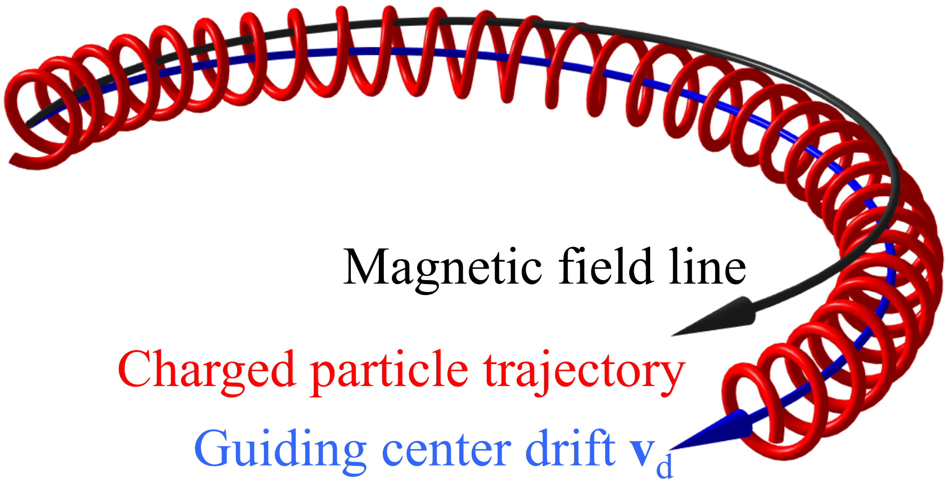
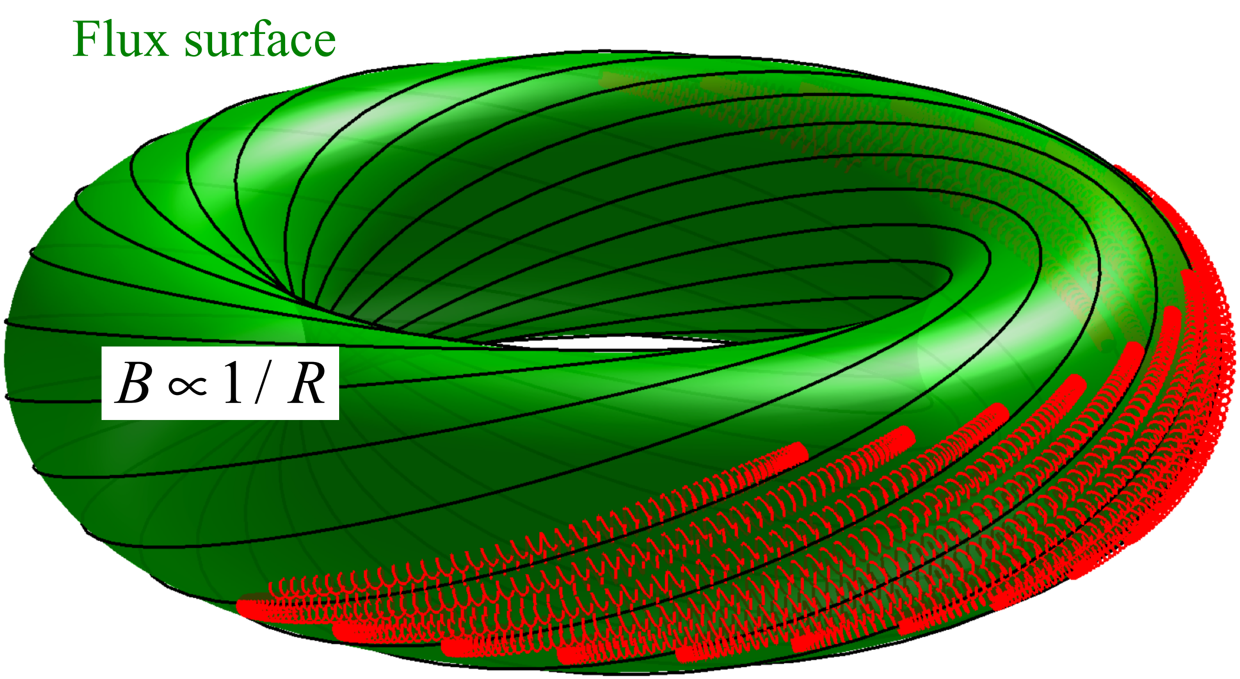
Omnigenity: A generalization of quasisymmetry

Perhaps quasisymmetry is unnecessary?

- H Mynick, T Chu, & A Boozer, *Physical Review Letters* **48**, 322 (1982).
- J Cary & S Shasharina, *Physics of Plasmas* **4**, 3323 (1997).
- J Cary & S Shasharina, *Physical Review Letters* **78**, 674 (1997).
- P Helander & J Nührenberg, *Plasma Physics and Controlled Fusion* 51, 055004 (2009).
- M Landreman & P J Catto, *Physics of Plasmas* **19**, 056103 (2012).

Review of particle trajectories in magnetic fields

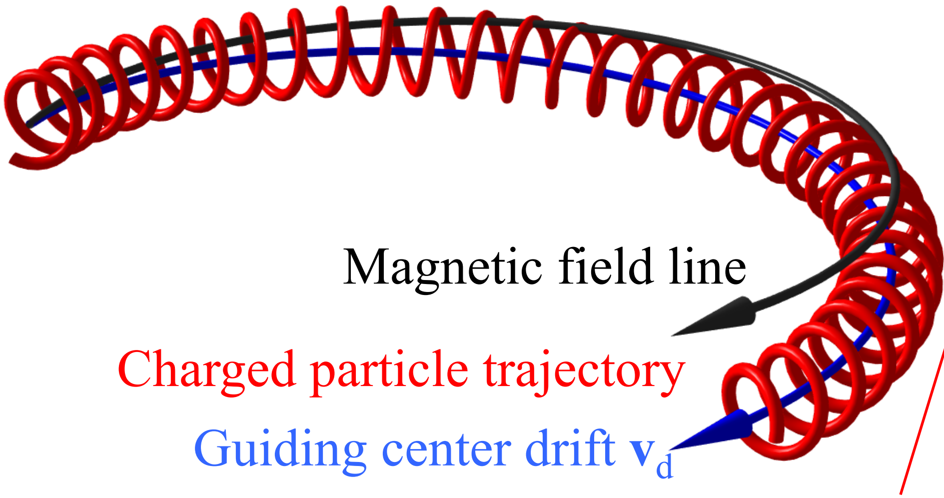




Particles with small $|v_{||}|/v$ are "trapped" in regions $B < B_{max}$.

Velocity along **B** slows, then reverses at "bounce points."

Omnigenity is a condition of good confinement: the bounce-averaged radial drift vanishes for all velocities.



Definition of omnigenity:

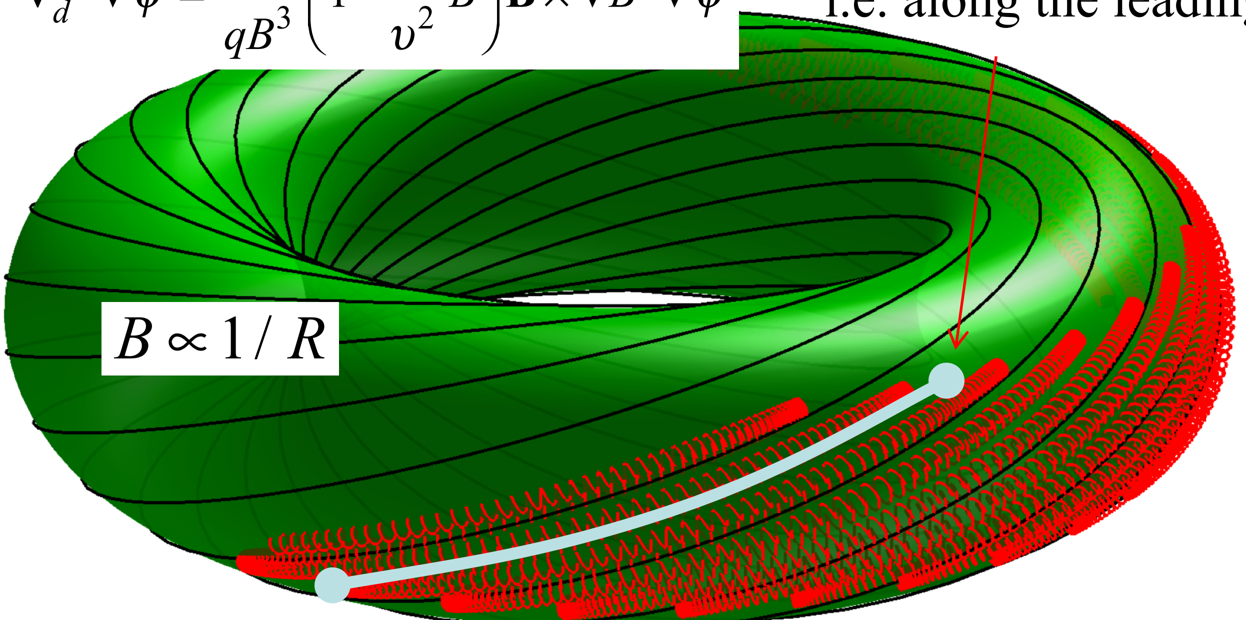
$$\oint \left(\mathbf{v}_d \cdot \nabla \psi \right) dt = 0$$

$\forall$ magnetic moments & energies.

ψ = Flux surface label

Integral along a field line between bounce points, i.e. along the leading-order motion, neglecting $\mathbf{v}_d$.

$$\mathbf{v}_d \cdot \nabla \psi = \frac{m\upsilon^2}{qB^3}\left(1 - \frac{\mu}{\upsilon^2} B \right) \mathbf{B} \times \nabla B \cdot \nabla \psi$$



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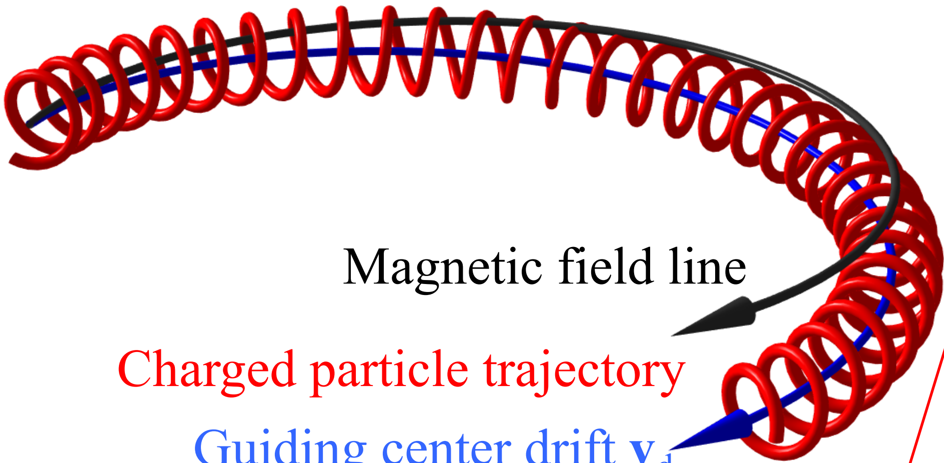

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Equivalent condition: For each magnetic moment, energy, and flux surface, the quantity J is uniform on the flux surface:

$$J = \oint v_{||} \, d\ell, \quad \ell = \text{distance along field line}$$

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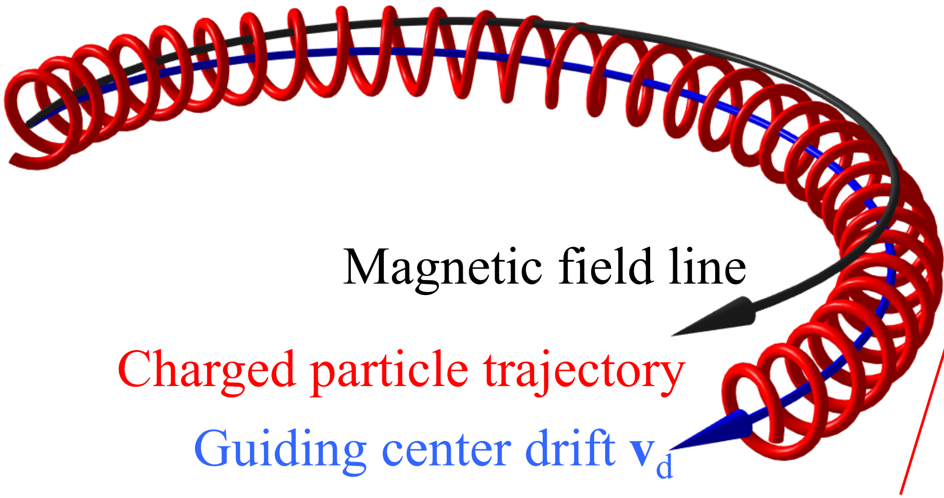

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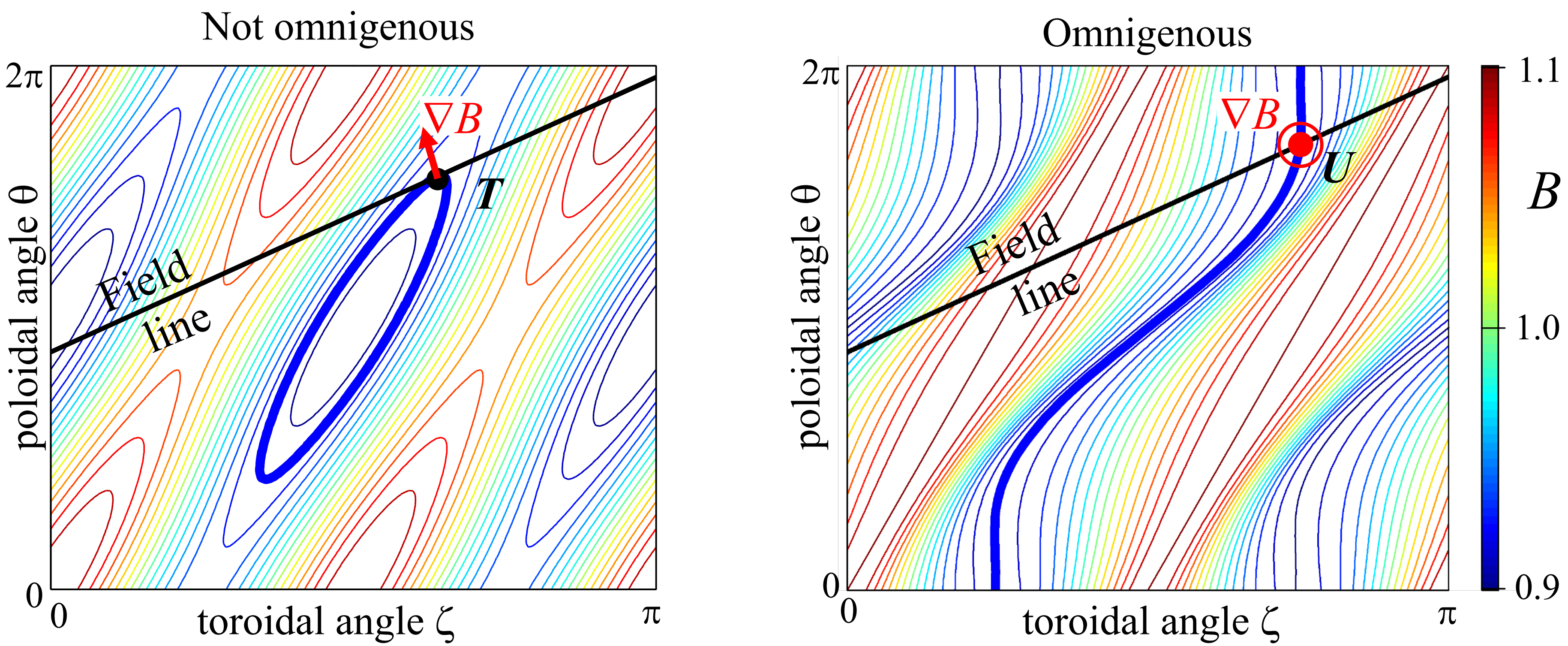
Quasisymmetry is sufficient but not necessary!

Omnigenity places strong constraints on *B*.

If $\oint \left(\mathbf{v}_d \cdot \nabla \psi \right) dt = 0$, then B contours can never be $\parallel$ to $\mathbf{B}$

Contours of *B* on a flux surface:



Deeply trapped particles at T would see a nonzero $\mathbf{v}_d \cdot \nabla \psi \propto \mathbf{B} \times \nabla B \cdot \nabla \psi$

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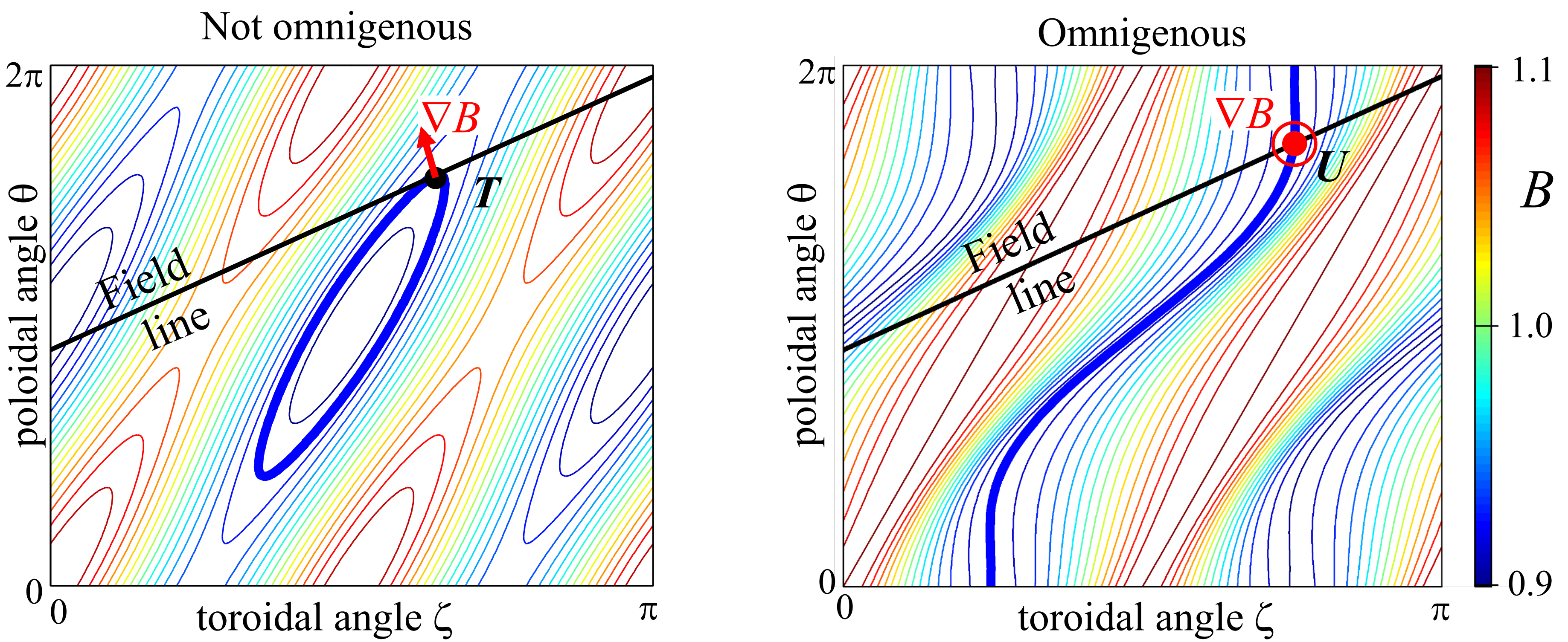


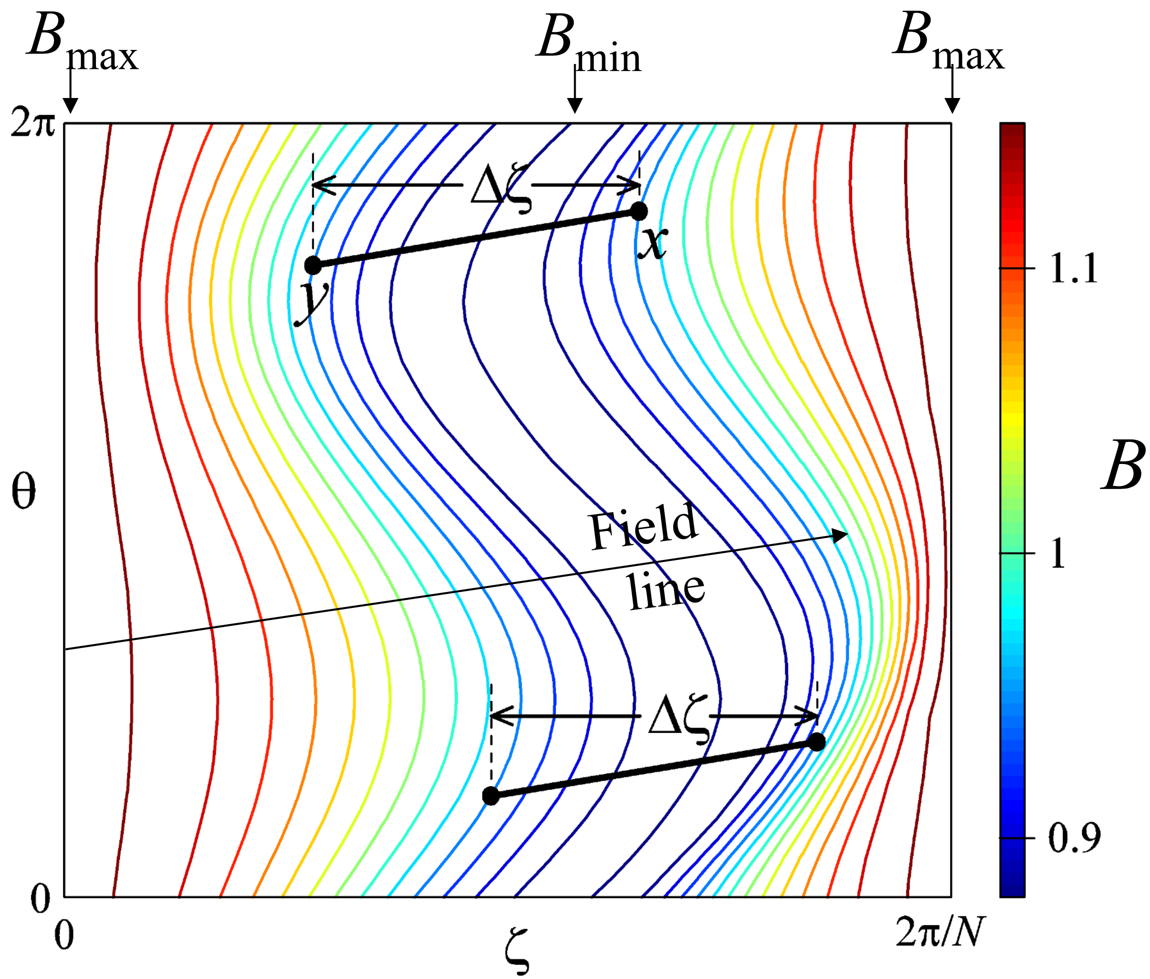
Deeply trapped particles at T would see a nonzero $\mathbf{v}_d \cdot \nabla \psi \;\propto\; \mathbf{B} \times \nabla B \cdot \nabla \psi$

Similar argument for maxima of B.

$\Rightarrow$ All B contours must link the torus toroidally, poloidally, or both.

Omnigenity places strong constraints on B.

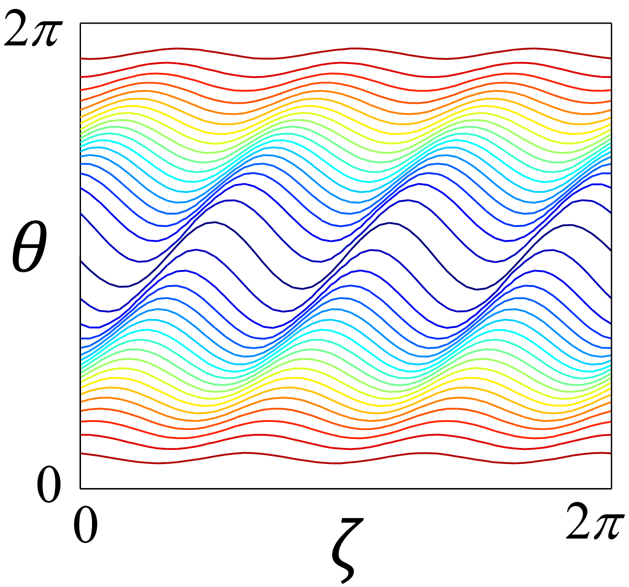
The width of a "well" in B must be the same everywhere on a flux surface. (Both in real distance and in Boozer coordinates.)



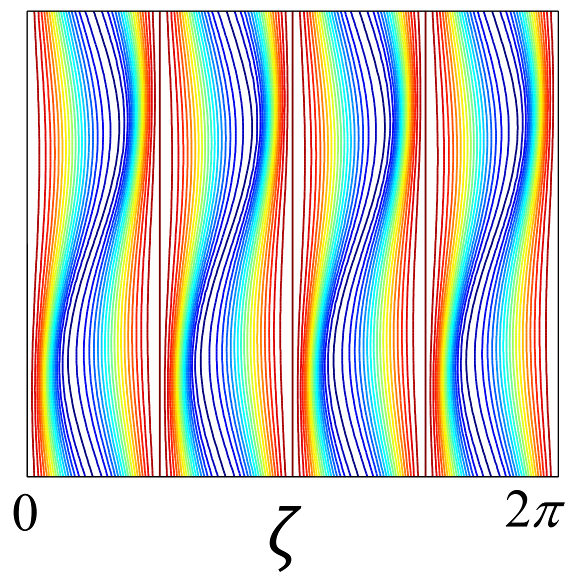
$\Rightarrow$ The B_{max} contour must be straight in Boozer coordinates.

Omnigenous fields come in toroidal/poloidal/helical variants, just like quasisymmetric fields.

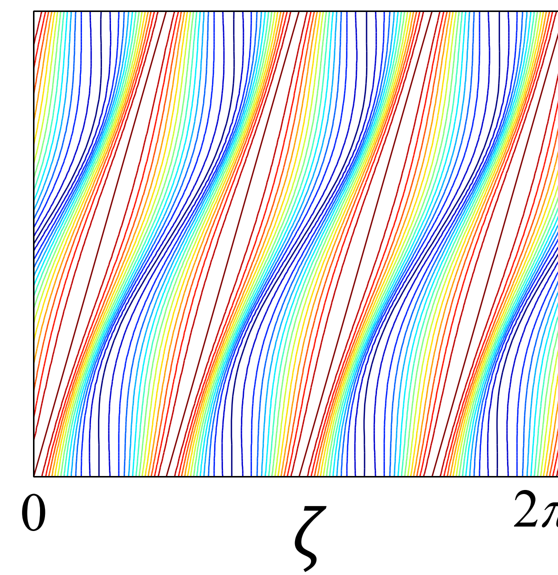
Generalized quasi-axisymmetry



Generalized quasi-poloidal symmetry



Generalized quasi-helical symmetry



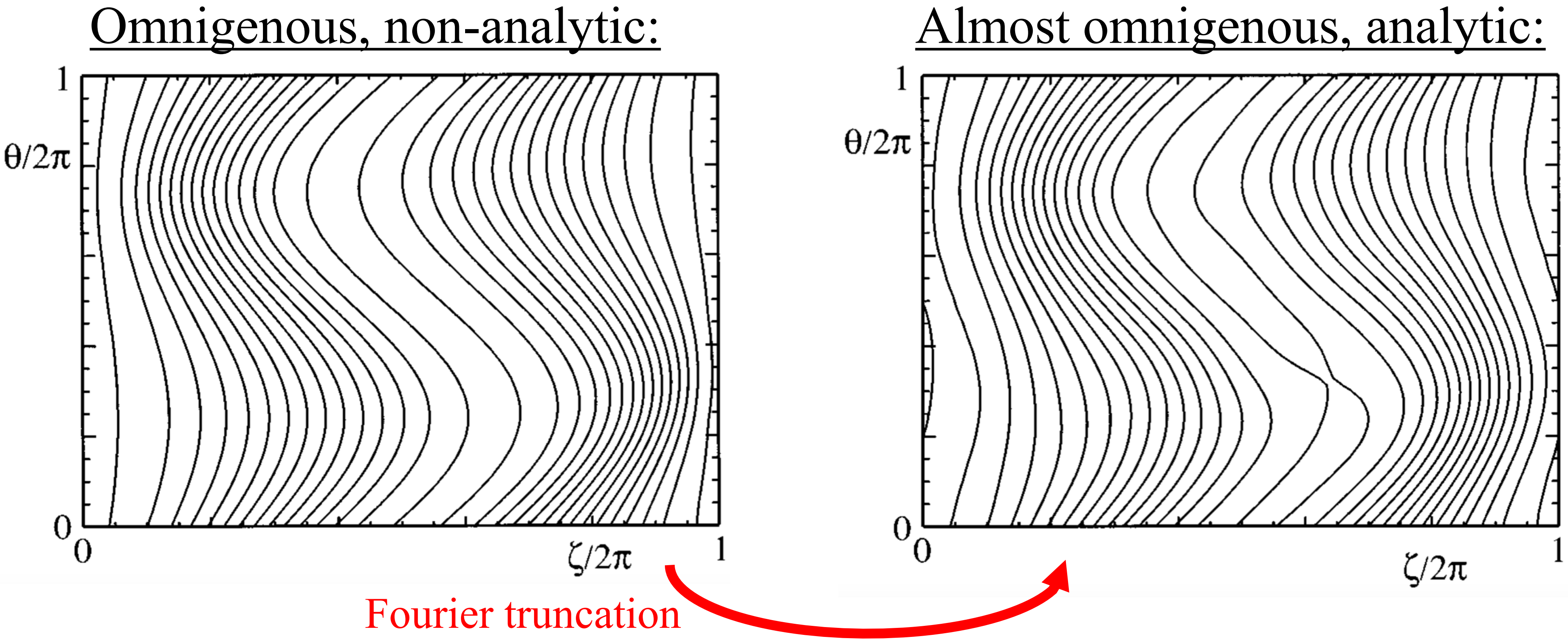
Omnigenity does not imply quasisymmetry in practice.

Cary & Shasharina (1997):

•If $B(\theta,\zeta)$ is omnigenous but not quasisymmetric, B will be non-analytic at the "seam" along the B_{max} contour.

•Thus, omnigenity + analyticity seems to imply quasisymmetry.

•But, there are analytic $B(\theta,\zeta)$ patterns that are nearly omnigenous but not close to quasisymmetry.



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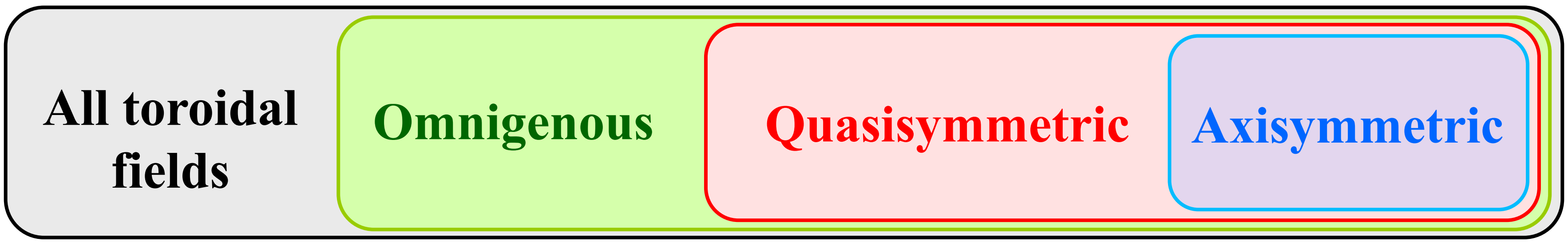


- There are no experiments far from quasisymmetry that approach these omnigenity conditions (especially for B_{max})
- Quasisymmetry is more restrictive than omnigenity, and is not obviously better for confinement.

Omnigenity is easier, so why might we want the more restrictive condition of quasisymmetry?

- Larger flows are permitted in quasisymmetry. These flows may improve stability and/or reduce turbulent transport?
- In quasisymmetric plasmas, the radial electric field and plasma flow are determined by turbulence, whereas they are determined by collisional processes in an omnigenous stellarator. Perhaps the former might turn out to be preferable?
- An outward collisional flux of impurities ("temperature screening") is predicted for quasisymmetry.

Extra slides

Omnigenity can also be understood in terms of *B* variation along field lines.

Tokamak or quasisymmetric stellarator:

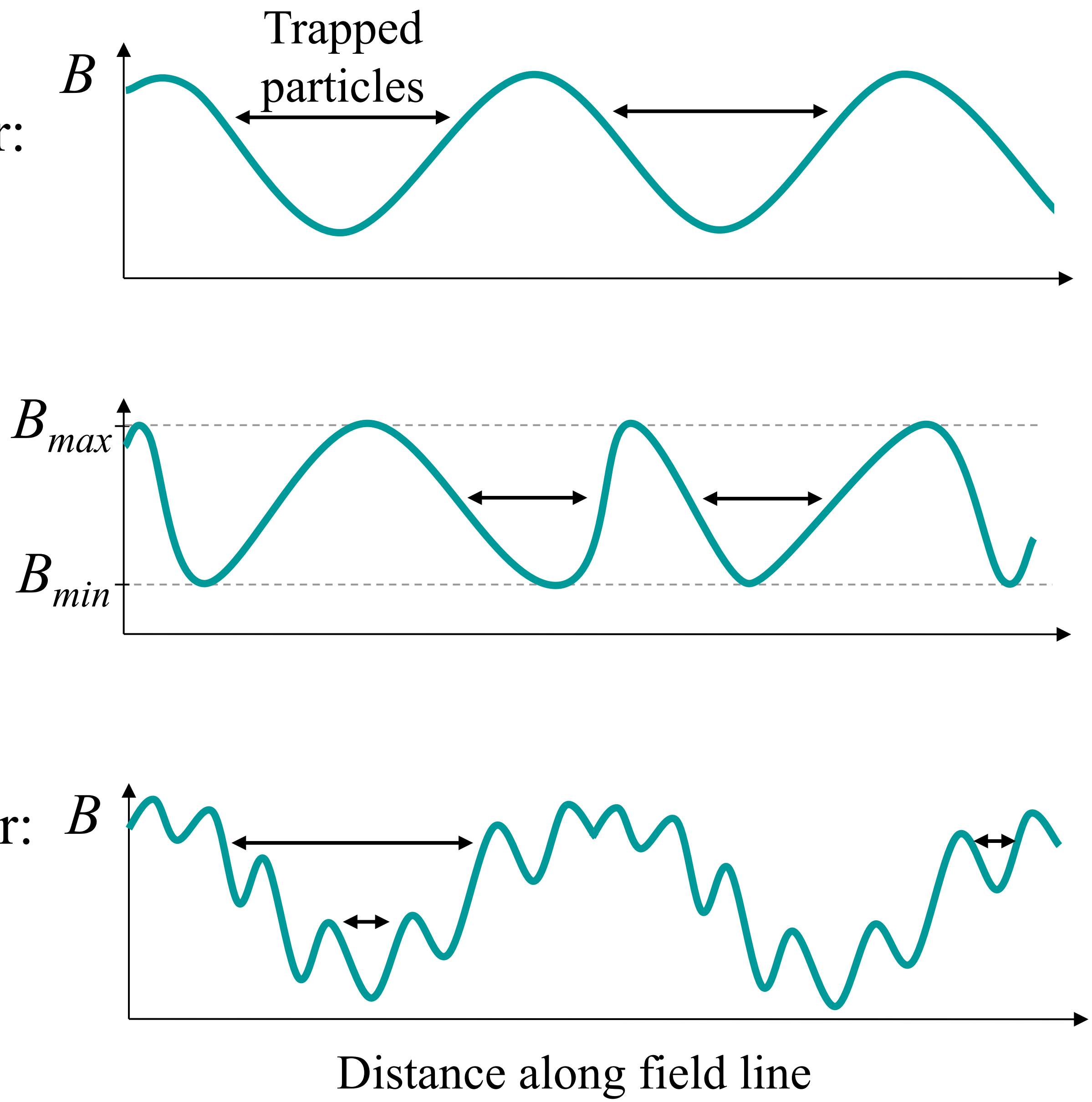
$\Delta\psi = 0$

Omnigenous stellarator:

$\Delta\psi = 0$

Non-optimized stellarator:

$\Delta\psi \neq 0$



The quasisymmetry helicity (M, N) can be generalized to omnigenity.

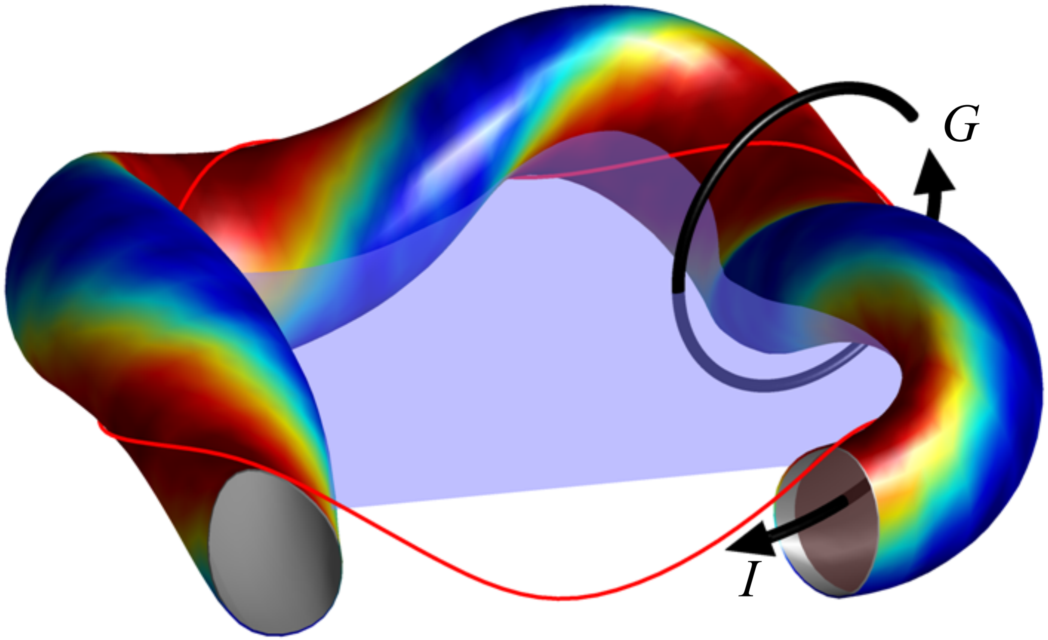
- Recall: all B contours encircle the torus poloidally, toroidally, or both.

- Define M and N: contours of B close after linking the torus M times toroidally and N times poloidally.

New geometric consequence of omnigenity:

Apply Ampère's Law to a B contour on a flux surface:

$$\int \mathbf{B}\cdot d\mathbf{r} = \frac{4\pi}{c}\times \underbrace{\left(\text{enclosed current}\right)}_{MG+NI}$$



$$\Rightarrow \quad \frac{\mathbf{B}\times\nabla\psi\cdot\nabla B}{\mathbf{B}\cdot\nabla B} = \frac{2q}{c}\left(\frac{MG+NI+H}{M-qN}\right) \quad \text{where } \langle H\rangle = 0.$$

E_r is determined by ambipolarity.

Non-quasisymmetric stellarators:

- Neoclassical radial current depends on E_r.
- $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla \psi \rangle \gg \langle \mathbf{j}_{\text{turbulence}} \cdot \nabla \psi \rangle$.

(*Helander & Simakov, Contrib. Plasma Phys. 2010*)

$\Rightarrow$ You can solve for E_r using $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla \psi \rangle = 0$.

Tokamaks & quasisymmetric stellarators:

- Neoclassical radial fluxes of ions and electrons are always equal, regardless of E_r ("intrinsic ambipolarity")

(*Helander & Simakov, PRL 2008*)

$\Rightarrow$ You cannot solve for E_r.

E_r is determined by ambipolarity.

Non-quasisymmetric stellarators:

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$\Rightarrow$ You can solve for E_r using $\langle \mathbf{j}_{\text{neoclassical}} \cdot \nabla \psi \rangle = 0$.

Omnigenous stellarators:

$$\langle \mathbf{j} \cdot \nabla \psi \rangle = \left(Zen_i \frac{d\Phi}{d\psi} + T_i \frac{dn_i}{d\psi} - 0.17 n_i \frac{dT_i}{d\psi} \right) \left\langle \left(\text{departure from quasisymmetry}\right)^2 \right\rangle$$

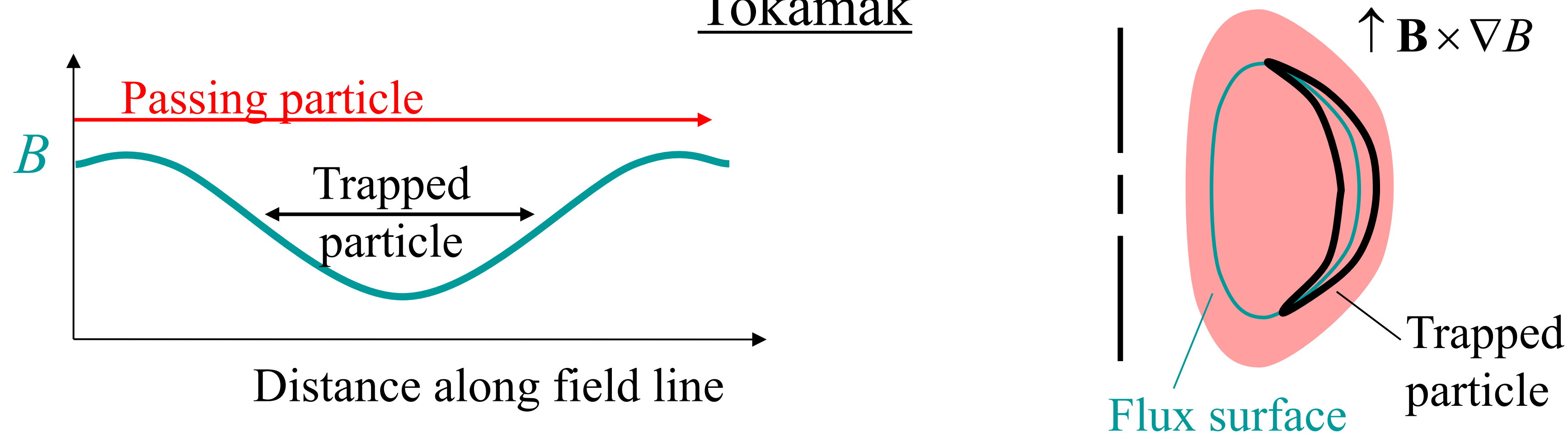
Universal result:

$$\boxed{\frac{d\Phi}{d\psi} = \frac{1}{Ze} \left(-\frac{T_i}{n_i} \frac{dn_i}{d\psi} + 0.17 \frac{dT_i}{d\psi} \right)}$$

Totally independent of the details of **B**.

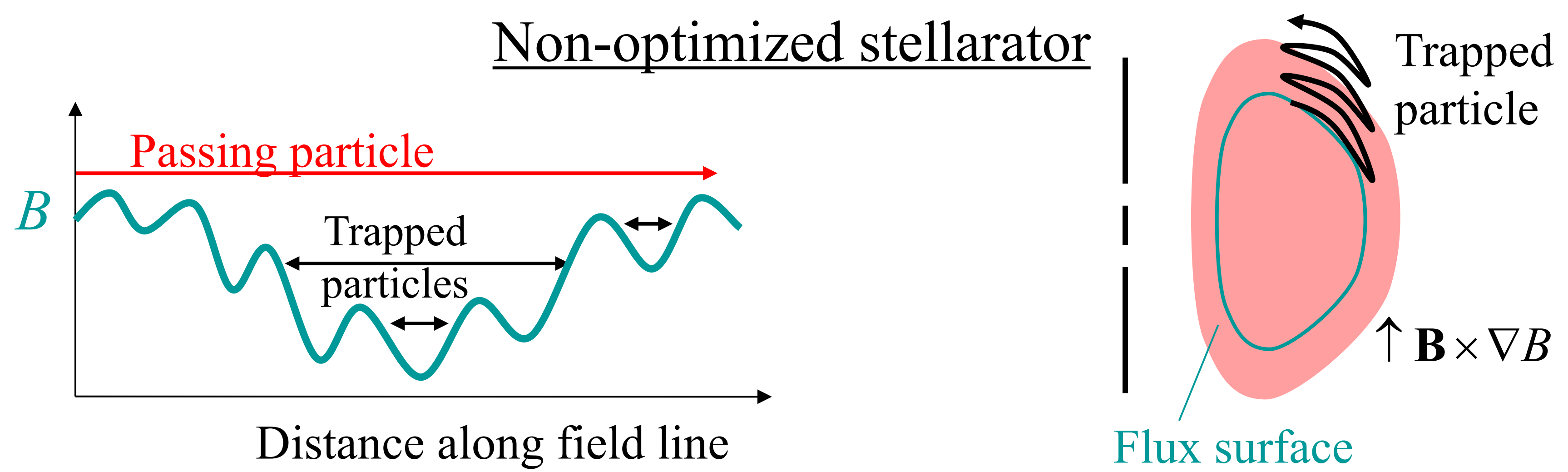
Single-particle confinement

Tokamak



Tamm's theorem: "In the absence of turbulence and collisions, all particles in a tokamak are confined."

Non-optimized stellarator



The ideal stellarator: omnigenity

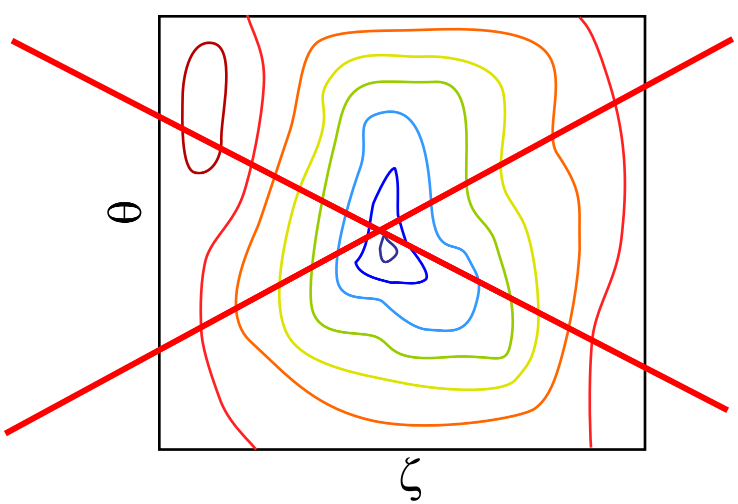
Definition: A field is "omnigenous" if the radial guiding-center drift averages over a bounce to zero for all trapped particles:

$$\Delta\psi \text{ per bounce} = \int_{\text{bounce}} \left(\mathbf{v}_d \cdot \nabla\psi\right) dt = 0.$$

Many properties can be proven about omnigenous fields.

E.g., all B contours link the torus toroidally, poloidally, or helically.

Contours of B



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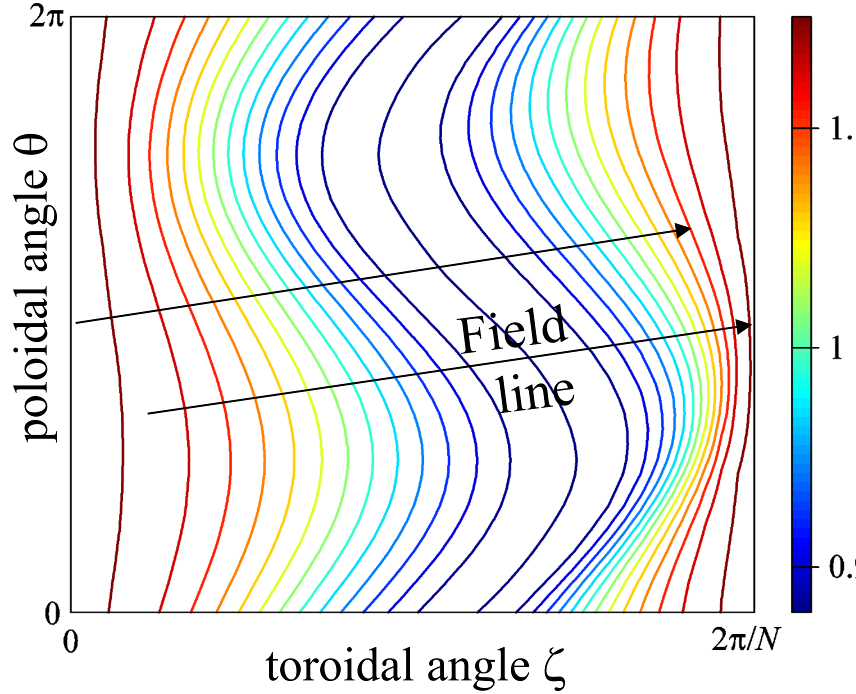
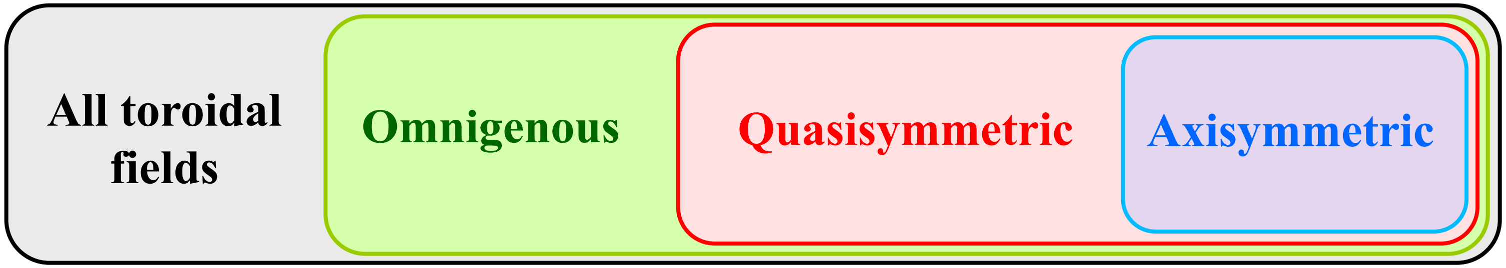
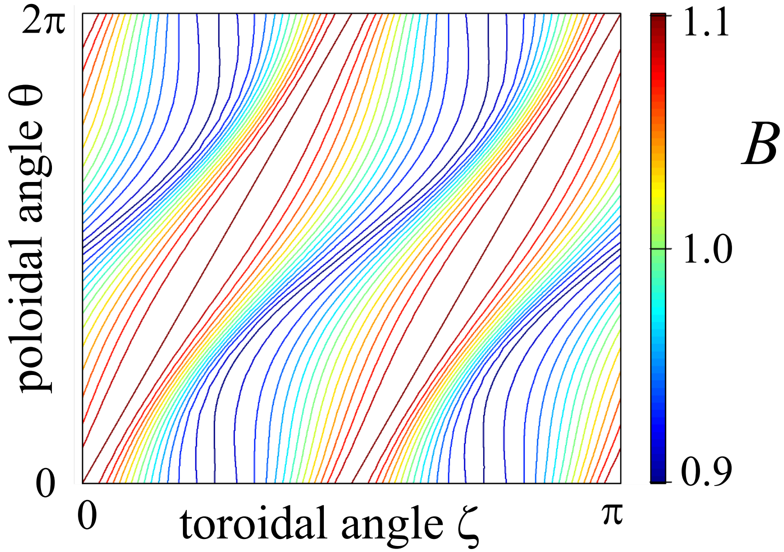


Omnigenity is more general than quasisymmetry.

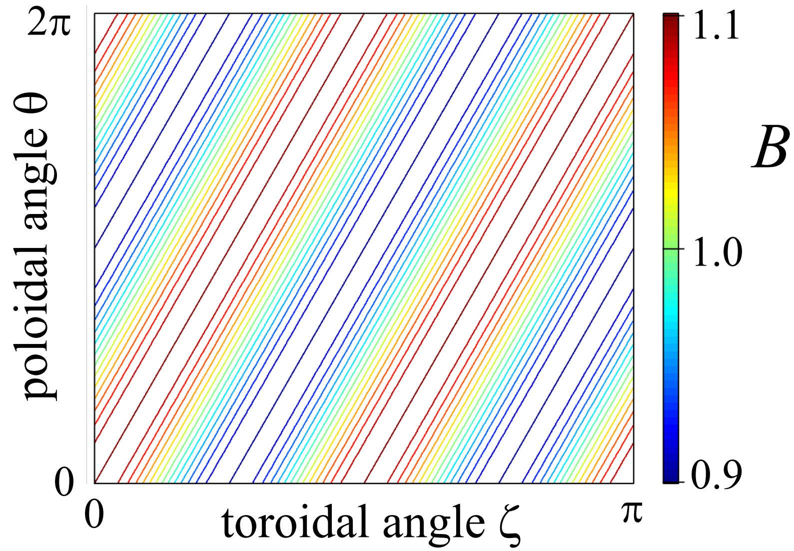
Cary & Shasharina, *PoP* (1997), *PRL* (1997)



Omnigenity: B contours may be curved



Quasisymmetry: $B = B(M\theta - N\zeta)$

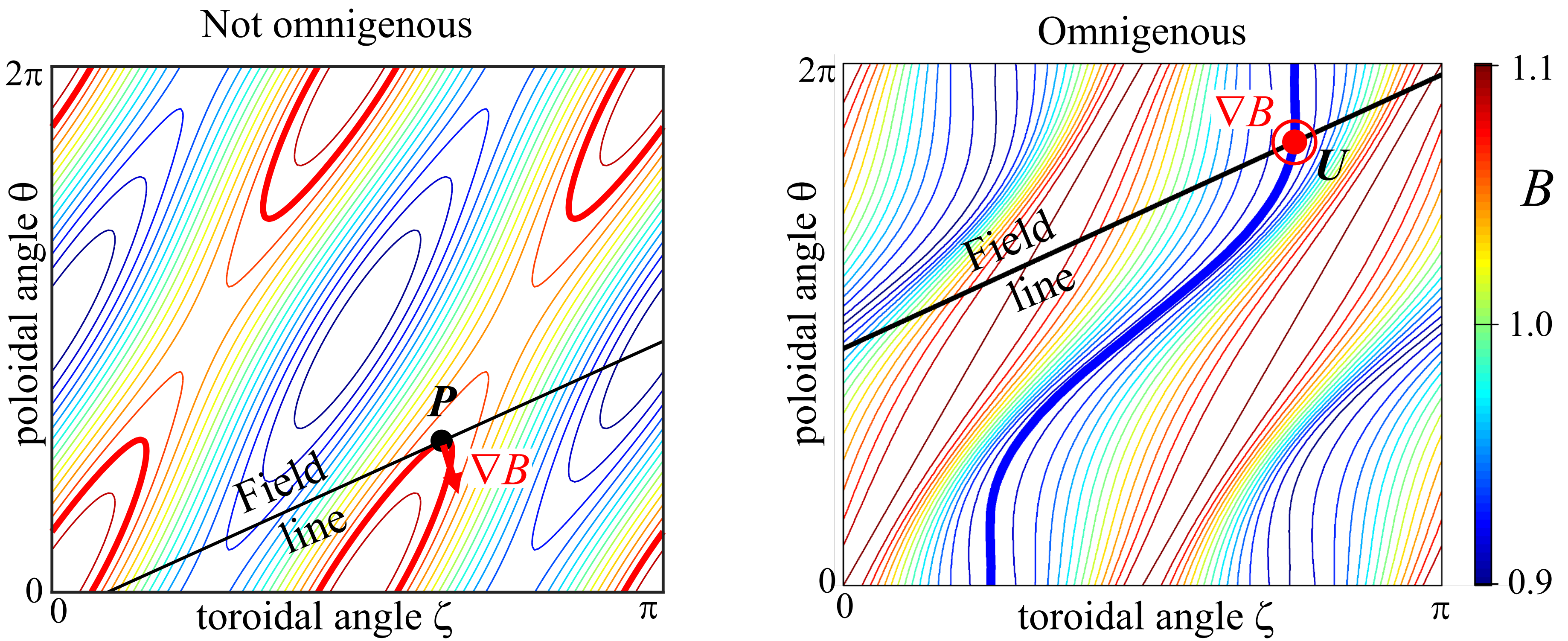


Quasisymmetry is more restrictive than omnigenity and does no more to reduce α-particle loss or neoclassical transport.

Omnigenity places strong constraints on *B*.

If $\oint \left(\mathbf{v}_d \cdot \nabla \psi \right) dt = 0$, then B contours can never be $\parallel$ to $\mathbf{B}$

Contours of *B* on a flux surface:



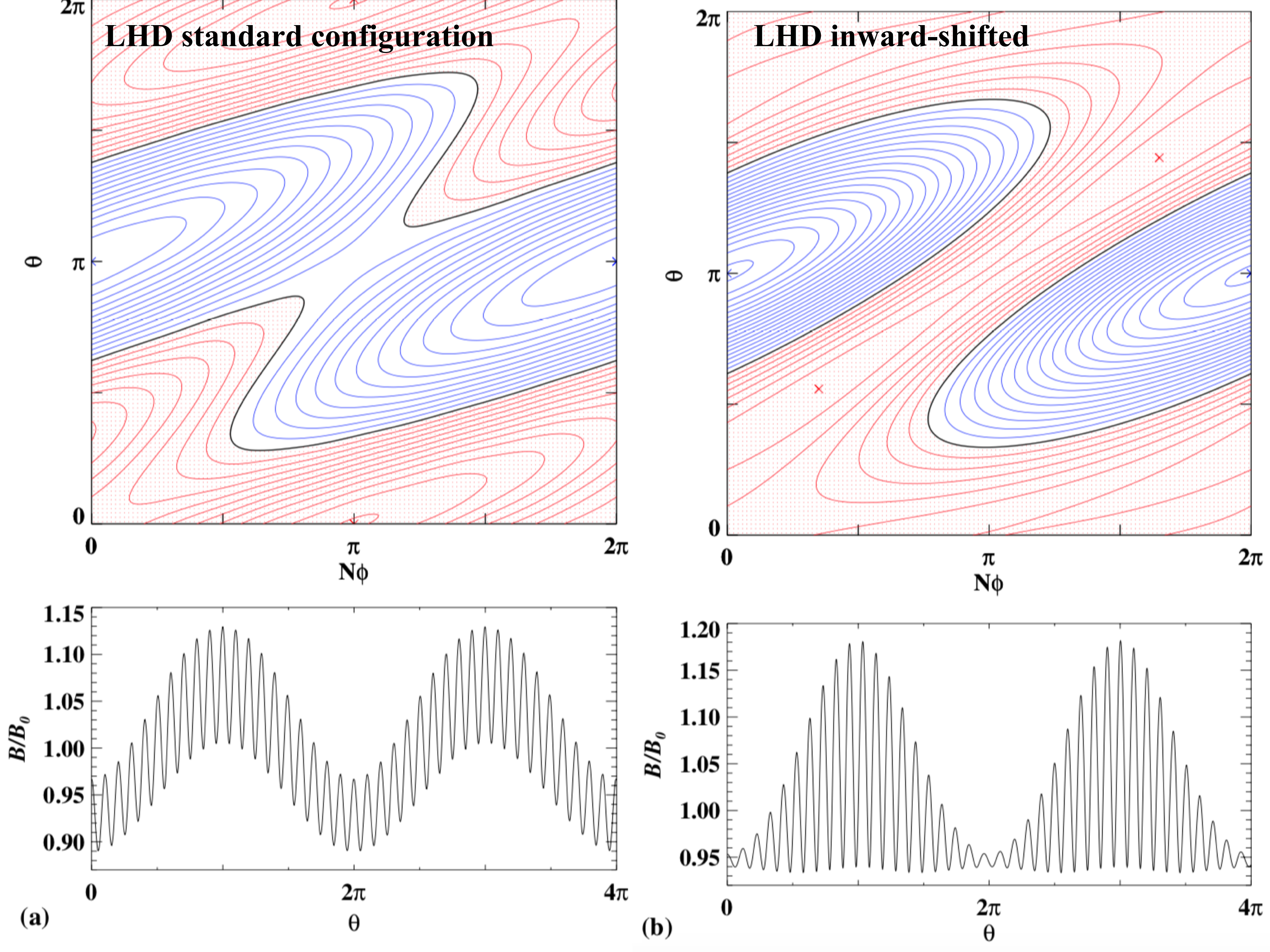
$J = \oint v_{\parallel} \, d\ell$ has different values above vs below P.

Also, for particles that bounce at ***P***, there is a nonzero radial $\mathbf{v}_\mathrm{d}$ there, and bounce time diverges, so the radial excursion is very large.

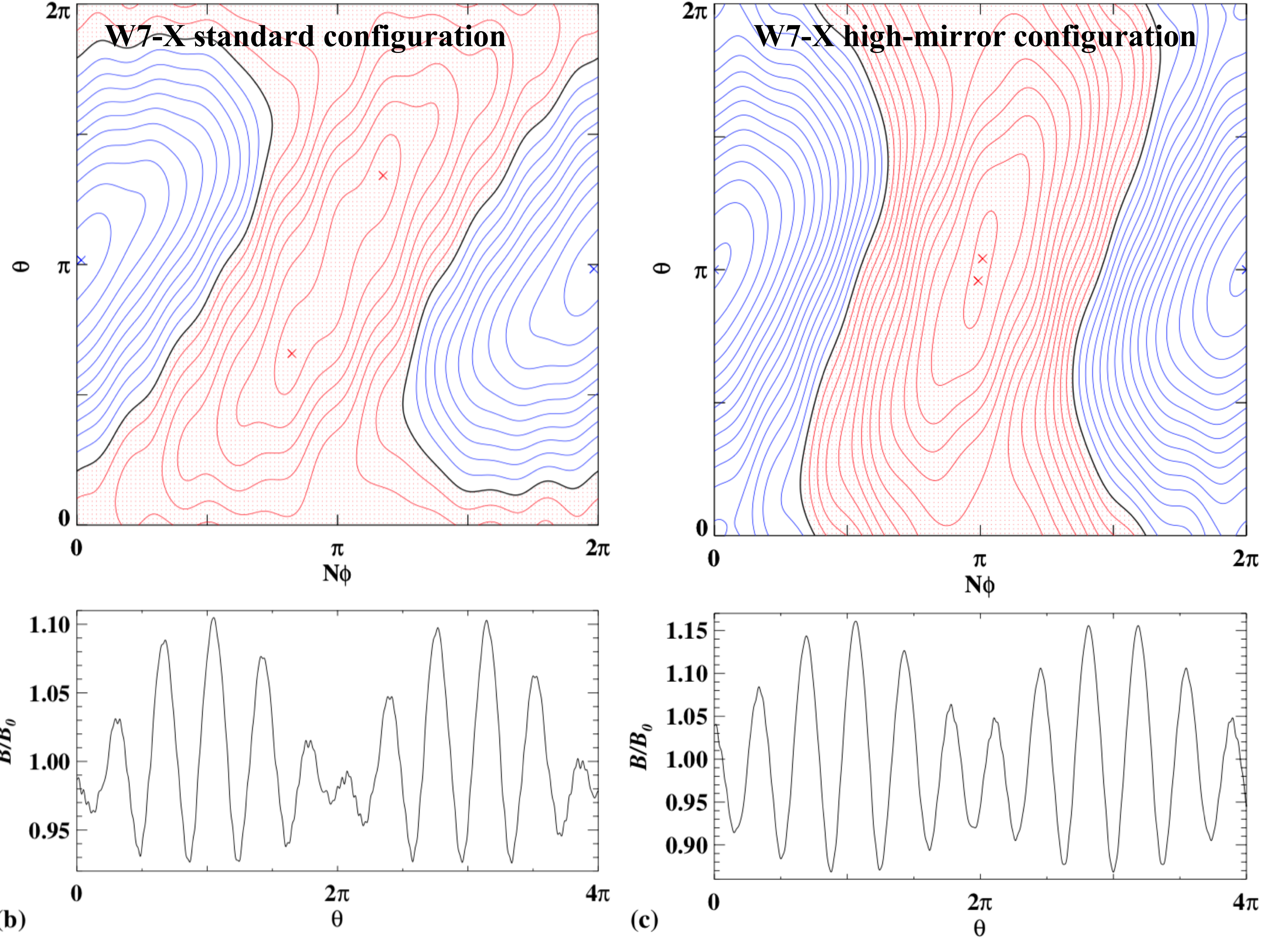
Remaining slides:

Patterns of field strength, from

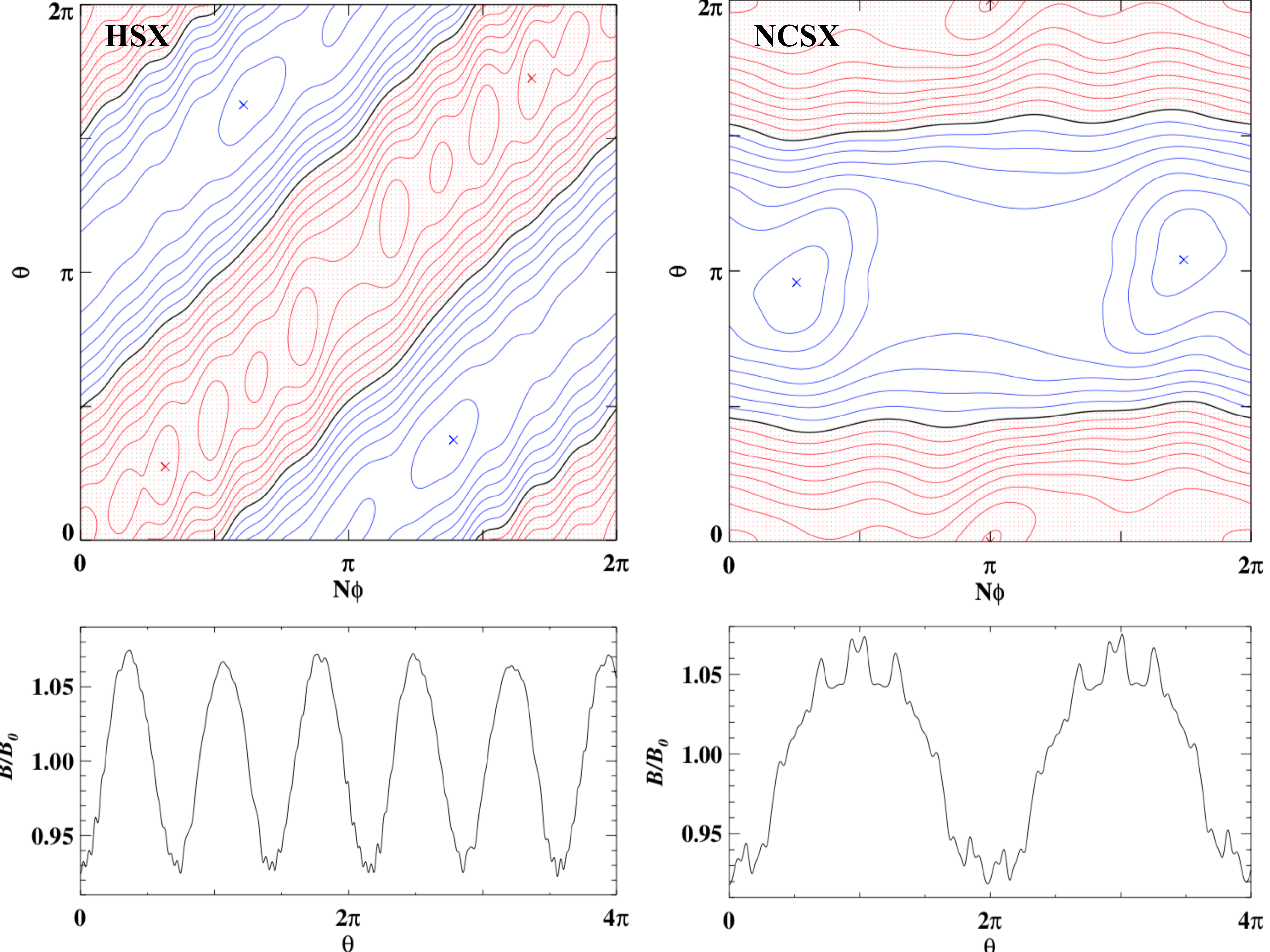
Beidler et al, Nuclear Fusion **51** (2011) 076001



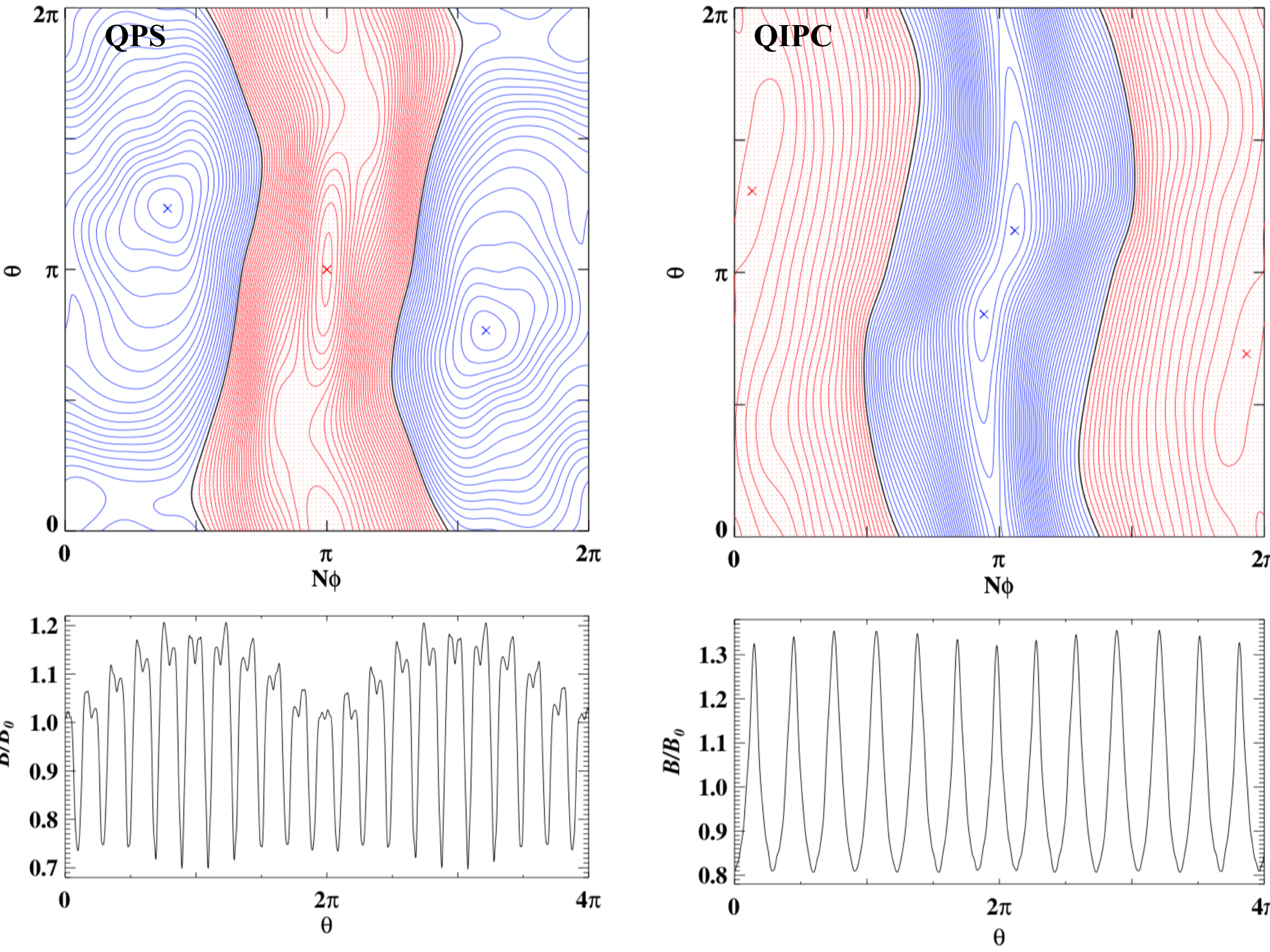
LHD standard configuration
LHD inward-shifted
θ
Nϕ
0
π
2π
B/B0
1.15
1.10
1.05
1.00
0.95
0.90
1.20
4π
(a)
(b)



W7-X standard configuration
W7-X high-mirror configuration
2π
π
0
θ
Nϕ
B/B0
1.10
1.05
1.00
0.95
1.15
0.90
4π
(b)
(c)



HSX
NCSX
2π
π
0
θ
Nφ
B/B0
1.05
1.00
0.95
4π


QPS
2π
π
0
θ
Nφ
QIPC
B/B0
1.2
1.1
1.0
0.9
0.8
0.7
1.3
2π
4π