confinement

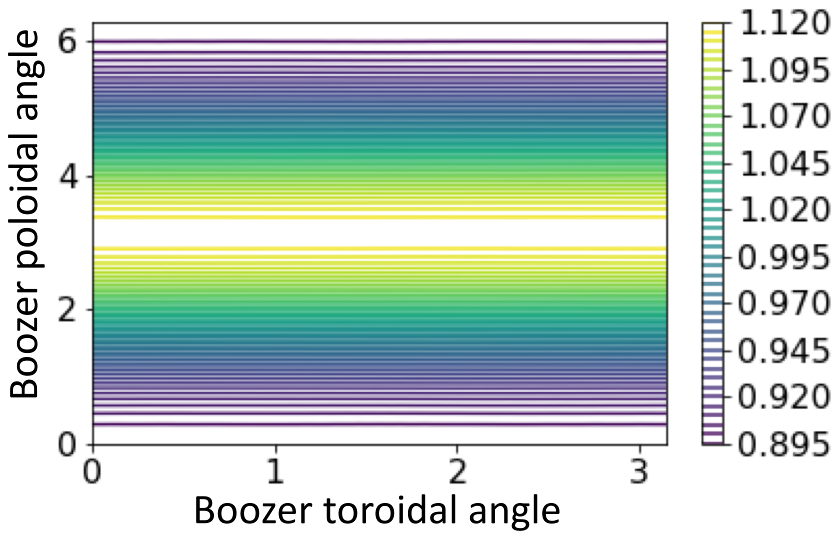




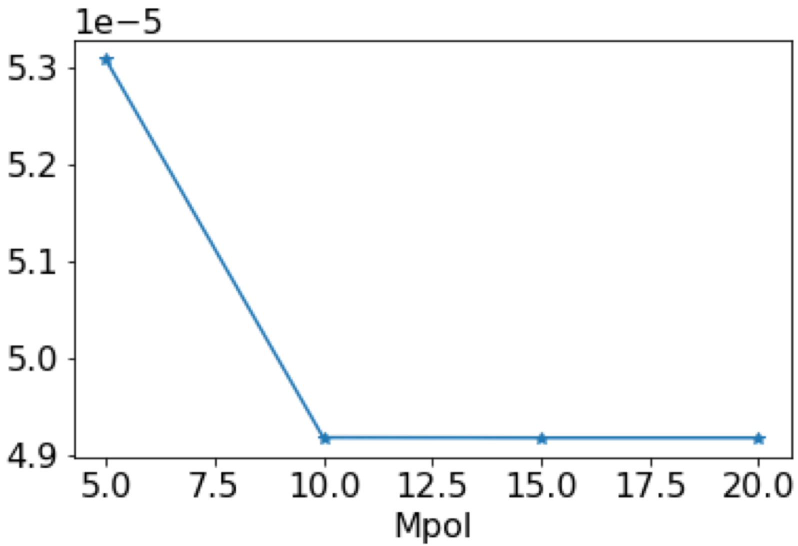
$$\langle\beta\rangle = 3\%, \qquad \varepsilon_{eff}^{3/2} < 7\times10^{-6}$$

α-particle losses < 1%

|B| in Boozer coordinates was verified by independent SPEC calculations



max $|B_{m,n}|$ [Tesla] with n ≠ 0

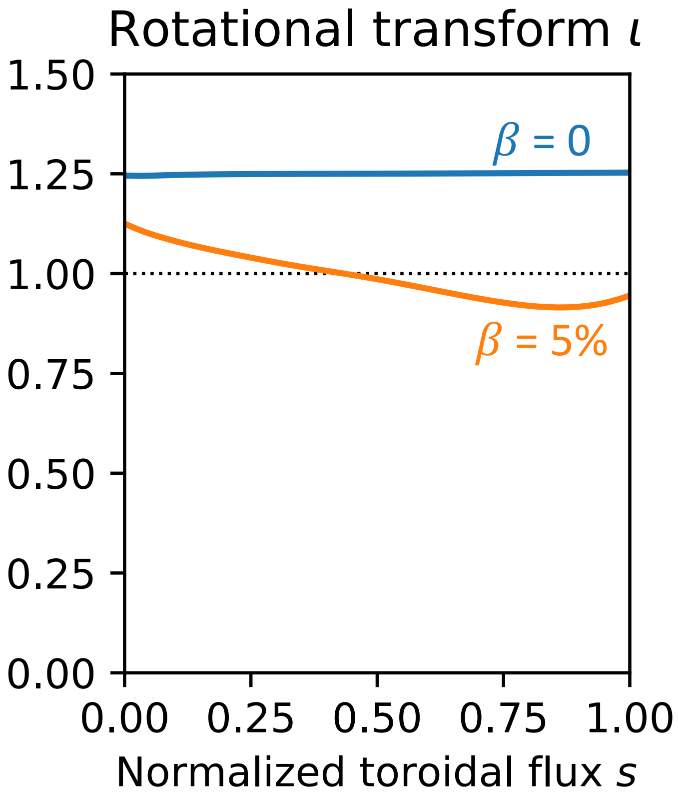


(Ntor = Mpol, Lrad = Mpol + 4)

By Elizabeth Paul

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

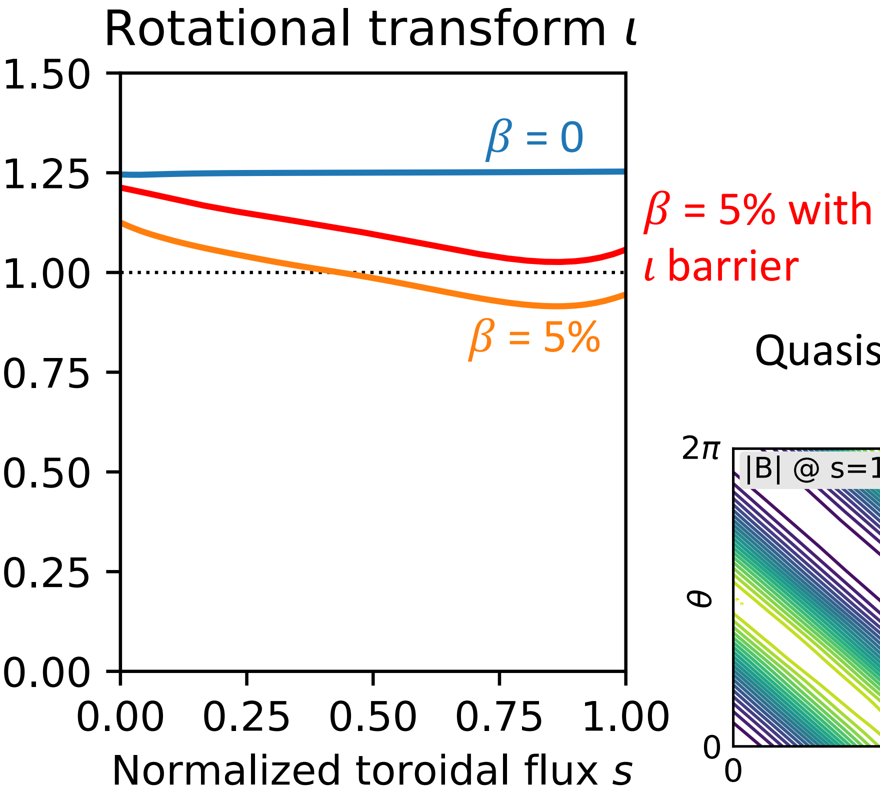
Crossing iota=1, the worst resonance, is probably unacceptable.



n_{e0} = 3e20/meters^3, $T_{e0} = T_{i0}$ = 15 keV

To reach reactor-relevant 5% beta in QH without crossing iota=1, a constraint on iota can be included

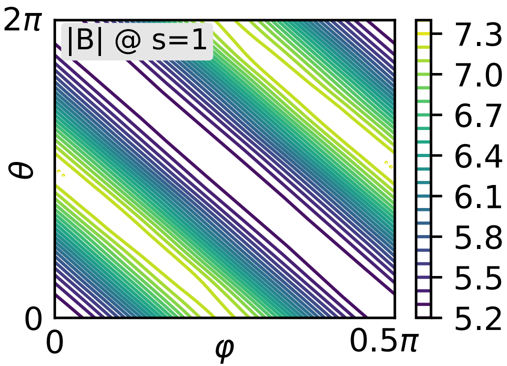
Crossing iota=1, the worst resonance, is probably unacceptable.

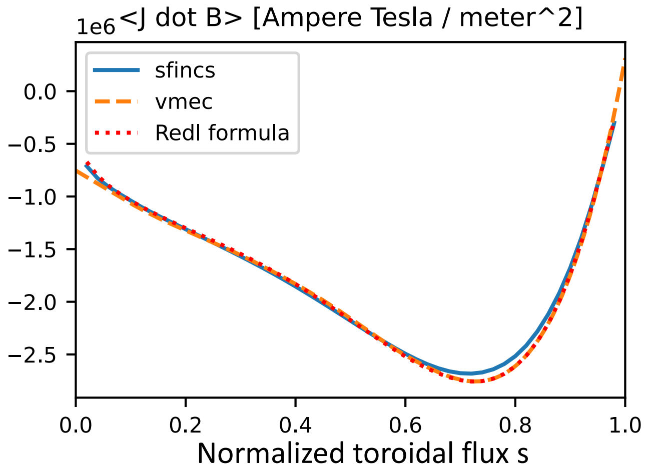


Solution: Add barrier term in objective

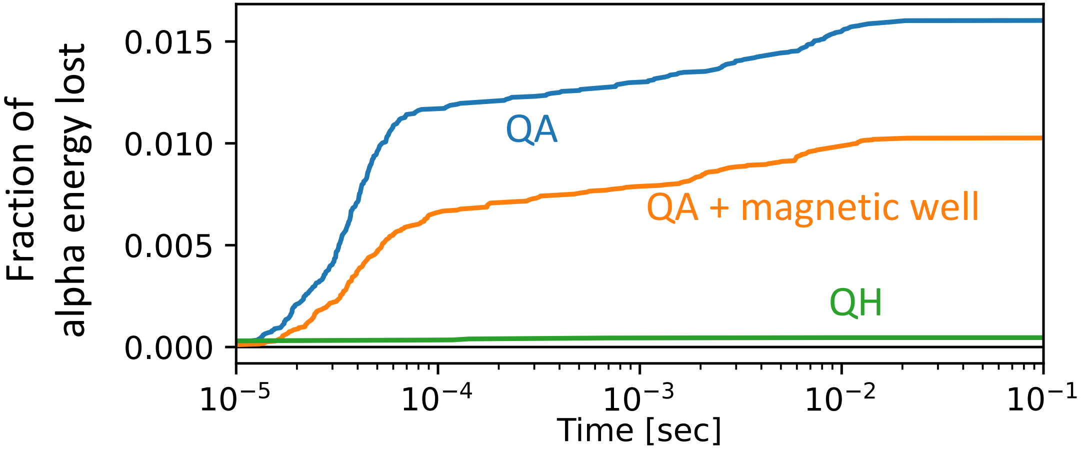
$$f \mathrel{+}= \int_0^1 ds \left[\min\left(\left| \iota(s) \right| - 1.03,\ 0 \right) \right]^2$$

Quasisymmetry & bootstrap consistency remain good:

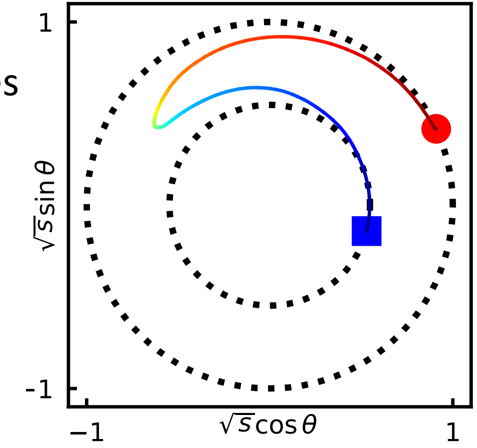




Why do the configurations with best quasisymmetry not have the best trajectory confinement?



Lost trajectories in the new QA look like this:



$$\text{Width of banana orbit } \Delta s \approx \left| \frac{mvR\sqrt{2r\bar{\eta}}}{(\iota - N)\psi_{edge} Ze} \right| \propto \frac{1}{|\iota - N|}$$

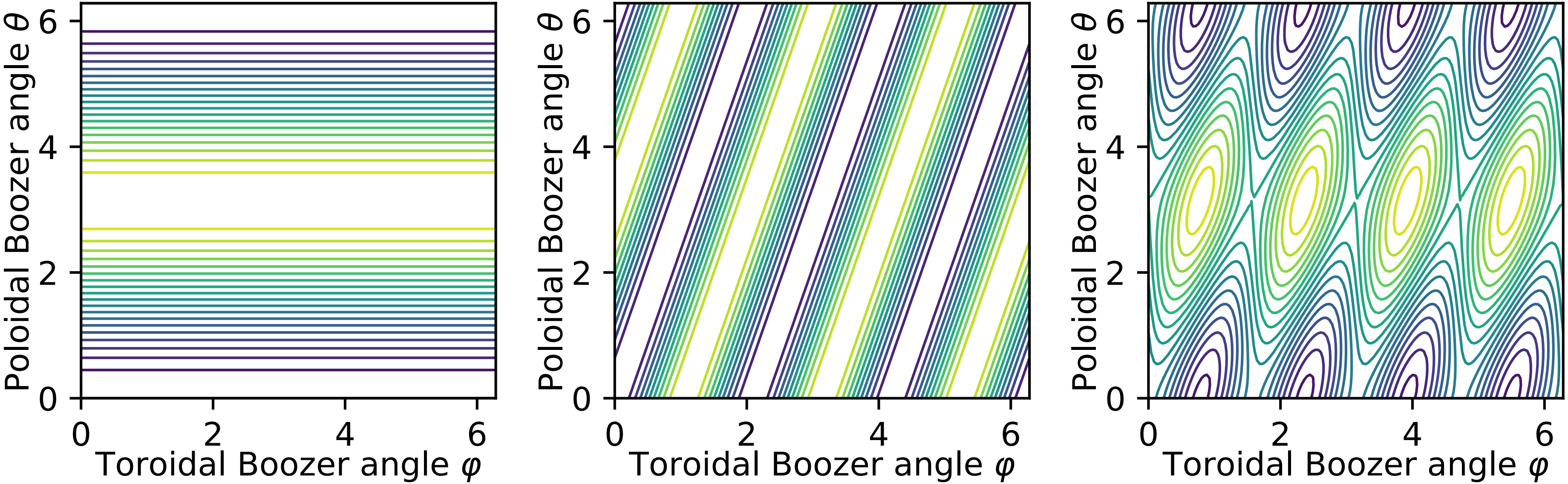
$$\text{For fixed minor radius, } \frac{\Delta s_{QA}}{\Delta s_{QH}} \sim 4$$

2 types of quasisymmetry

Quasi-axisymmetry (QA): $B = B(r, \theta)$

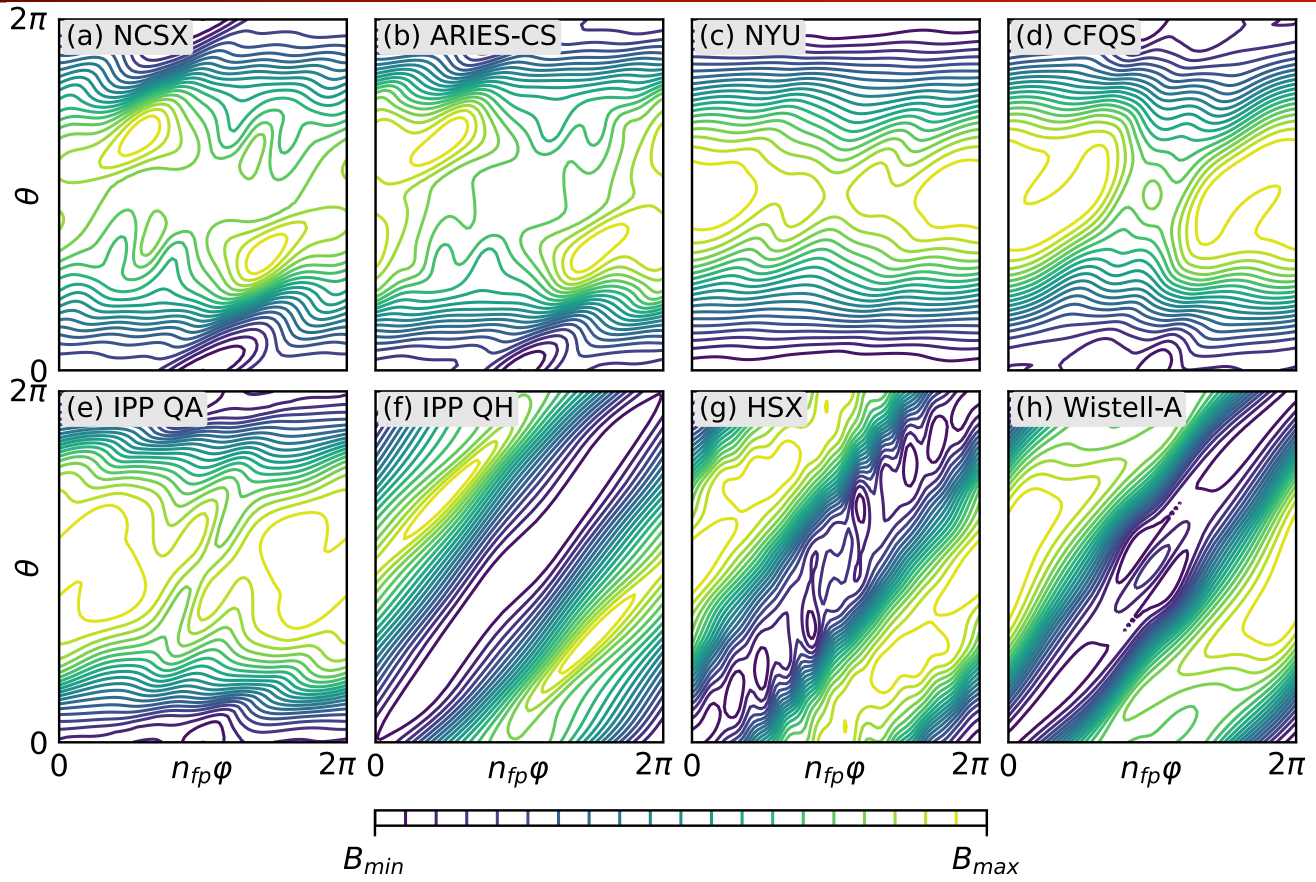
Quasi-helical symmetry (QH): $B = B(r, \theta - N\varphi)$

General stellarator (not symmetric)



Contours of $B = |\mathbf{B}|$: B_{min}  B_{max}

Previous quasisymmetric configurations

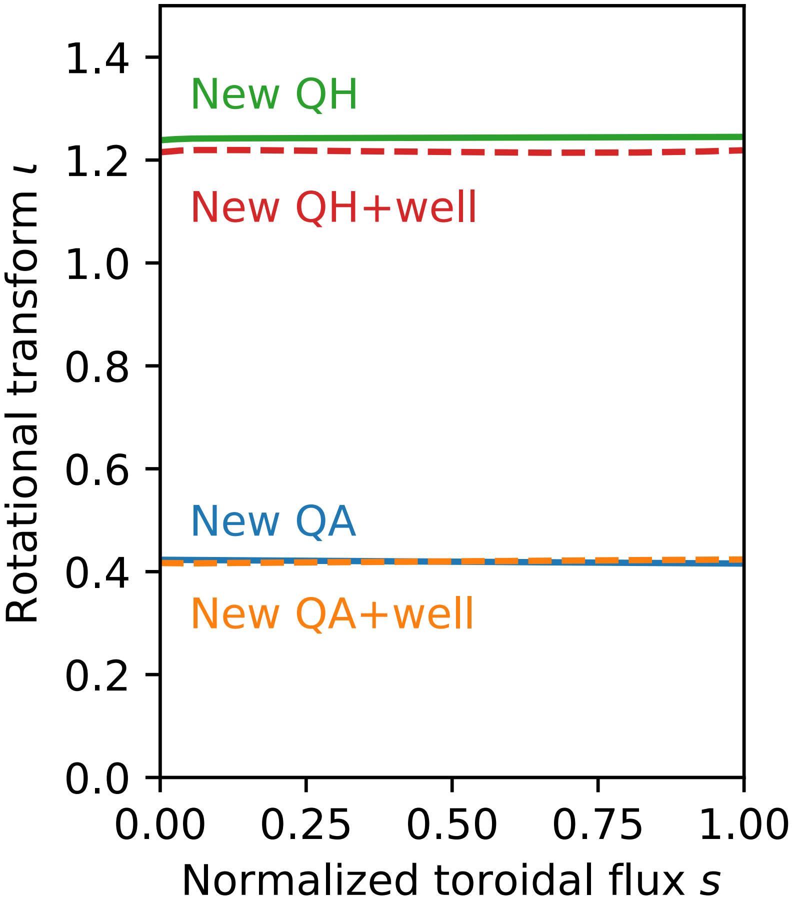


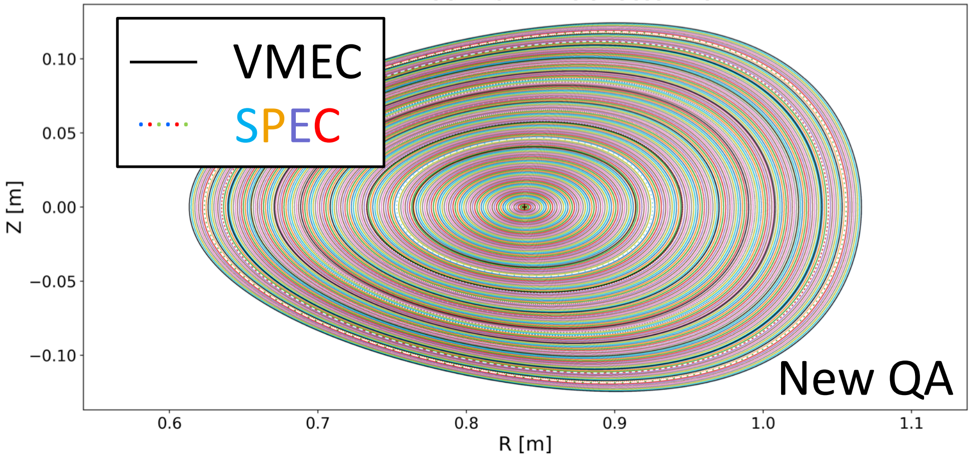
(a) Zarnstorff et al (2001)
(b) Najambadi et al (2008)
(c) Garabedian (2008)
(d) Liu et al (2018)
(e) Henneberg et al (2019)
(f) Nuhrenberg & Zille (1988)
(g) Anderson et al (1995)
(h) Bader et al (2020)

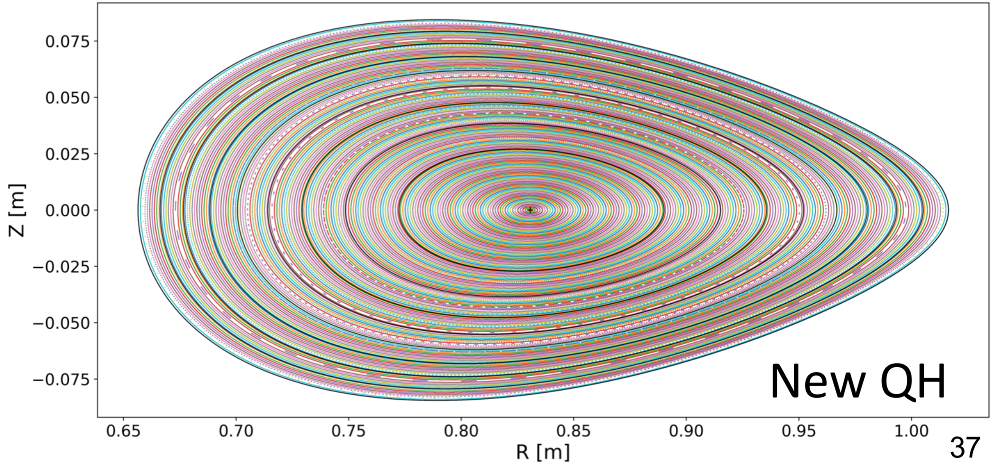
We want
$B = B(r, \theta - N\,\varphi)$

Is there an optimization recipe that can give consistently straight |B| contours?

The new configurations have small magnetic shear

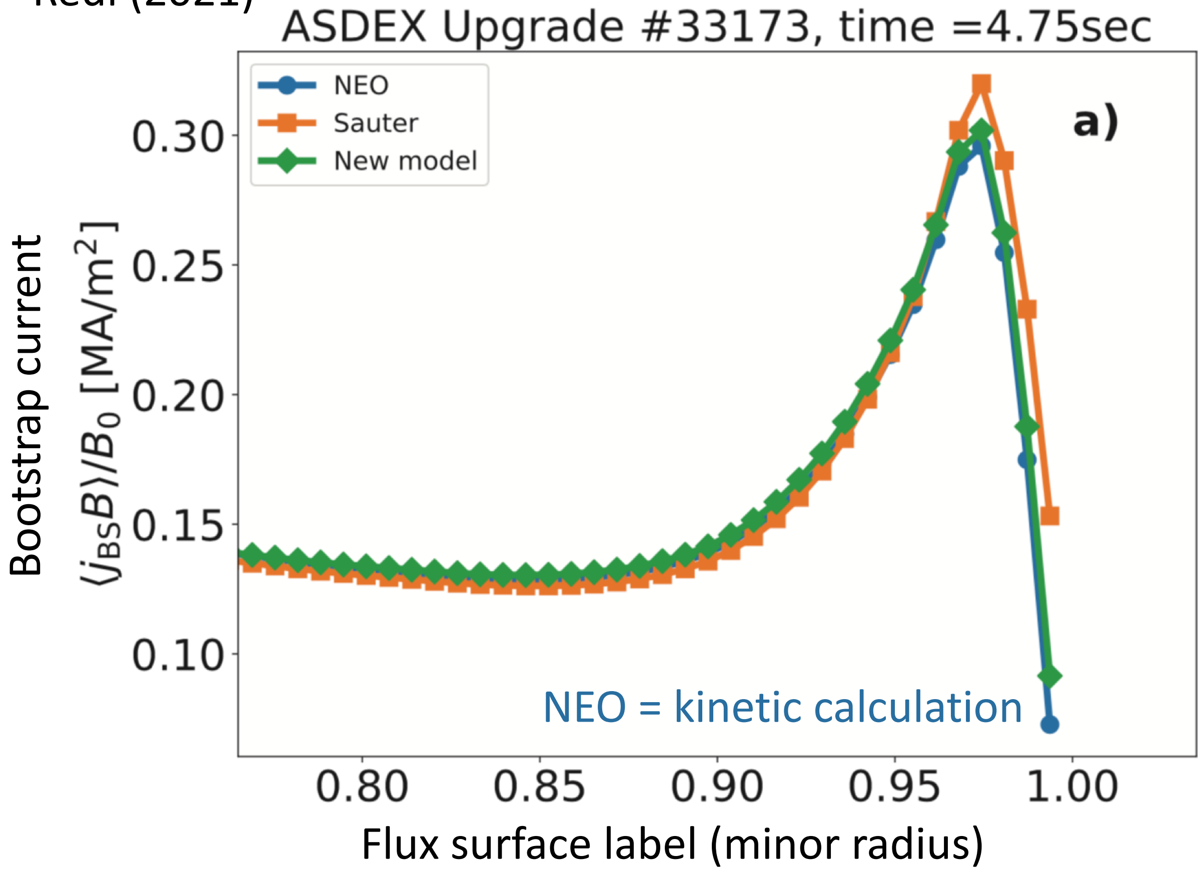






New idea: exploit quasisymmetry & use analytic expressions for tokamaks

Redl (2021)



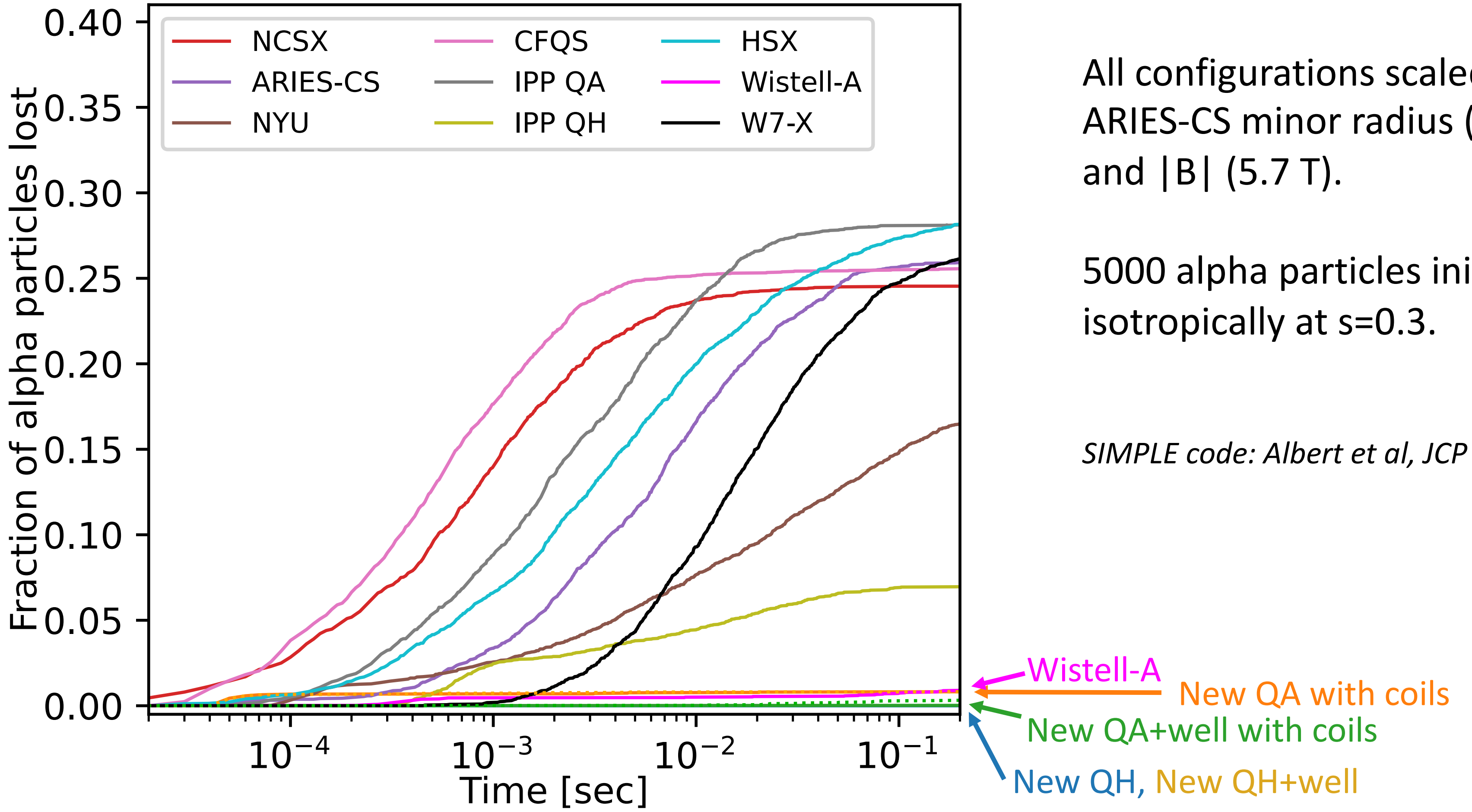
Geometry enters through

$$f_t = 1 - f_c = 1 - \frac{3}{4}\langle B^2\rangle \int_0^{1/B_{\max}} \frac{\lambda d\lambda}{\langle \sqrt{1-\lambda B}\rangle}.$$

$$\nu_{e*} = 6.921 \cdot 10^{-18} \frac{qRn_e Z \ln \Lambda_e}{T_e^2 \epsilon^{3/2}},$$

$$\nu_{i*} = 4.90 \cdot 10^{-18} \frac{qRn_i Z^4 \ln \Lambda_{ii}}{T_i^2 \epsilon^{3/2}},$$

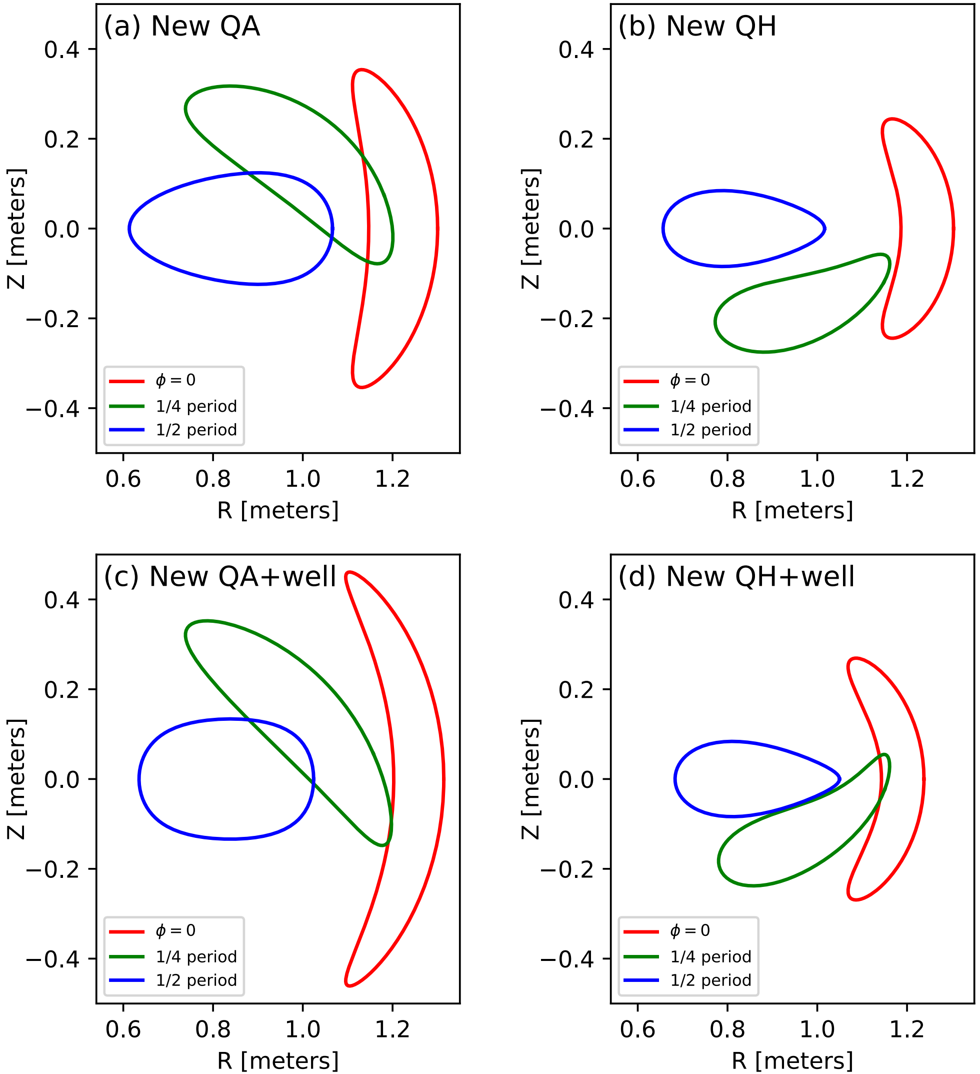
The symmetry yields extremely good confinement of collisionless trajectories



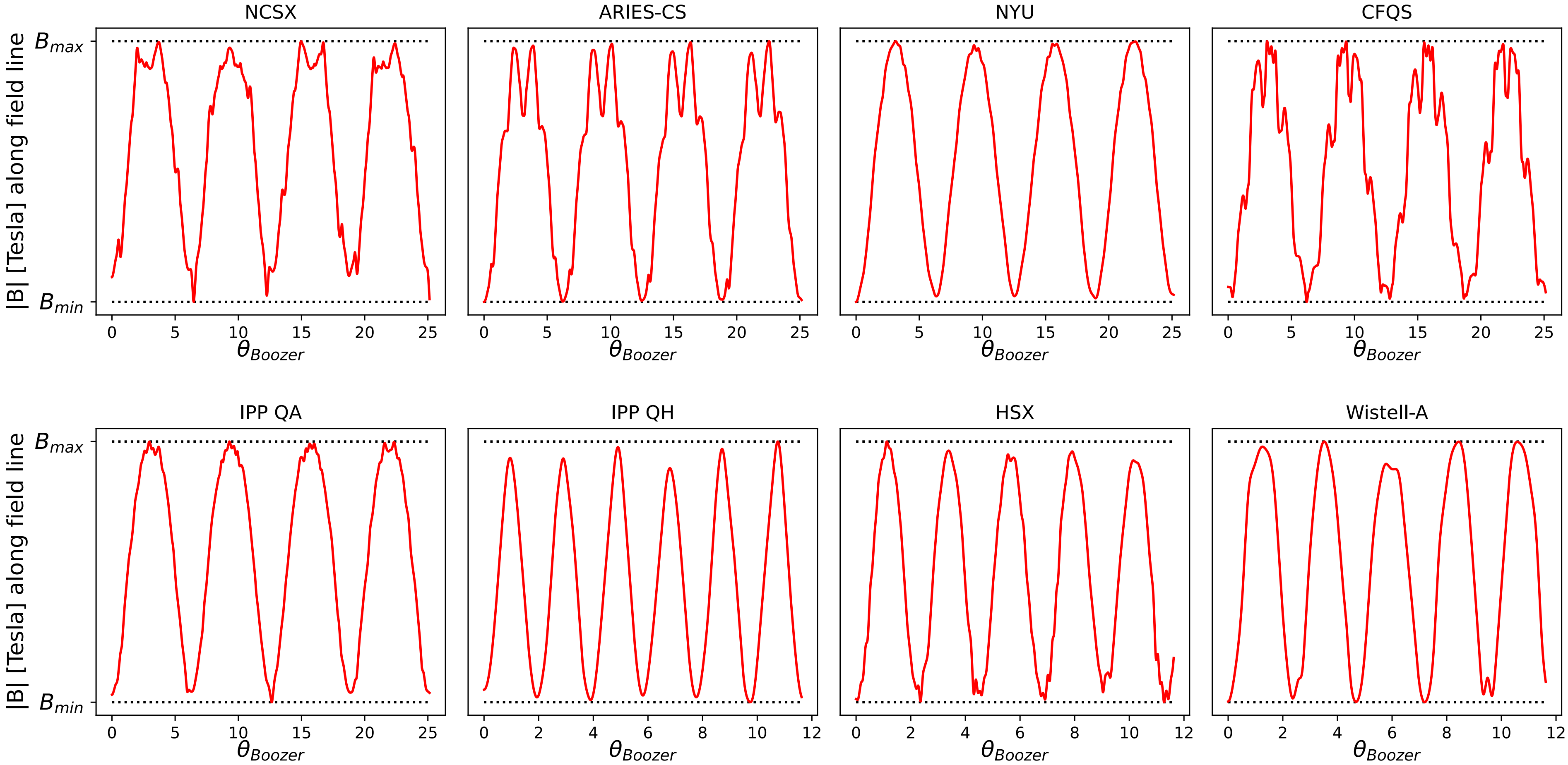
All configurations scaled to ARIES-CS minor radius (1.7 m) and |B| (5.7 T).

5000 alpha particles initialized isotropically at s=0.3.

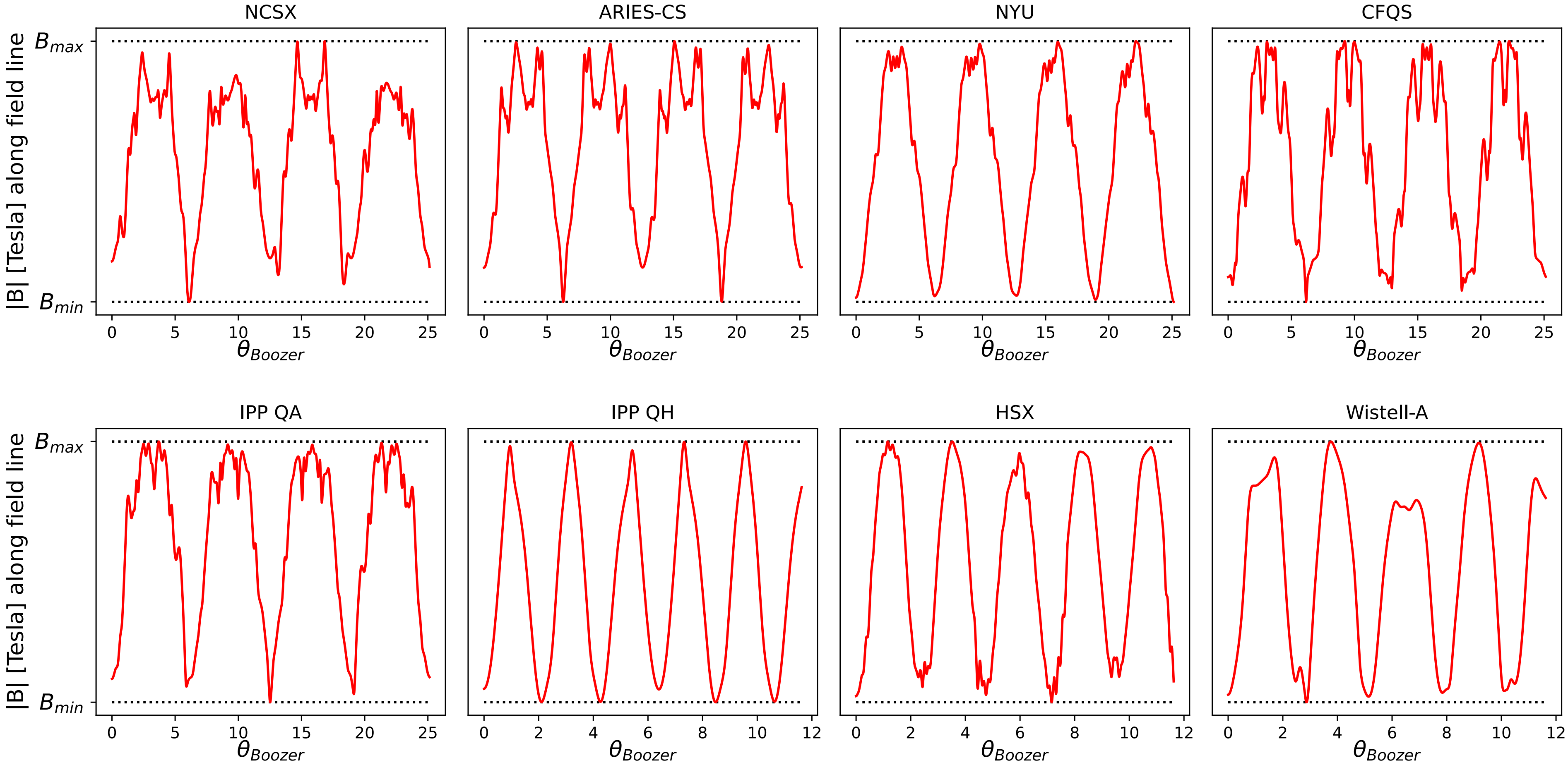
SIMPLE code: Albert et al, JCP (2020).


(a) New QA
(b) New QH
(c) New QA+well
(d) New QH+well
$\phi = 0$
1/4 period
1/2 period
Z [meters]
R [meters]
0.4
0.2
0.0
−0.2
−0.4
0.6
0.8
1.0
1.2

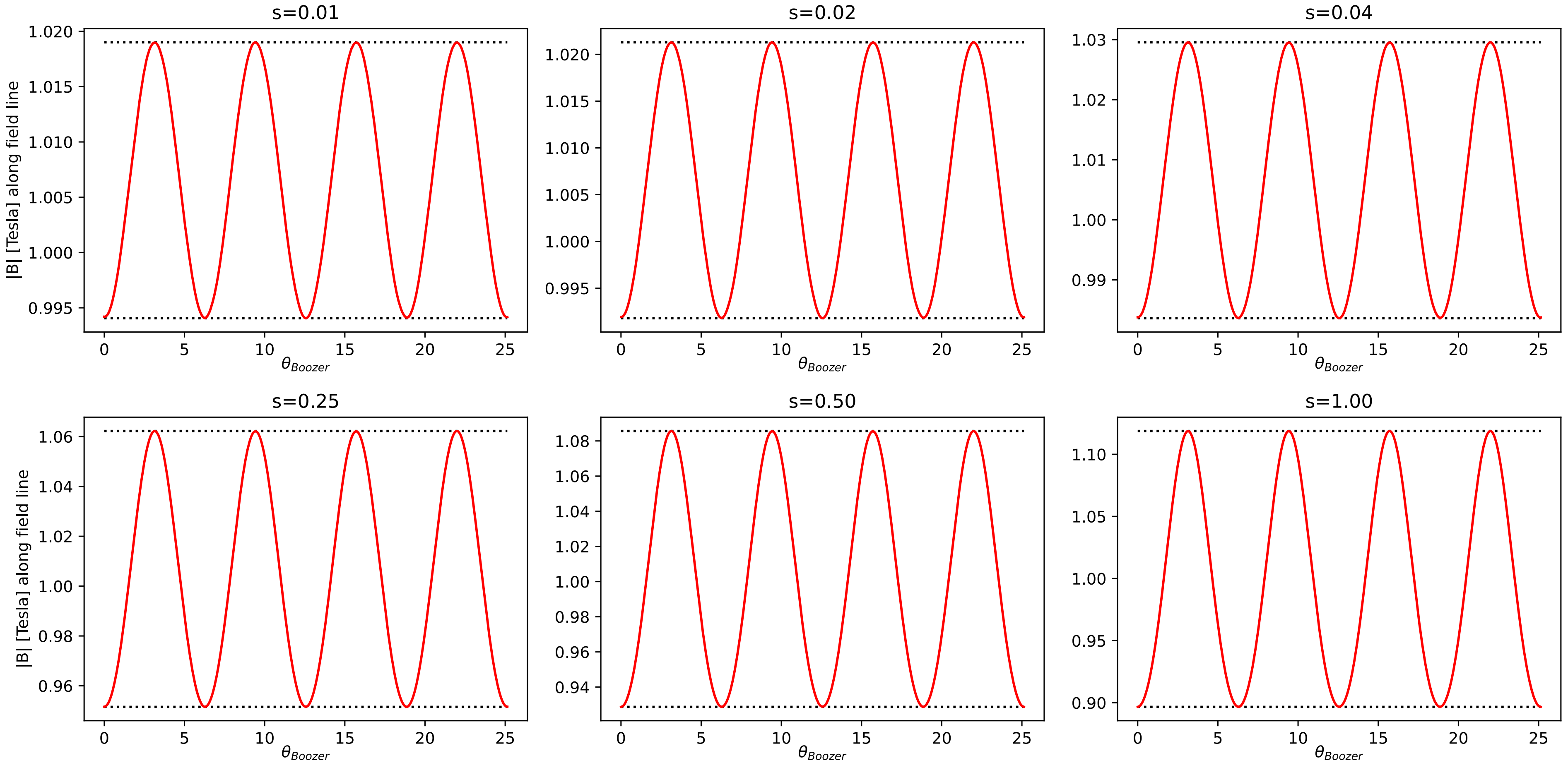
Previous quasisymmetric configurations (s=0.5)



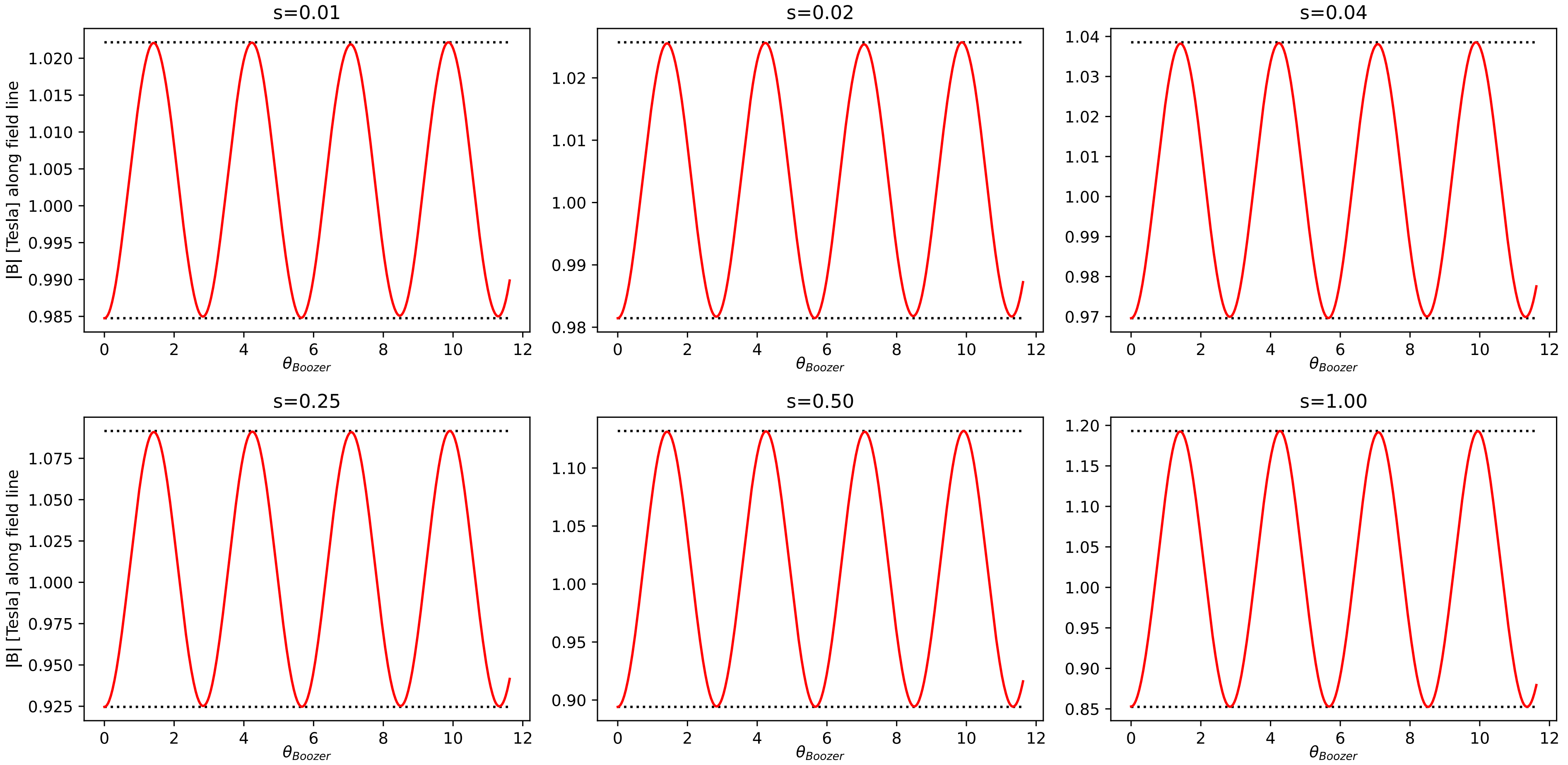
Previous quasisymmetric configurations (s=1)



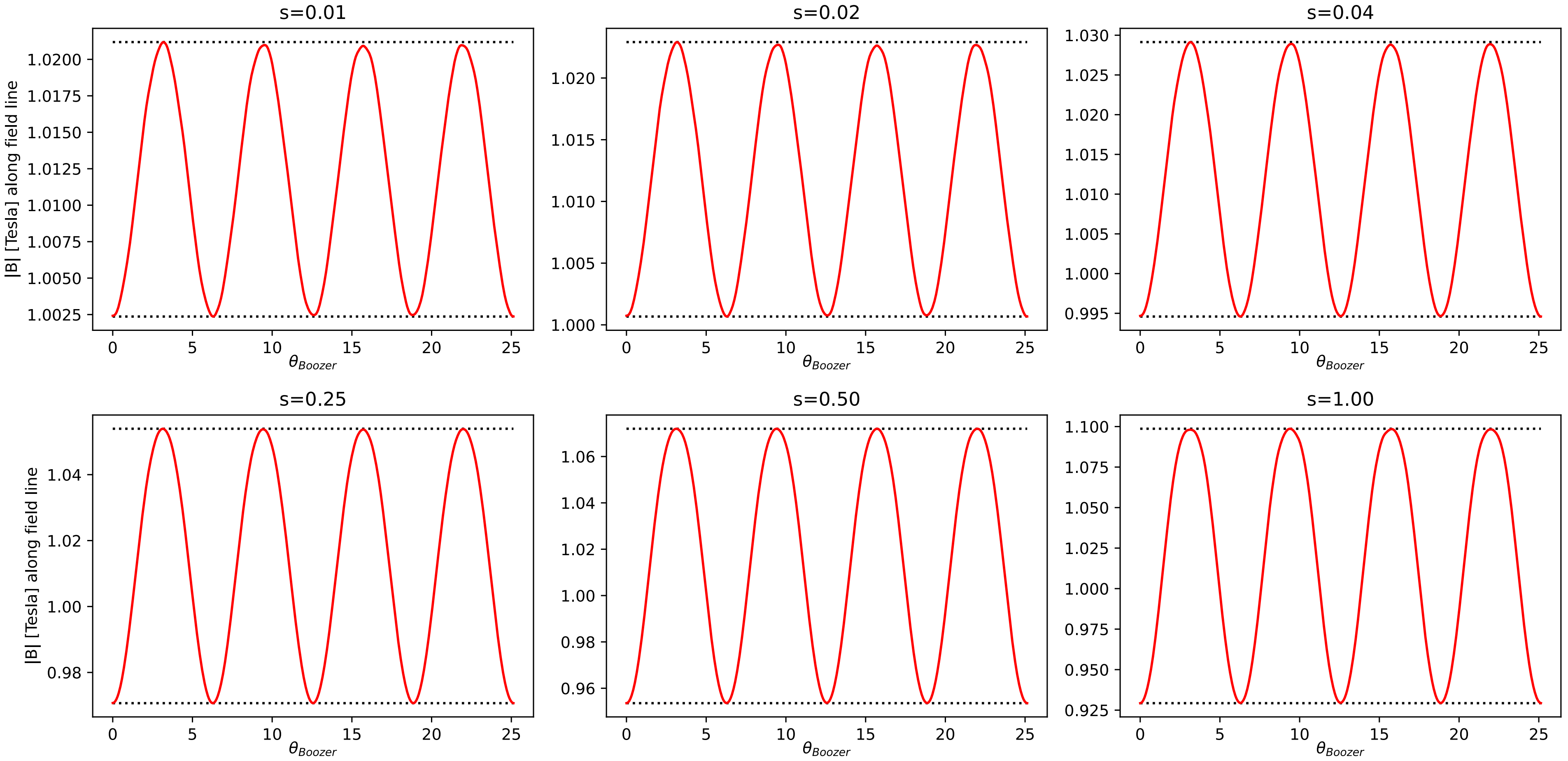
|B| along a field line for new QA



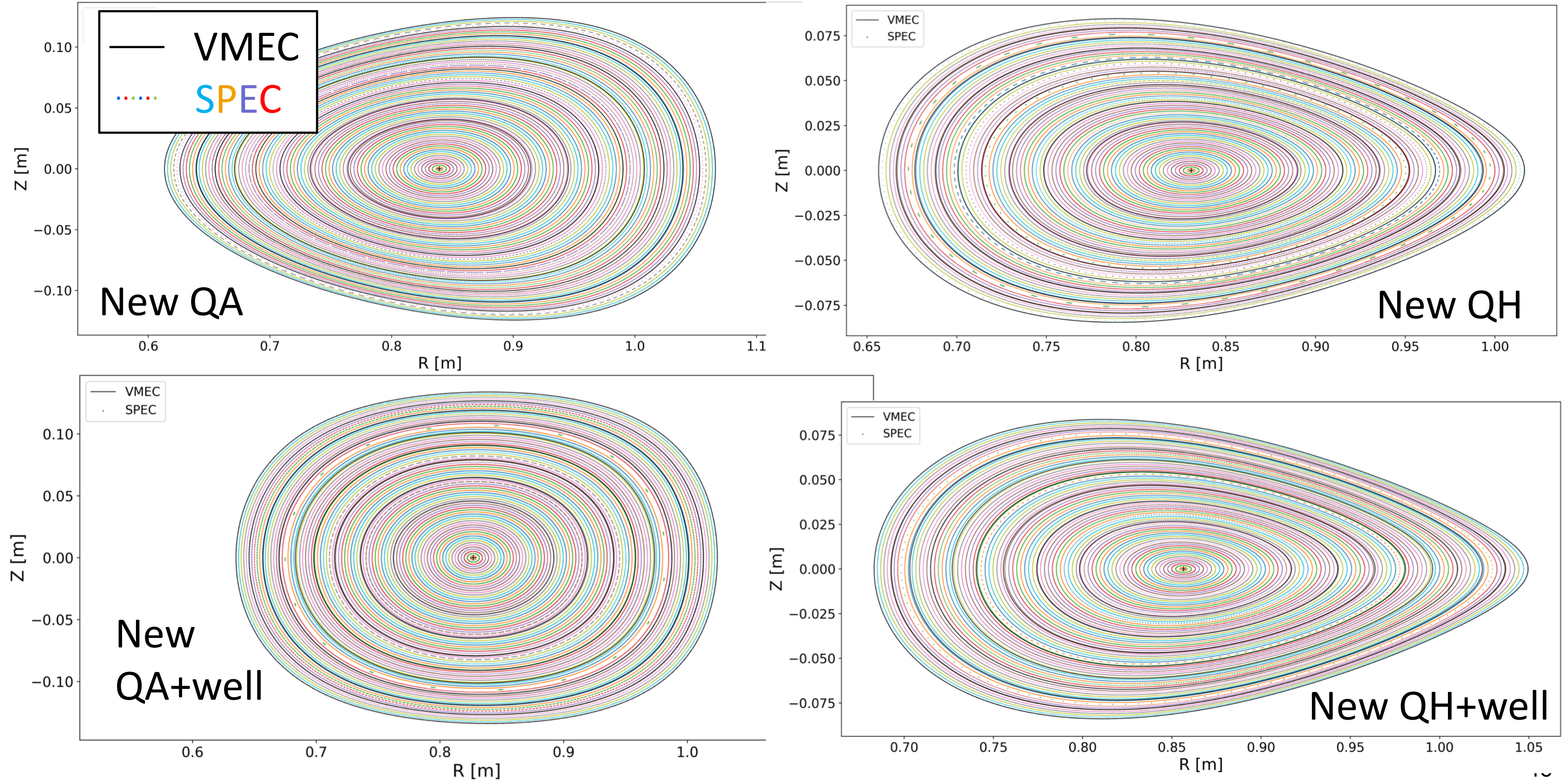
|B| along a field line for new QH



|B| along a field line for new QA with magnetic well

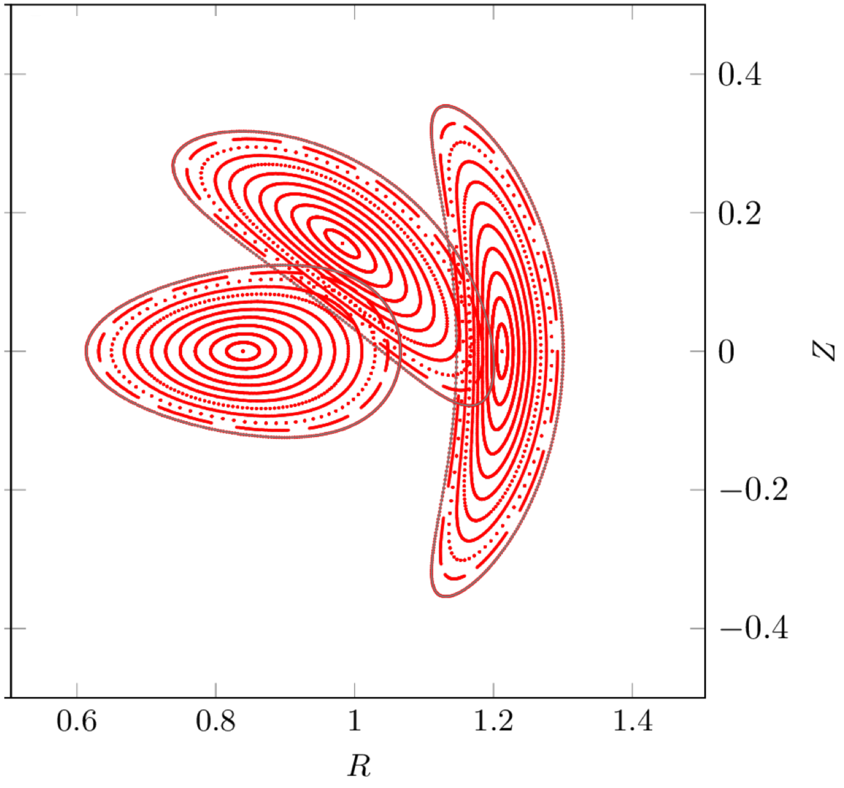


SPEC confirms the new QA/QH configurations have good surfaces

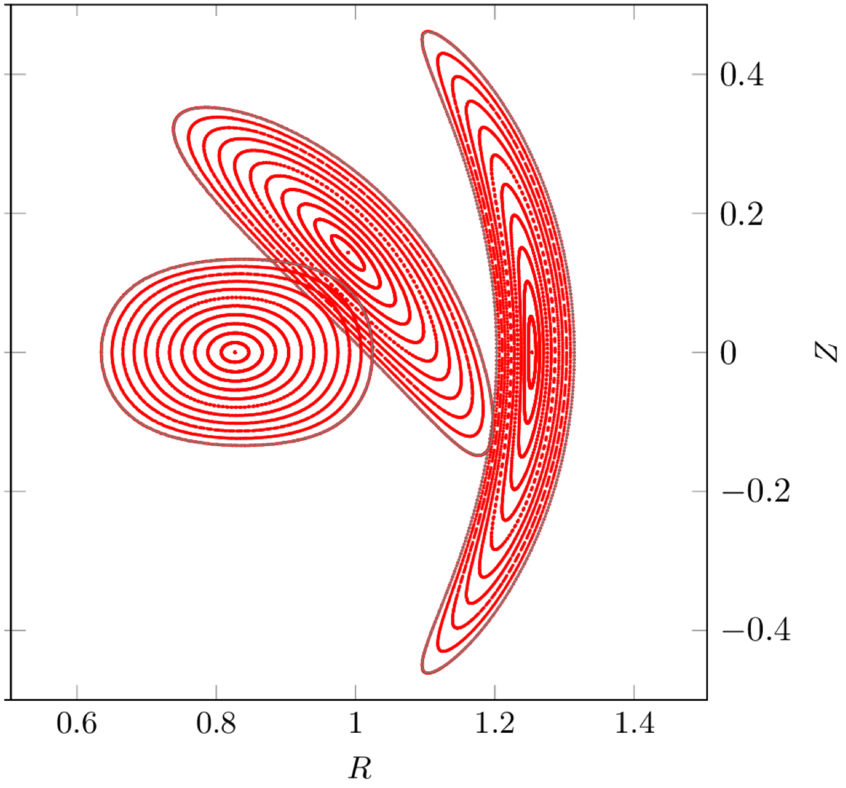


Good flux surface exist with coils

New QA

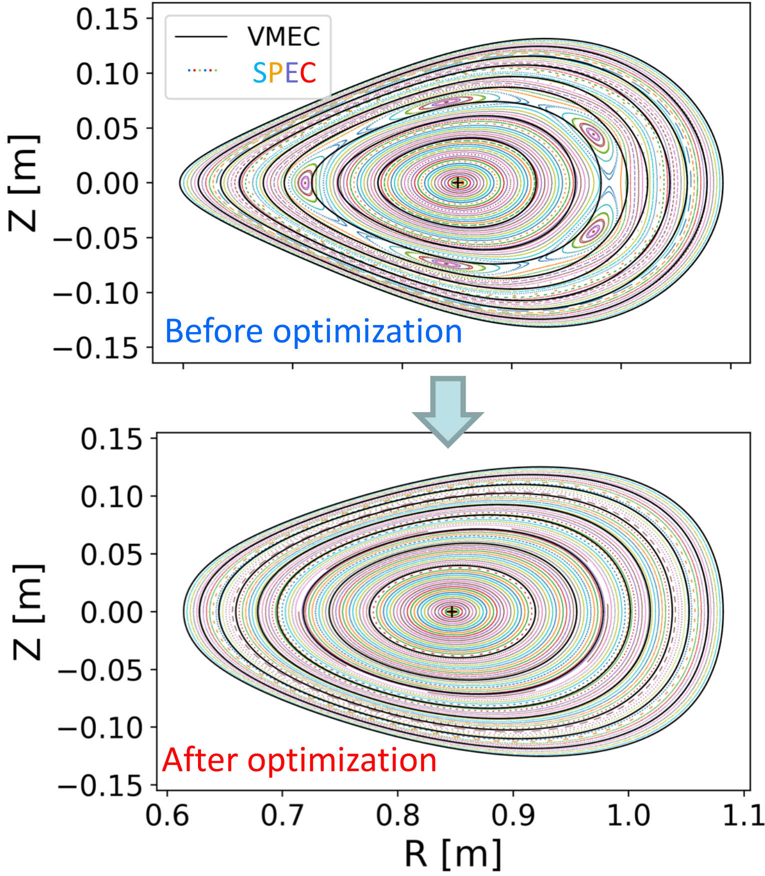


New QA+well



Future directions

- Eliminate islands at high β (plasma pressure & current)
- Coils & MHD stability for the high β configurations
- Quasi-isodynamic (|B| contours close poloidally)
- Understand trade-offs using multi-objective optimization
- Robustness to uncertainty in the pressure profile
- Interact with systems codes & reactor studies
- Combined coil + confinement (1-stage) optimization
- Optimization of turbulence & divertor



ML, Medasani & Zhu (2021)