New approaches to stellarator optimization using expansion in aspect ratio

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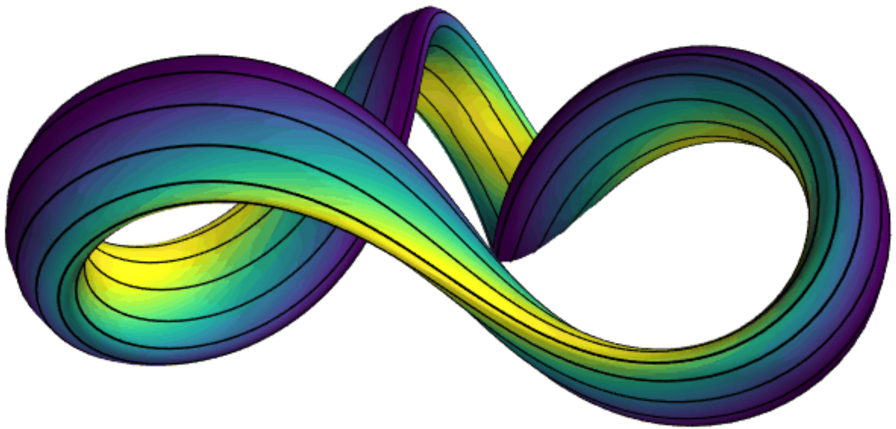

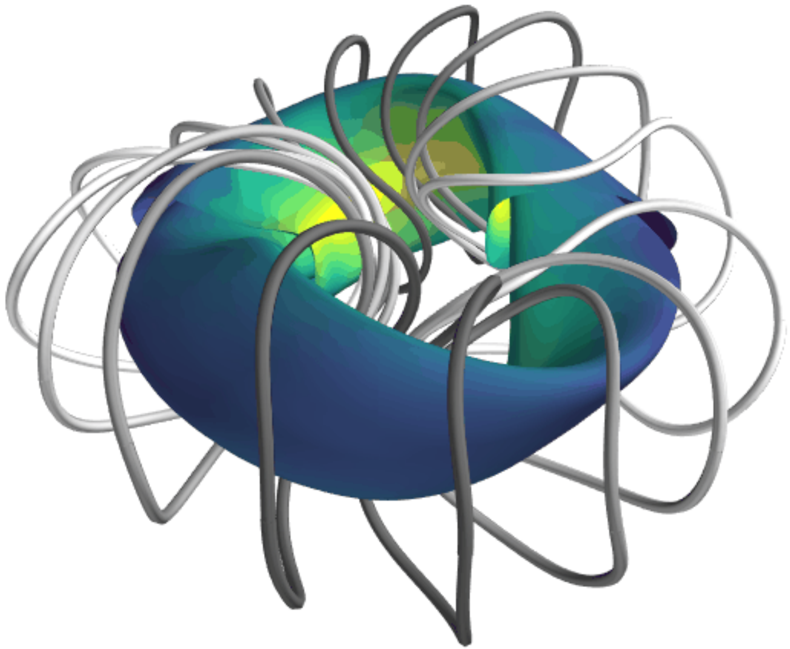

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- Advantages of stellarators: steady-state, no disruptions, no Greenwald density limit, no power recirculated for current drive.

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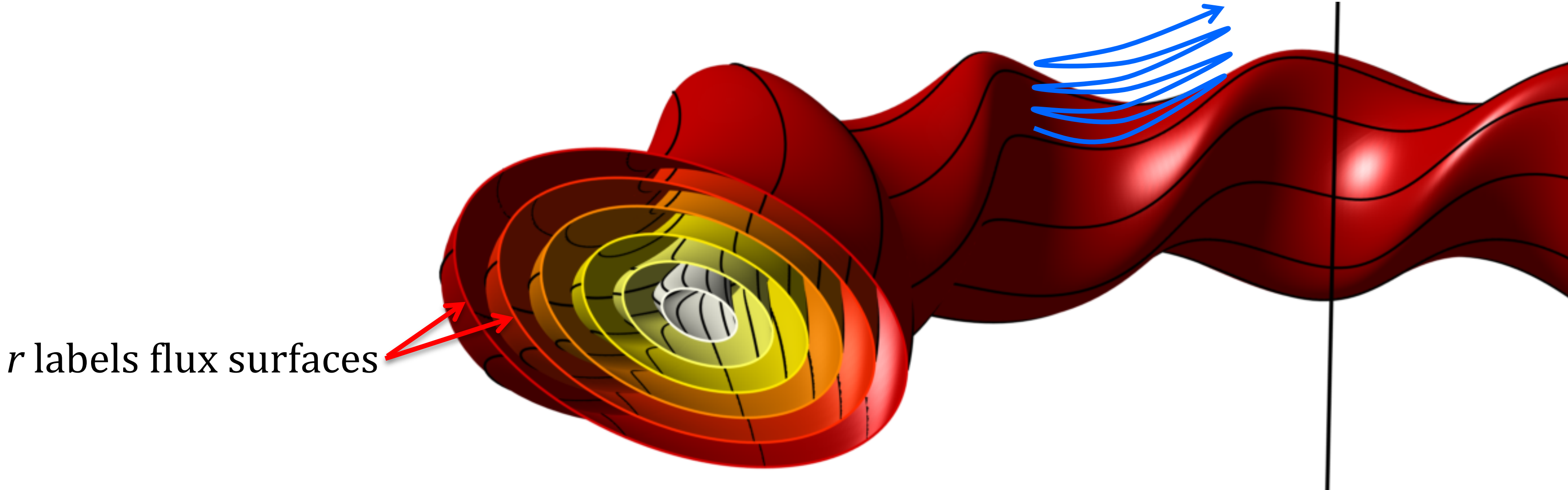
- But, alpha losses & neoclassical transport would be too large unless you carefully choose the geometry.

$$\oint \left(\mathbf{v}_d \cdot \nabla r \right) dt = 0 \text{ in axisymmetry,} \quad \neq 0 \text{ in a general stellarator.}$$



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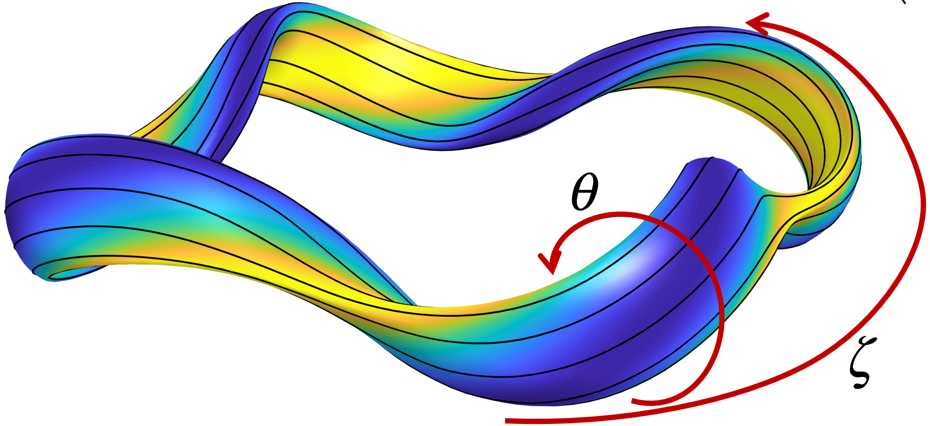
- But, alpha losses & neoclassical transport would be too large unless you carefully choose the geometry.

$$\oint \left(\mathbf{v}_d \cdot \nabla r \right) dt = 0 \text{ in axisymmetry, } \quad \neq 0 \text{ in a general stellarator.}$$

- A solution: quasisymmetry

$$B = B\left(r,\ \theta - N\zeta\right) \quad \Rightarrow \quad \oint \left(\mathbf{v}_d \cdot \nabla r \right) dt = 0.$$

Boozer angles

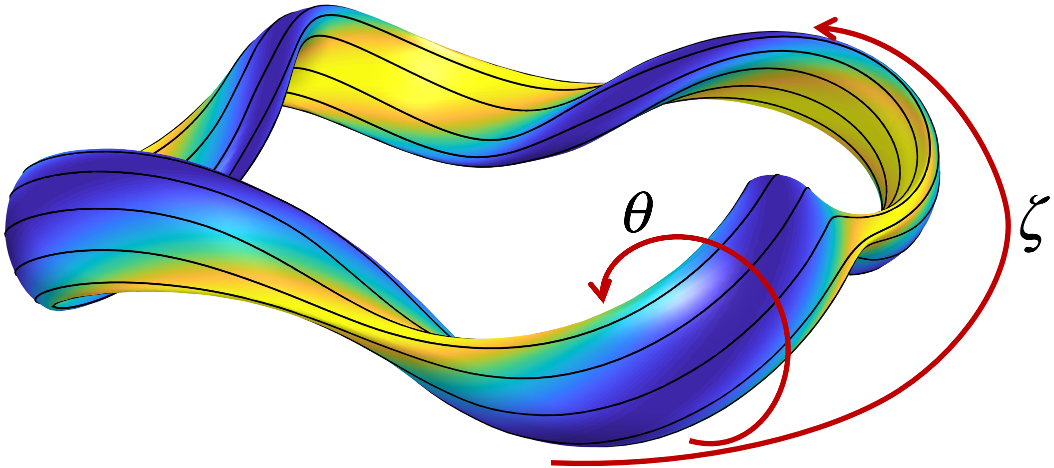


Guiding-center Lagrangian in Boozer coordinates depends on (θ,ζ) only through B=$|\mathbf{B}|$.

- How do you find configurations with quasisymmetry?

Until now, understanding of quasisymmetric plasmas has been limited by the method of finding them numerically

Want $\mathbf{J}\times\mathbf{B}=\nabla p$ and $B=B\left(r,\,\theta-N\zeta\right)$.



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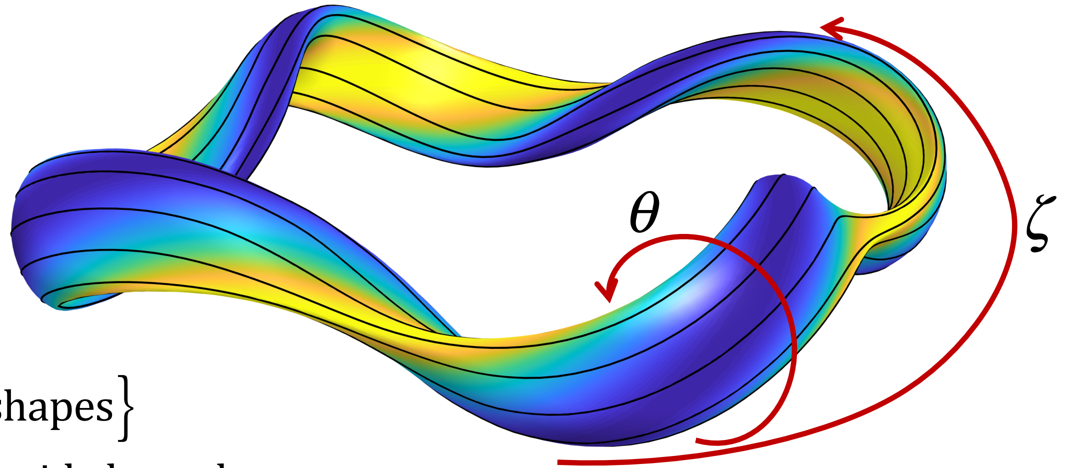

$$\text{minimize } f\left(x\right)$$

Parameter space: $x\in\left\{\text{toroidal boundary shapes}\right\}$

Objective: Solve $\mathbf{J}\times\mathbf{B}=\nabla p$ numerically inside boundary,

$f=$ departure from quasisymmetry in the result.

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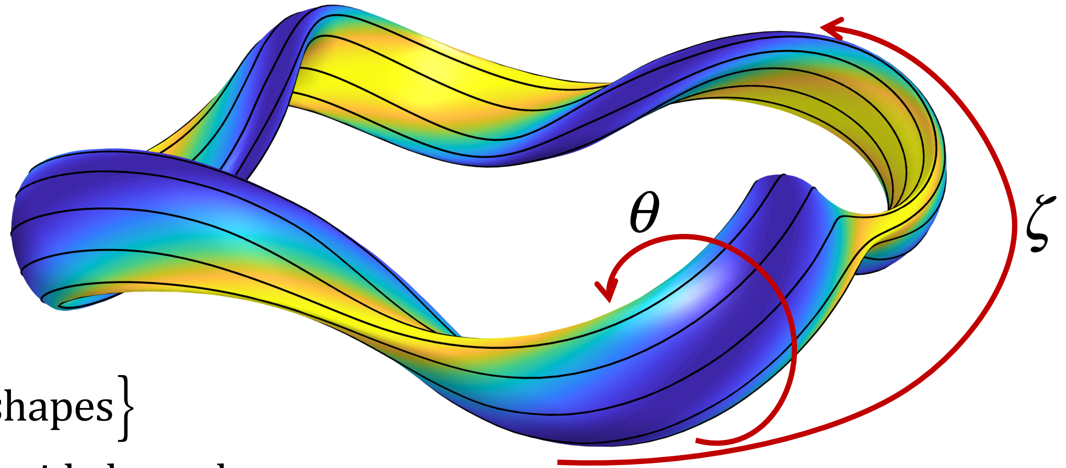


minimize $f\left(x\right)$

Parameter space: $x\in\left\{\text{toroidal boundary shapes}\right\}$

Objective: Solve $\mathbf{J}\times\mathbf{B}=\nabla p$ numerically inside boundary,

$f=$ departure from quasisymmetry in the result.

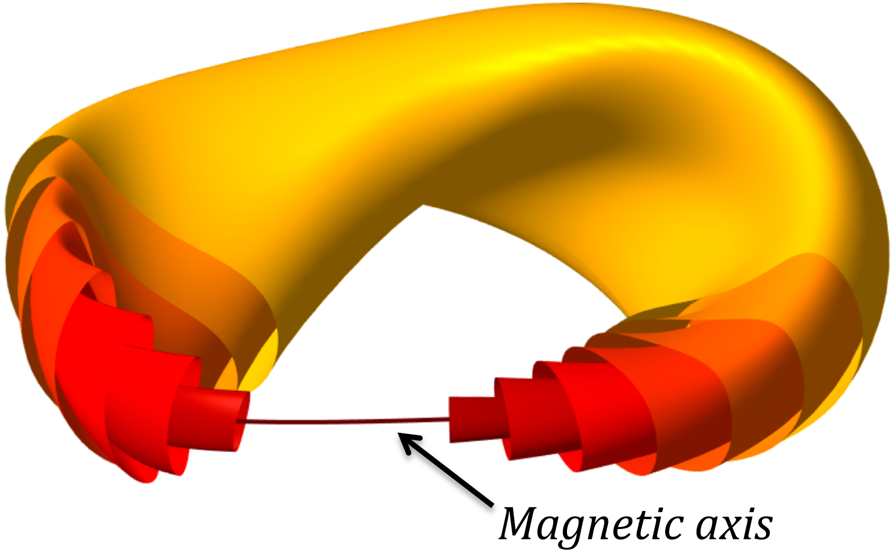
- Computationally expensive.
- What is the size & character of the solution space?
- Result depends on initial condition. $\Rightarrow$ Cannot be sure you've found all solutions.

Expansion about the magnetic axis can be a powerful practical tool for generating quasisymmetric stellarator configurations

- Accurate at least in the core of any configuration.

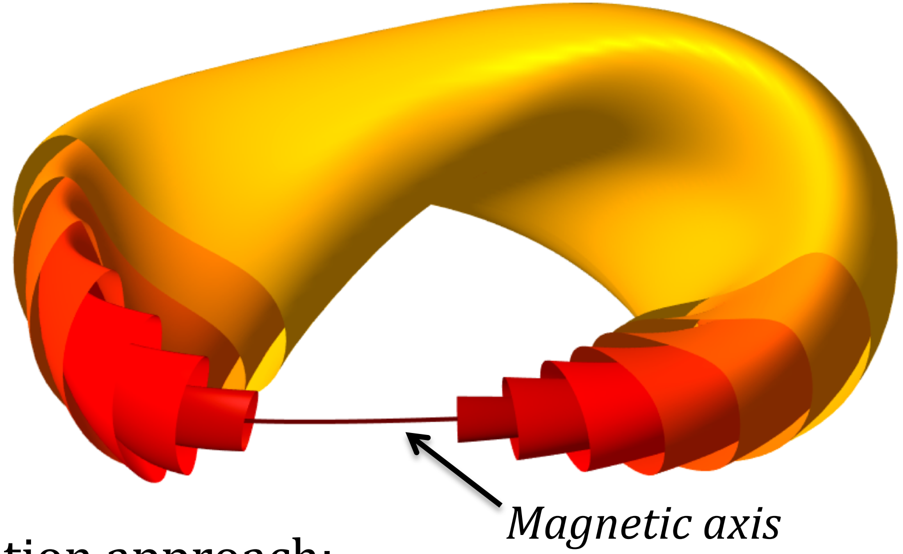
- Hasn't been considered much since numerical optimization began.

Mercier (1964),
Solov'ev & Shafranov (1970),
Lortz & Nührenberg (1976),
Garren & Boozer (1991)



Revisit expansions with modern concepts (e.g. quasisymmetry, gyrokinetics), computing, & optimization.

Expansion about the magnetic axis can be a powerful practical tool for generating quasisymmetric stellarator configurations



- Accurate at least in the core of any configuration.

- Hasn't been considered much since numerical optimization began.

- Complements the traditional optimization approach:
  - Many orders of magnitude faster.
  - Opportunities for analytic insights.
  - Can generate new initial conditions that can be refined by optimization.

Outline

- Constructing quasisymmetric stellarator shapes
- Evaluating other physics properties
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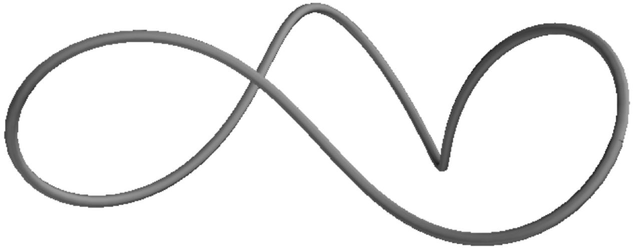
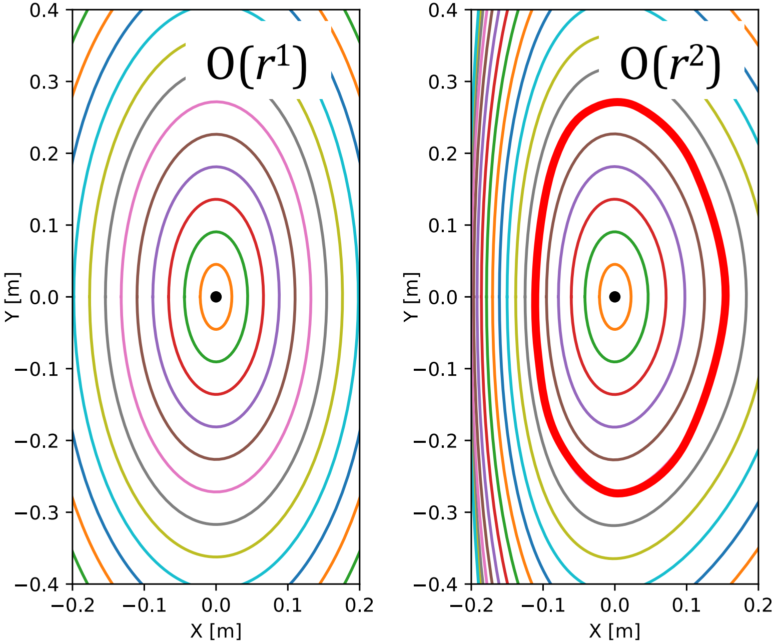
The expansion by Garren & Boozer (1991) has been converted into a practical algorithm for generating stellarator shapes

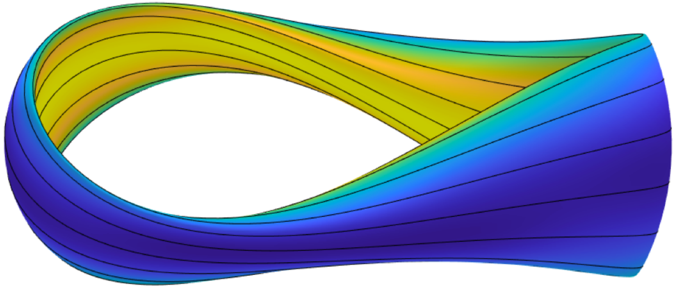
- Inputs:
  - Shape of the magnetic axis.
  - 3-5 other numbers (e.g. current on the axis).
- Outputs:
  - Shape of the surfaces around the axis.
  - Rotational transform on axis.
  - ...
- Quasisymmetry guaranteed in a neighborhood of axis.
- Can pick any surface to pass to traditional 3D MHD fixed-boundary solve.





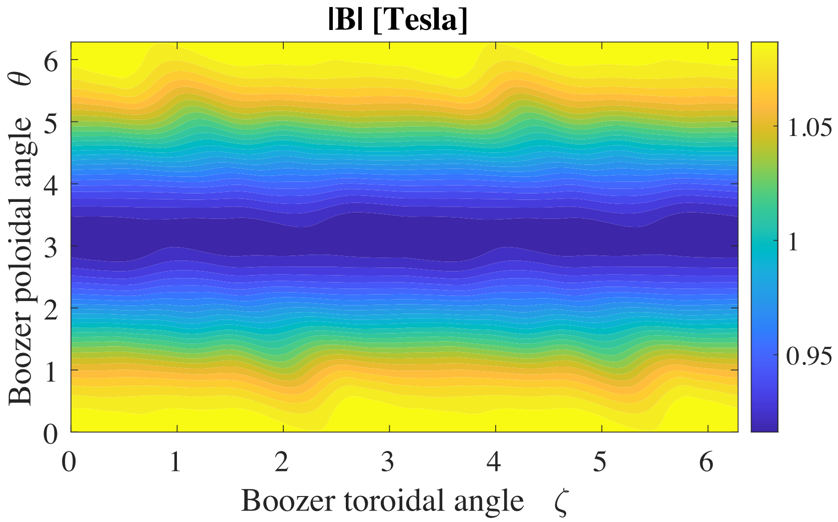
The construction can be verified by running an MHD equilibrium code (e.g. VMEC) which does not make the expansion.

Quasi-axisymmetry (QA)

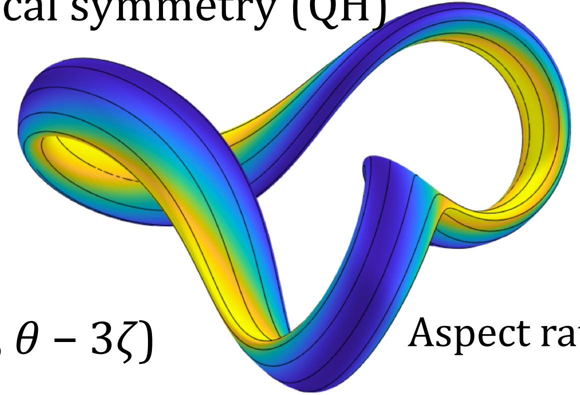


$B = B(r, \theta)$

Aspect ratio 6.0

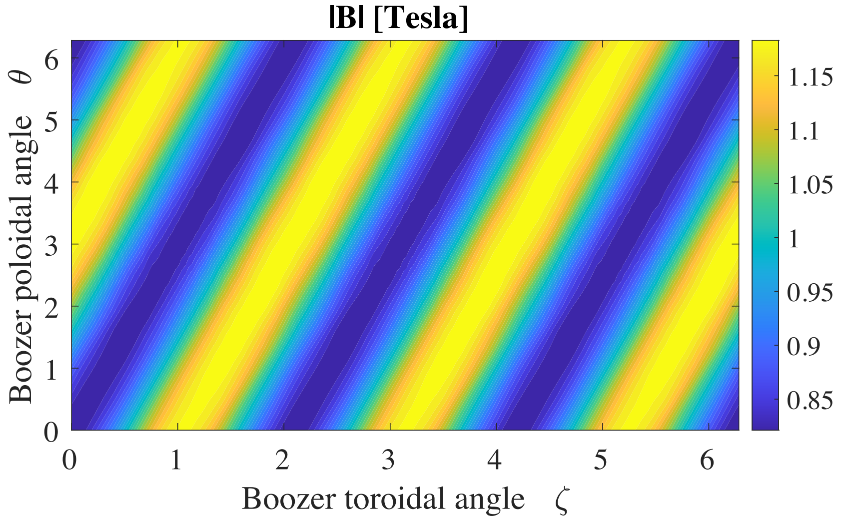


Quasi-helical symmetry (QH)



$B = B(r, \theta - 3\zeta)$

Aspect ratio 5.0



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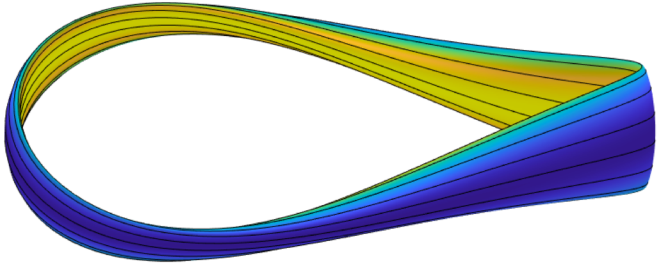
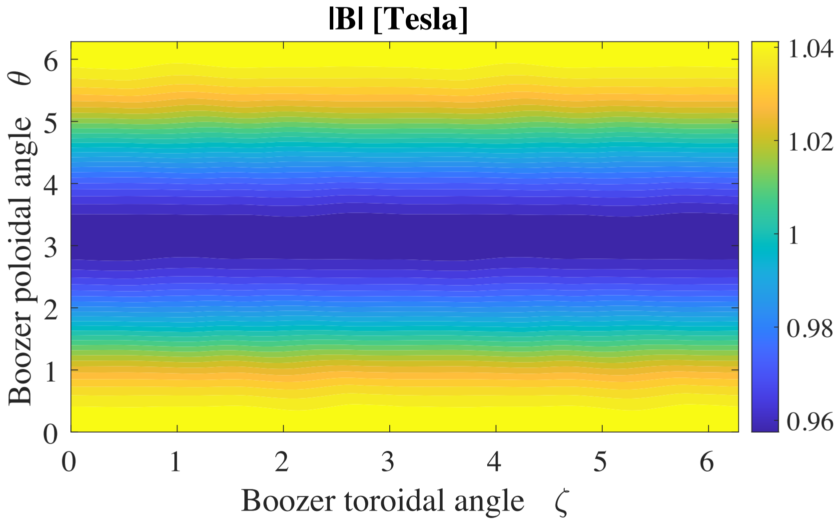


$B = B(r, \theta)$

Aspect ratio 12.0



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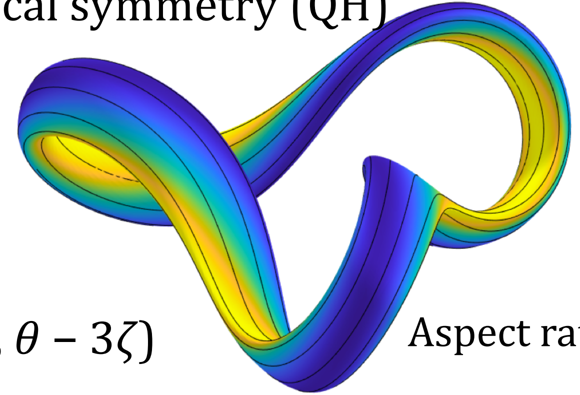

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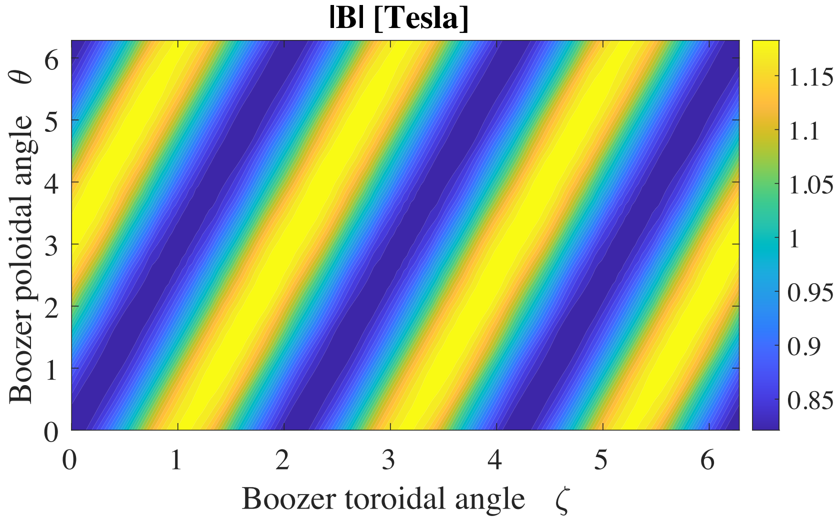
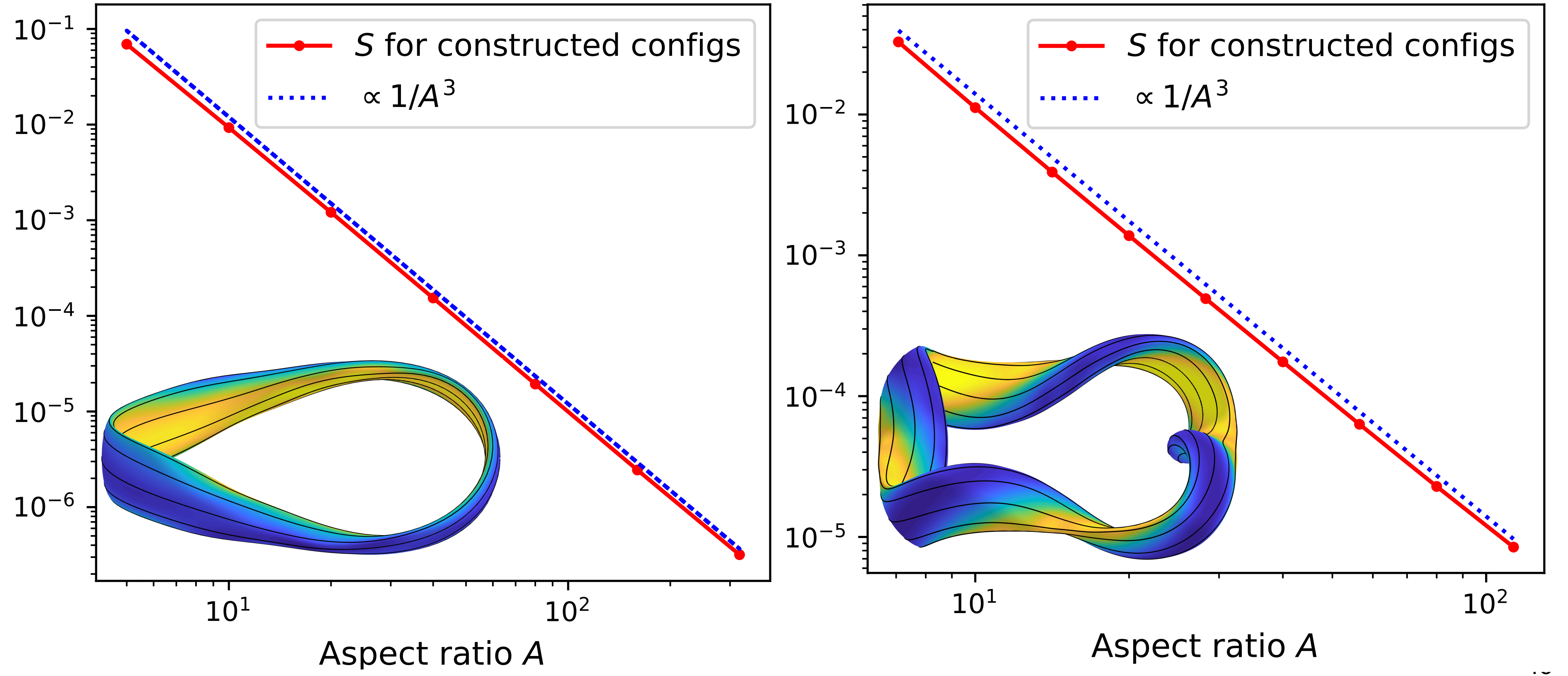
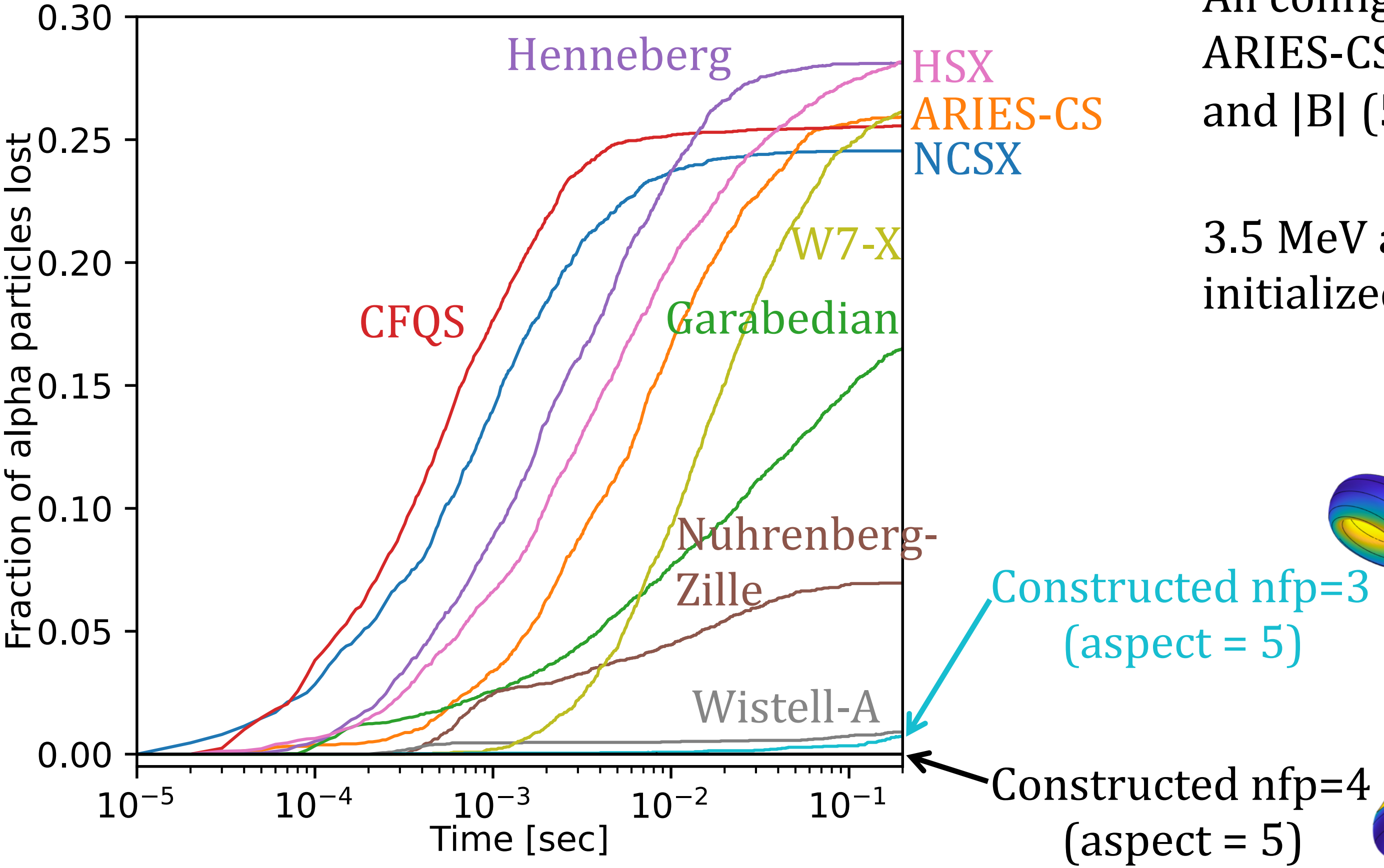


We can now numerically demonstrate Garren & Boozer's scaling: $B_{nonsymm} \sim 1/A^3$

$$S = \frac{1}{B_0}\sqrt{\sum_{m,n\neq Nm} B_{m,n}^2} = \text{Symmetry-breaking}$$

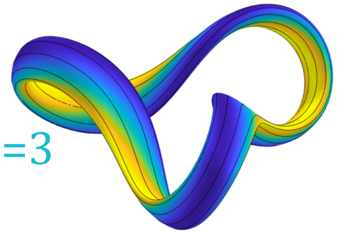


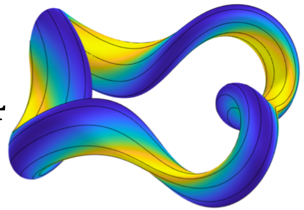
Accurate quasisymmetry is effective at curing alpha particle losses



All configurations scaled to ARIES-CS minor radius (1.7m) and |B| (5.7 T)

3.5 MeV alpha particles initialized at ψ/ψ_{edge} = 0.3.





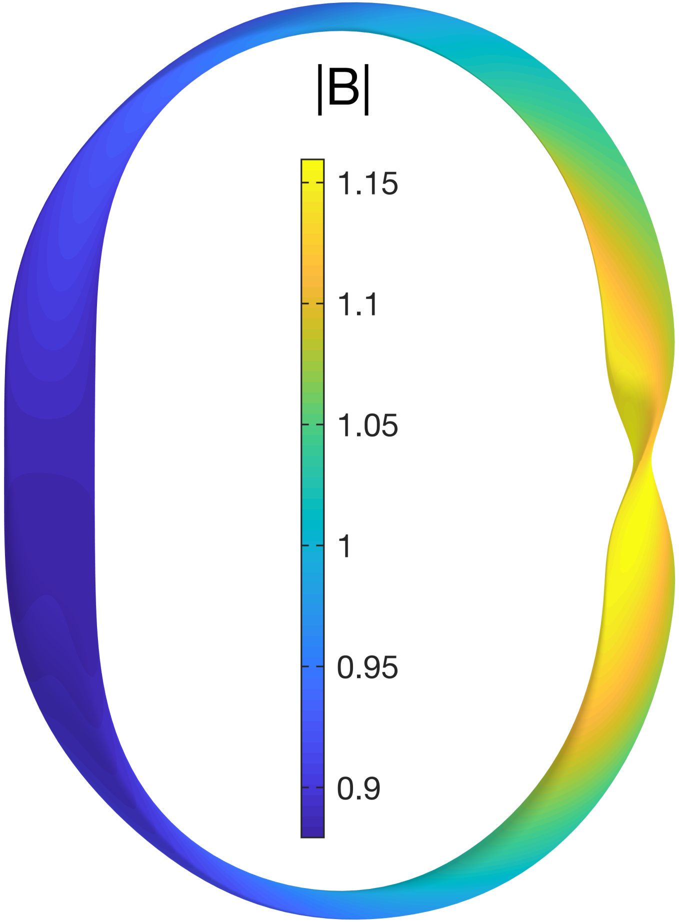
The near-axis analysis can be generalized to construct configurations with *omnigenity*

G G Plunk, ML, and P Helander, JPP (2019)

Omnigenity:

$$\oint_{\text{bounce}} \left(\mathbf{v}_d \cdot \nabla r\right) dt = 0 \quad \forall \text{ magnetic moments \& energies.}$$

- Weaker condition than quasisymmetry.
- *B* contours can close poloidally.



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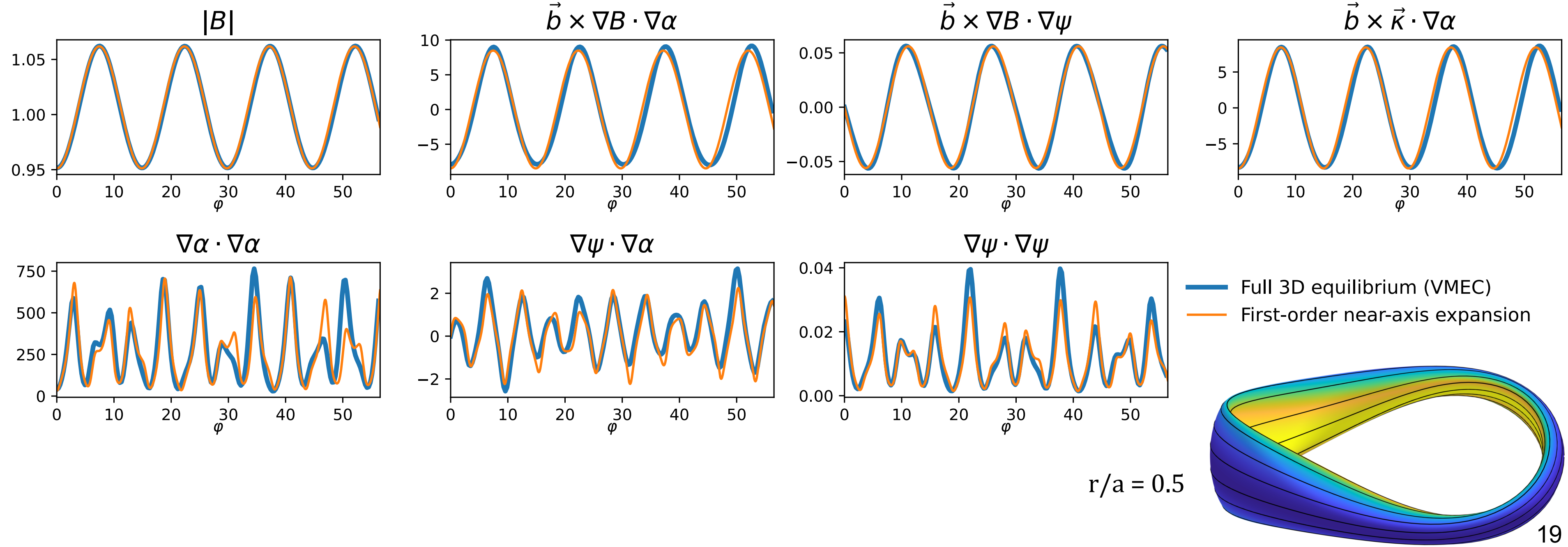
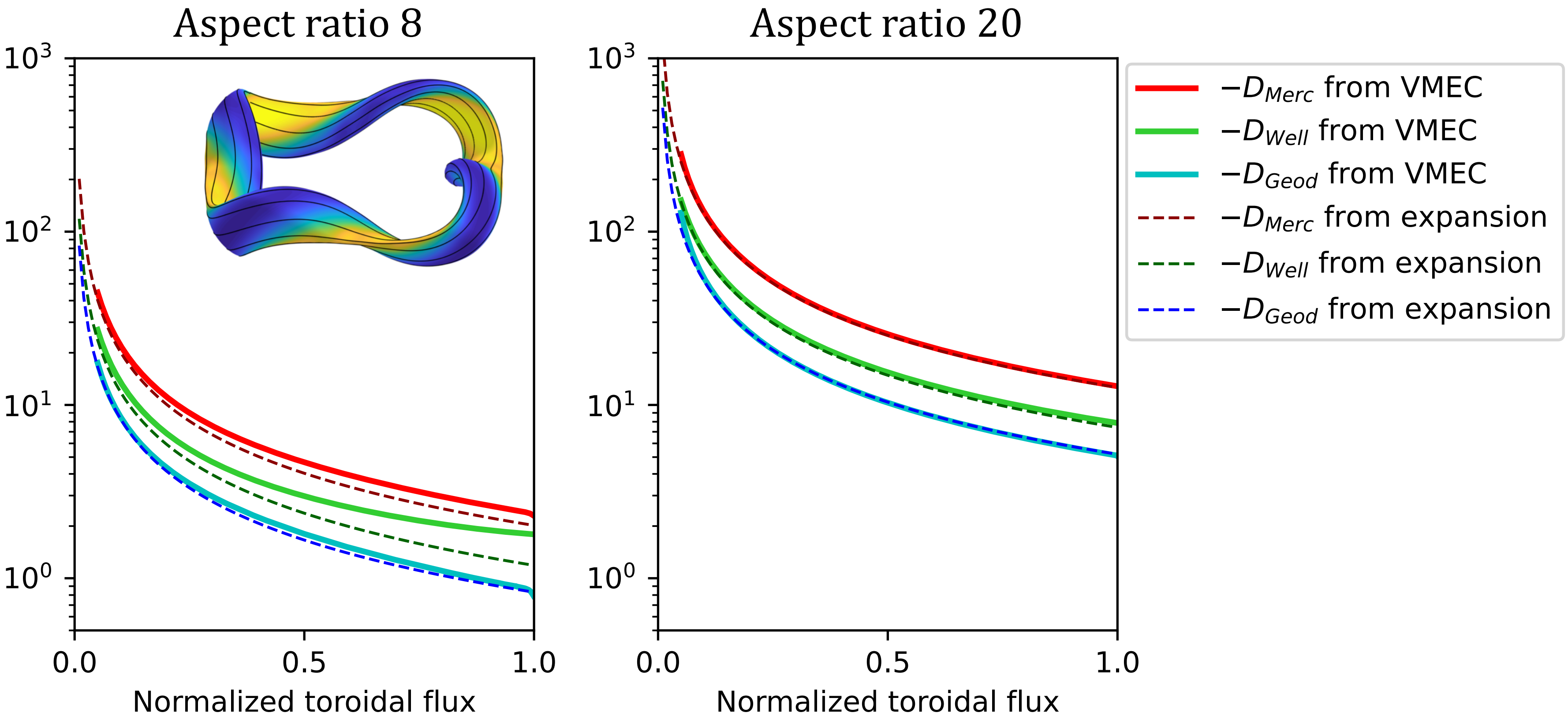
From a near-axis solution, analytic expressions exist for all the geometric quantities in gyrokinetics

Rogerio Jorge & ML, PPCF 63, 014001 (2021), Rogerio Jorge & ML, PPCF 63, 074002 (2021)

$$\text{E.g.}\quad \nabla\alpha\cdot\nabla\alpha = \frac{1}{r^2\bar{\eta}^2\kappa^2}\left[\bar{\eta}^4\cos^2\theta + \kappa^4\left(\sigma\cos\theta + \sin\theta\right)^2\right] \quad \text{where } \mathbf{B} = \nabla\psi\times\nabla\alpha$$



Mercier stability can be evaluated directly from a near-axis solution



ML & Jorge, JPP (2020)

Many properties of a stellarator can be computed in ~ 1 ms directly from a solution of the near-axis equations

<u>Available so far:</u>

- Surface shapes
- Rotational transform
- Magnetic well
- Mercier stability
- All the geometric factors in the gyrokinetic equation & MHD ballooning equation
- **B** vector & 2 gradients
- Scale length in **B** (proxy for coil complexity?)

<u>Not available now:</u>

- Magnetic shear
- Ballooning growth rates
- Low-n MHD growth rates
- Gyrokinetic growth rates

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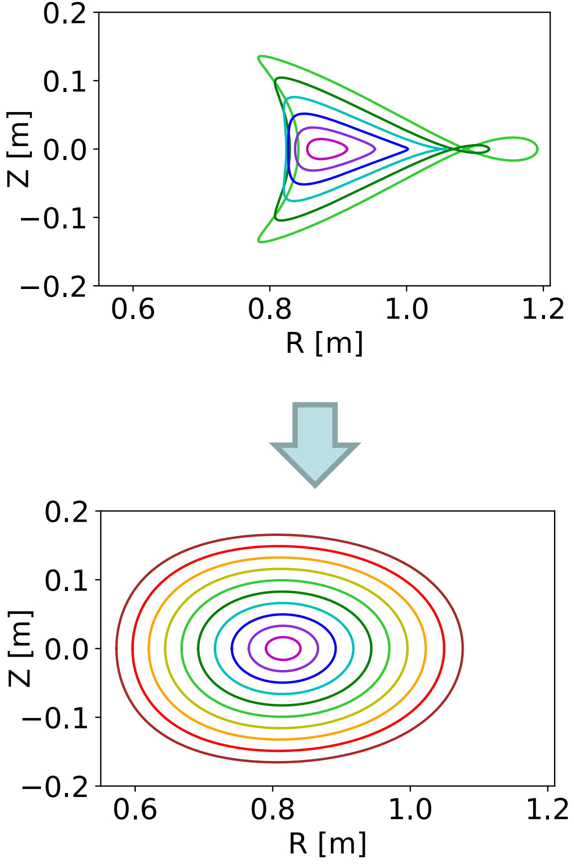
Though quasisymmetry is guaranteed in a neighborhood of the axis, optimization can greatly increase the volume of good symmetry



Objective function to minimize:

Average along magnetic axis — Axis length — Desired axis length — Desired rotational transform

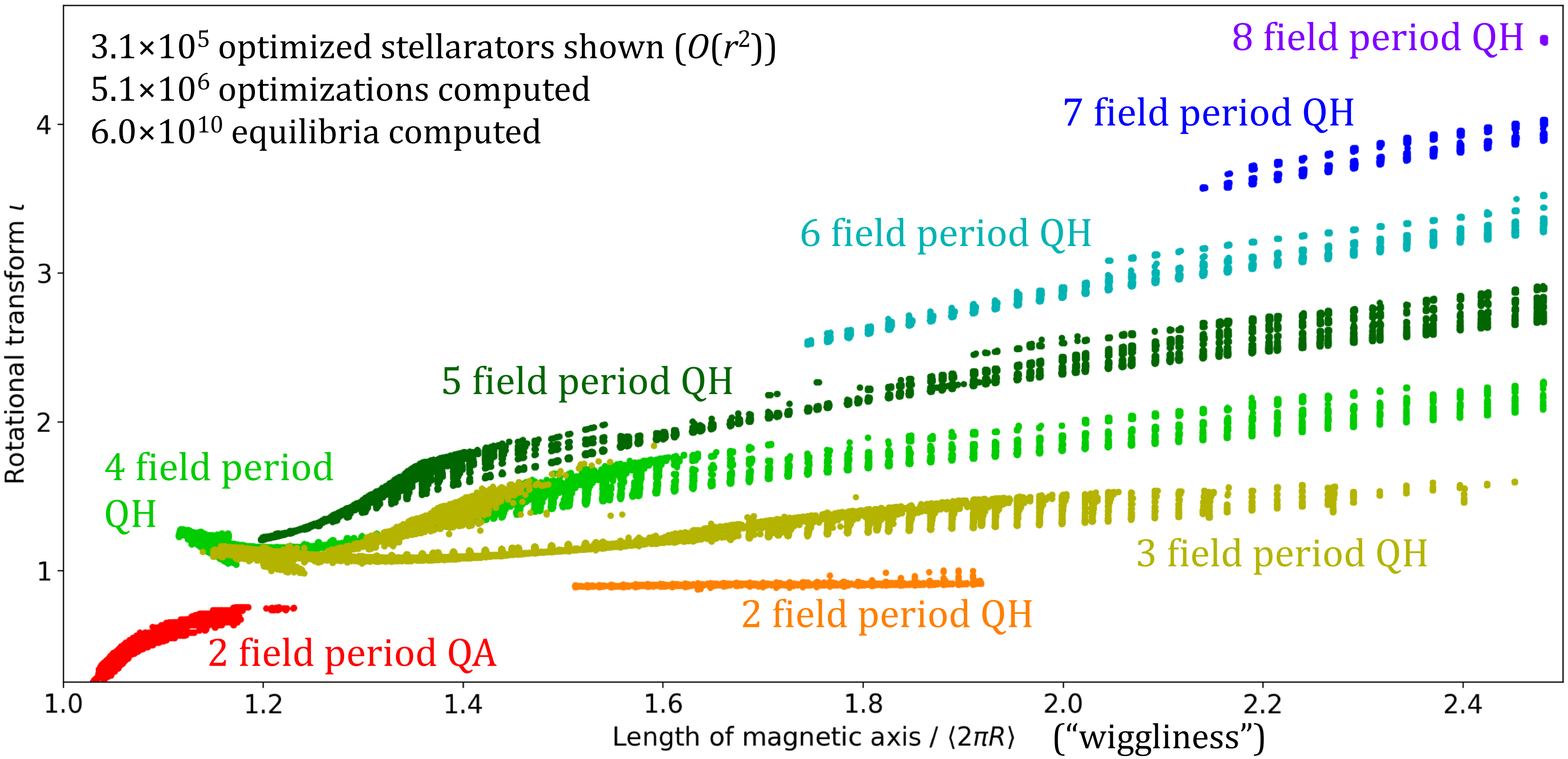
$$f = \frac{1}{L}\int d\ell \left|\left|\nabla \mathbf{B}\right|\right|^2 + \frac{w_{\nabla\nabla}}{L}\int d\ell \left|\left|\nabla\nabla\mathbf{B}\right|\right|^2 + w_L\left(L - L_*\right)^2 + w_\iota\left(\iota - \iota_*\right)^2$$

$$+ \frac{w_{B20}}{L}\int d\ell \left(B_{20} - \frac{1}{L}\int d\ell' B_{20}\right)^2 + w_{well}\left[\max\left(0,\ \frac{d^2V}{d\psi^2} - W_*\right)\right]^2$$

Deviation from quasisymmetry at $O(r^2)$ — Magnetic well — Desired well

$w_{\nabla\nabla},\ w_L,\ w_\iota,\ w_{B20},\ w_{well}$: Weights chosen by user

The near-axis equations can be solved so quickly that tensor-product scans over many parameters are feasible



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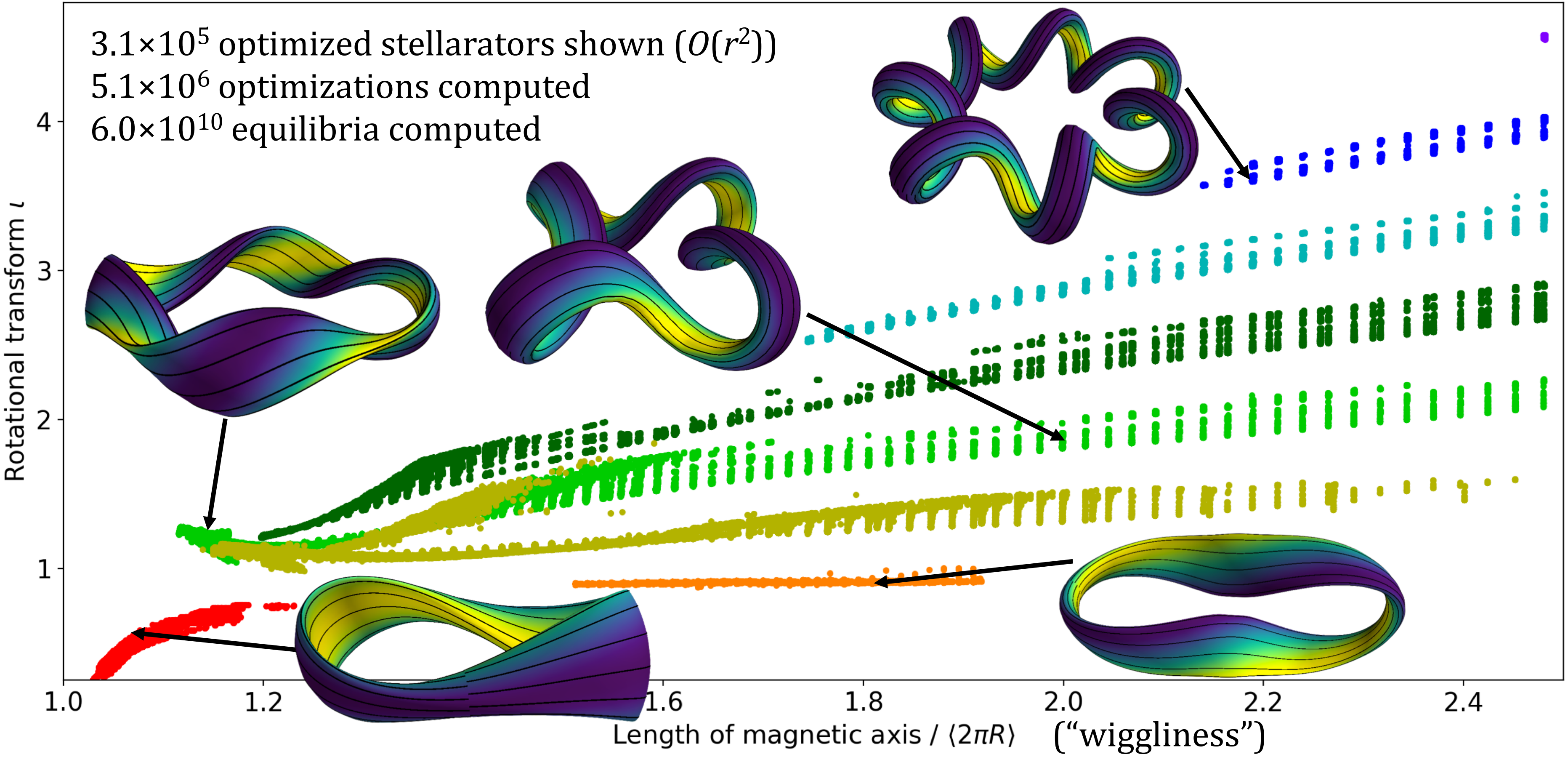
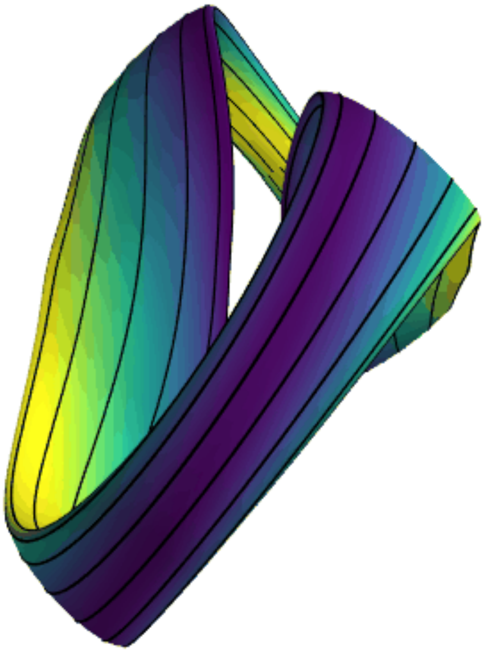


Discovery: quasi-helical symmetry with 2 field periods



Spitzer, 1958

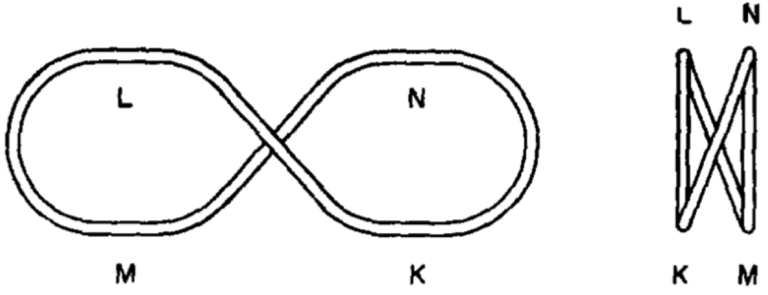
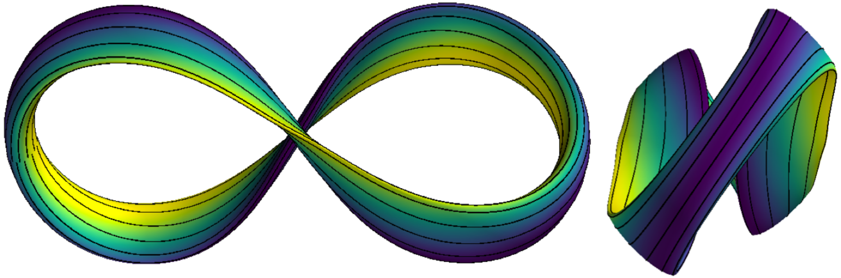
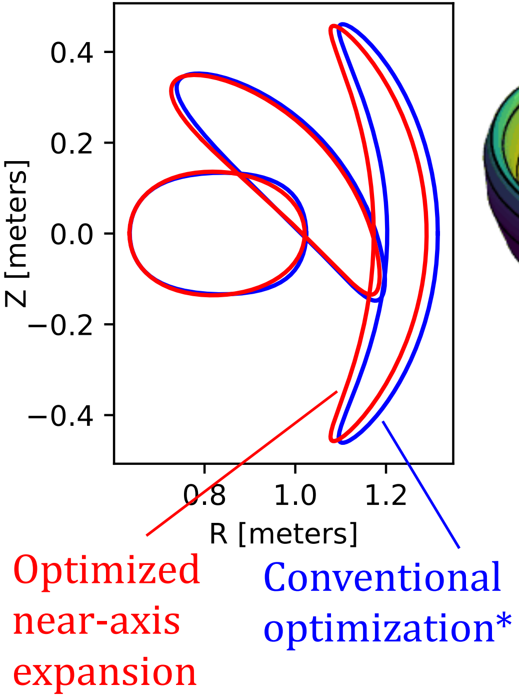


FIG. 2. Top and end views of a figure-eight stellarator.

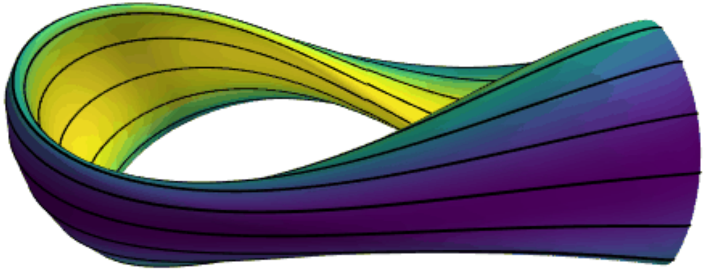
But now with quasisymmetry:



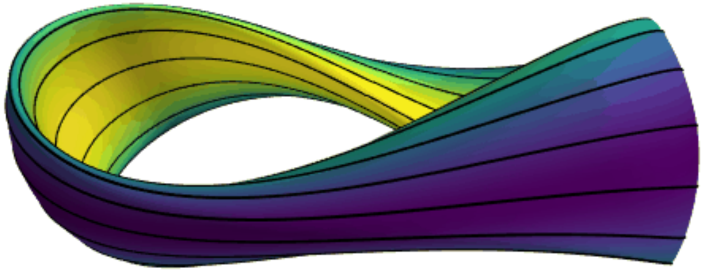
The near-axis expansion can yield configurations very similar to conventional optimization, but much faster



Conventional optimization



Optimized near-axis expansion



	Conventional optimization	Optimized near-axis expansion
Time for 1 equilibrium solve	50 CPU-sec	5e-4 CPU-sec
Total time for optimization (cold start)	8e+5 CPU-sec	1 CPU-sec

** ML & Paul, arXiv (2021)*

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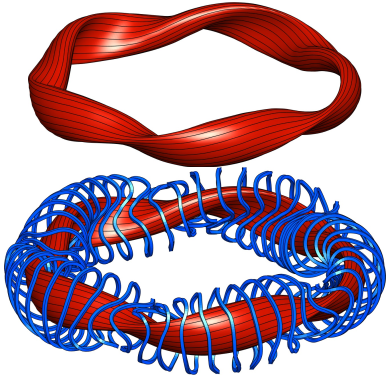
The near-axis expansion can form the basis of a new design formulation for *coils*



Previous designs (HSX, W7-X) used a 2-stage approach:

1. Optimize plasma shape, ignoring coils.
2. Find coils to make the plasma shape from stage 1.

Downside: The result of stage 1 may be hard to produce with practical coils.

New approach: Directly optimize coil shapes for consistency with near-axis quasisymmetry. *A Giuliani et al, arXiv (2020)*

- Derivatives are available.
- Stochastic optimization yields wider coil tolerances $\Rightarrow$ lower cost. *F Wechsung et al, arXiv (2021)*

Wechsung BO08.00011

Giuliani BO08.00012

The near-axis expansion can form the basis of a new design formulation for *coils*

$$\min f(X)$$

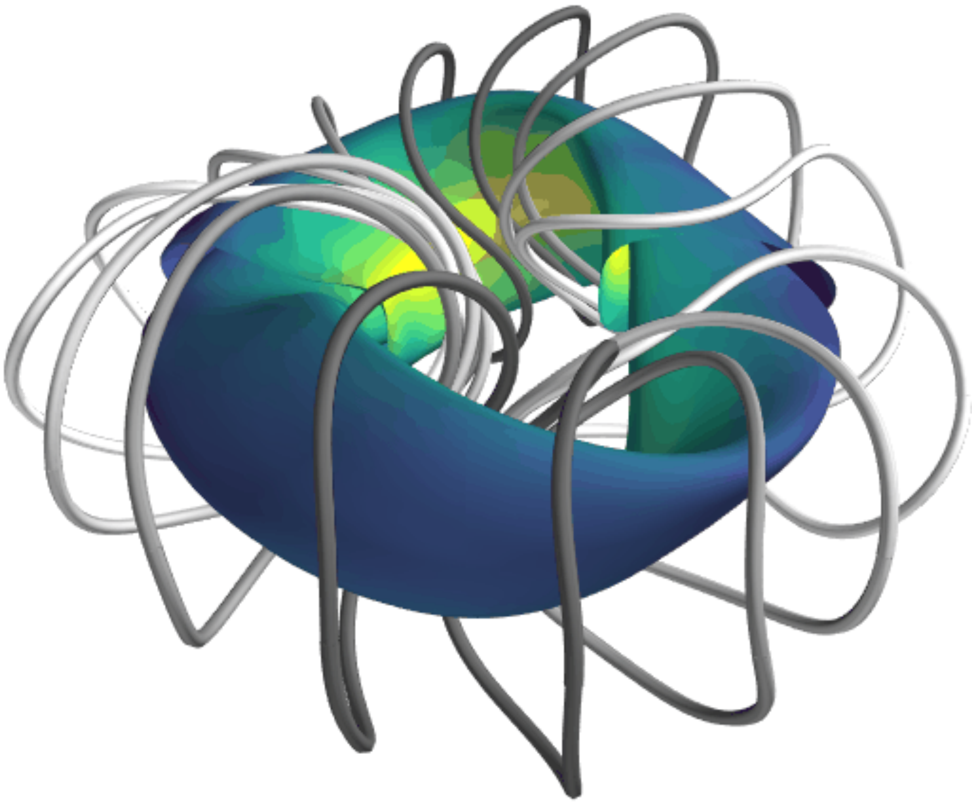
$$X = \{\text{Coil shapes, magnetic axis shape}\}$$

$$f = \oint_{\text{axis}} d\ell \left| \mathbf{B}_{\substack{\text{Biot-}\\\text{Savart}}} - \mathbf{B}_{\substack{\text{Near-axis}\\\text{quasisymmetry}}} \right|^2 + \oint_{\text{axis}} d\ell \left| \nabla\mathbf{B}_{\substack{\text{Biot-}\\\text{Savart}}} - \nabla\mathbf{B}_{\substack{\text{Near-axis}\\\text{quasisymmetry}}} \right|^2$$

$$+ \left(\frac{\iota_{\text{axis}} - \iota_{\text{axis}}^{\text{target}}}{\iota_{\text{axis}}^{\text{target}}} \right)^2 + \left(\frac{L_{\text{coils}} - L_{\text{coils}}^{\text{target}}}{L_{\text{coils}}^{\text{target}}} \right)^2 + \left(\begin{matrix}\text{other}\\ \text{terms}\end{matrix} \right)$$

Analytic $\partial f / \partial X$ is available!

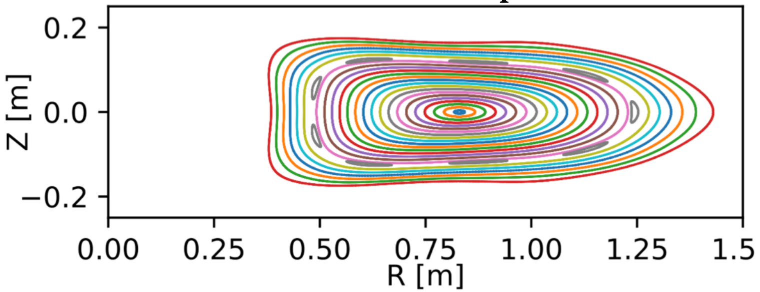
Combined coil + quasisymmetry optimization using analytic derivatives successfully achieves flux surfaces & QA



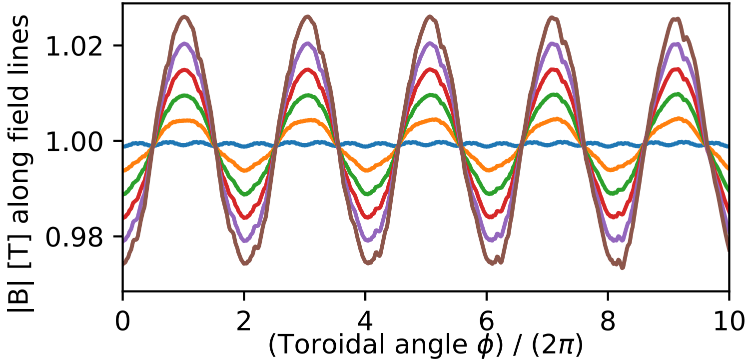
< 2 minutes on a laptop

$0.4 < \iota < 0.5$,

Decent surfaces out to aspect ratio 3.2

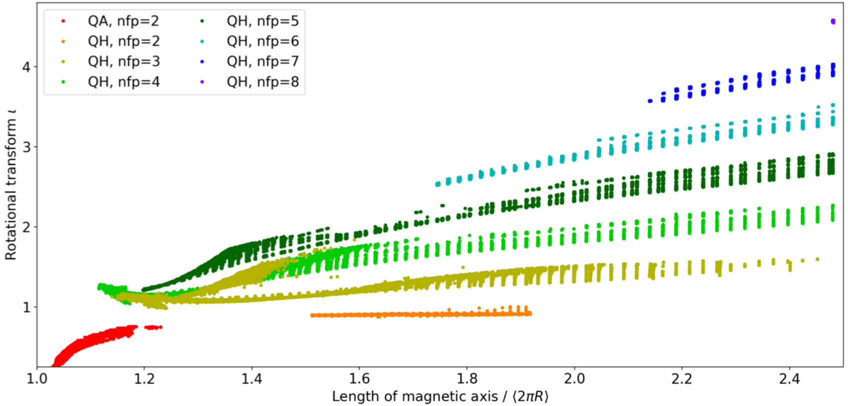


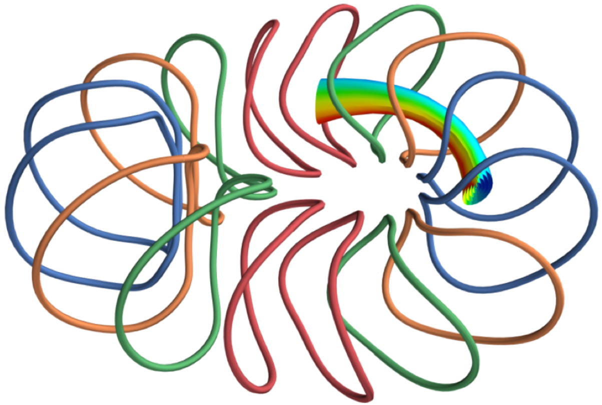
|B| is quasisymmetric



Ongoing & future work using this expansion

- Better understand the landscape of $O(r^2)$ quasisymmetric configurations.
- Evaluate more aspects of MHD & gyrokinetic stability.
- Higher order: compute magnetic shear & symmetry-breaking B_3.
- Construction to give quasisymmetry at an off-axis surface.
- Include off-axis quasisymmetry in the 1-stage coil optimization.





Closing thoughts

- The high-aspect-ratio expansion enables multiple new stellarator design paradigms:
  - Directly construct plasma geometries with good confinement.
  - Brute force parameter scans, enabled by orders-of-magnitude speed-up.
  - Derivative-based 1-stage optimization of coils for quasisymmetry.

- There is hope of definitively identifying all regions of parameter space with practical quasisymmetric fields.

- We may still discover qualitatively new magnetic confinement configurations.